A Novel Stabilizing Control for Neural Nonlinear Systems with Time Delays by State and Dynamic Output Feedback

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Abstract: A novel neural network model, termed the standard neural network model (SNNM), similar to the nominal model in linear robust control theory, is suggested to facilitate the synthesis of controllers for delayed (or non-delayed) nonlinear systems composed of neural networks. The model is composed of a linear dynamic system and a bounded static delayed (or non-delayed) nonlinear operator. Based on the global asymptotic stability analysis of SNNMs, Static state-feedback controller and dynamic output feedback controller are designed for the SNNMs to stabilize the closed-loop systems, respectively. The control design equations are shown to be a set of linear matrix inequalities (LMIs) which can be easily solved by various convex optimization algorithms to determine the control signals. Most neural-network-based nonlinear systems with time delays or without time delays can be transformed into the SNNMs for controller synthesis in a unified way. Two application examples are given where the SNNMs are employed to synthesize the feedback stabilizing controllers for an SISO nonlinear system modeled by the neural network, and for a chaotic neural network, respectively. Through these examples, it is demonstrated that the SNNM not only makes controller synthesis of neural-network-based systems much easier, but also provides a new approach to the synthesis of the controllers for the other type of nonlinear systems.

Keywords: Asymptotic stability, chaotic neural network, feedback control, linear matrix inequality (LMI), nonlinear system, standard neural network model (SNNM), time delay.

1. INTRODUCTION

Applications of neural networks (NNs) in system identification and control have been extensively studied in the past ten years. It has been shown that successful identification and control may be possible using NNs for complex nonlinear dynamic systems whose mathematical models are not available from first principles, see for instance [1-3] for an overview. In practical applications, it is desirable to have a systematic method of ensuring stability, robustness, and performance properties of the overall system. Recently, several good NN control approaches have been proposed based on Lyapunov's stability theory [3-6]. Suykens *et al.* introduce NLq-theory to design neural controller for nonlinear systems modeled by neural state space model via the linear matrix inequality (LMI) approach in [3]. The NLq theory is widely applied to the neuro-control field. Tanaka represented a sigmoid MLP as a linear difference inclusion (LDI) system in the context of stability analysis of neural networks in [4]. However, the representation was not rigorously discussed and was obtained only for a few sigmoid MLP examples. Limanond *et al.* [5] rigorously show that the sigmoid MLP does admit an LDI state-space representation. Then, based on the LDI representation, they designed a state-feedback controller for the nonlinear discrete-time system which is approximated by a sigmoid MLP, such that the overall closed-loop system is globally asymptotically stable. The resulting equations are in the form of a set of LMIs. Based on the LDI representation of MLP in [4] and [5], Lin *et al.* [6] design an H_∞ state-feedback controller for the continuous-time nonlinear system to eliminate the effect of reconstruction error and external disturbances via an LMI approach. However, the approaches in [3-6] can't be applied to the time-delayed systems.

Time-delay is frequently encountered in various dynamical systems. Such time-delayed systems

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generally arise as a result of delay in transmission of information between different parts of the system or the finite switching speed of electric component or mechanical device. It is well known that feedback control in the presence of time-delay leads to particular difficulties since a delay places a limit on the achievable control signal. Recently, stability analysis and application of recurrent neural networks with time delays is receiving much attention. Liao *et al*. have summarized the research results about stability analysis of various time-delayed recurrent neural networks in many published literature, and employed Lyapunov-Krasovskii stability theory for functional differential equations and the linear matrix inequality (LMI) approach to investigate the problems about asymptotical stability in [7] and exponential stability in [8] of neural networks with constant or time-varying delays. However, there does not seem to be much (if any) study on the stabilization for neural nonlinear systems with time delays via dynamic output feedback control.

At present, the controller synthesis approaches are also varied as the different neural-network-based nonlinear systems. There are no unified methods to deal with these problems. In this paper, similar to the nominal models in linear robust control theory, we advance a standard neural network model (SNNM) with time delays. By the Lyapunov-Krasovskii stability theory, we analysis the global asymptotic stability of delayed SNNM. Based on the analysis results, we design static state-feedback controllers and dynamic output feedback controllers for the delayed SNNMs with inputs and outputs such that the closed-loop systems are globally asymptotically stable, respectively. The resulting design equations are in the form of a set of LMIs which allow for the application of convex optimization algorithms to be possible. Here, it is shown that most delayed (or non-delayed) neural nonlinear systems can be transformed into SNNMs with inputs and outputs to be stabilization synthesized in a unified way.

In this paper, the following notations are used. $\Re^n$ denotes n dimensional Euclidean space, $\Re^{n\times m}$ is the set of all $n\times m$ real matrices, I denotes identity matrix of appropriate order, $\|x\|$ denotes the Euclid norm of the vector x, $*$ denotes the symmetric parts. The notation $X > Y$ and $X \geq Y$, respectively, where X and Y are matrices of same dimensions, means that the matrix $X-Y$ is positive definite and positive semi-definite, respectively. If $X\in\Re^p$ and $Y\in\Re^q$, $C(X;\ Y)$ denotes the space of all continuous functions mapping $\Re^p \to \Re^q$.

2. STANDARD NEURAL NETWORK MODEL

In robust control, systems with uncertainty can be transformed into a standard form known as linear fractional transformation (LFT) [9]. Similar to the LFT, and referring to [10] and [11], we can synthesize controllers for the delayed nonlinear systems composed of neural network by transforming them into SNNMs. The SNNM represents a neural network model as the interconnection of a linear dynamic system and static delayed (or non-delayed) nonlinear operators consisting of bounded activation functions. Here, we discuss only the continuous-time SNNM, since there are similar architecture and results for corresponding discrete-time model [12]. The continuous-time SNNM with inputs and outputs is shown in Fig. 1. The block Φ is a block diagonal operator composed of nonlinear activation functions $\phi_i(\xi_i(\cdot))$, which are typically continuous, differentiable, monotonically increasing, slope-restricted, and bounded. The matrix N represents a linear mapping between the inputs and outputs of the integrator $\int$ (or time delay $z^{-1}I$ in the discrete-time case) and the operator Φ. The vectors $\xi(\cdot)$ and $\phi(\xi(\cdot))$ are the inputs and outputs of the nonlinear operator Φ, respectively. The block D represents the delayed element. $\kappa(\cdot)$ is the time-varying delay satisfying $0<\kappa(\cdot)\leq h$, where h is the maximal delay. The vectors $u(\cdot)$ and $y(\cdot)$ are the inputs and outputs of the SNNM, respectively.

If N in Fig. 1 is partitioned as

$$N = \begin{bmatrix} A & B_p & B_{pd} & B_u \\ C_q & D_p & D_{pd} & D_{qu} \\ C_y & D_{yp} & D_{ypd} & D_u \end{bmatrix}, \tag{1}$$

then the input-output SNNM can be depicted as an linear difference inclusion (LDI):

$$\begin{cases} \dot{x}(t) = Ax(t) + B_p\phi(\xi(t)) \\ \qquad + B_{pd}\phi(\xi(t-\kappa(t))) + B_u u(t), \\ \xi(t) = C_q x(t) + D_p\phi(\xi(t)) \\ \qquad + D_{pd}\phi(\xi(t-\kappa(t))) + D_{qu}u(t), \\ y(t) = C_y x(t) + D_{yp}\phi(\xi(t)) \\ \qquad + D_{ypd}\phi(\xi(t-\kappa(t))) + D_u u(t). \end{cases} \tag{2}$$

with the initial condition function

$$\phi(\xi(t_0+\theta)) = \phi(\xi(t_0)),\ \forall\theta\in[-h,0], \tag{3}$$

where $x\in\Re^n$ is the state vector, $A\in\Re^{n\times n}$, $B_p\in\Re^{n\times L}$, $B_{pd}\in\Re^{n\times L}$, $B_u\in\Re^{n\times m}$, $C_q\in\Re^{L\times n}$, $C_y\in\Re^{l\times n}$, $D_p\in\Re^{L\times L}$, $D_{pd}\in\Re^{L\times L}$, $D_u\in\Re^{l\times m}$, $D_{qu}\in\Re^{L\times m}$, $D_{ypd}\in\Re^{l\times L}$ and $D_{yp}\in\Re^{l\times L}$ are the corresponding state-space matrices, $\xi\in\Re^L$ is the input vector of nonlinear operator Φ, $\phi\in C(\Re^L;\ \Re^L)$ is the output vector of nonlinear

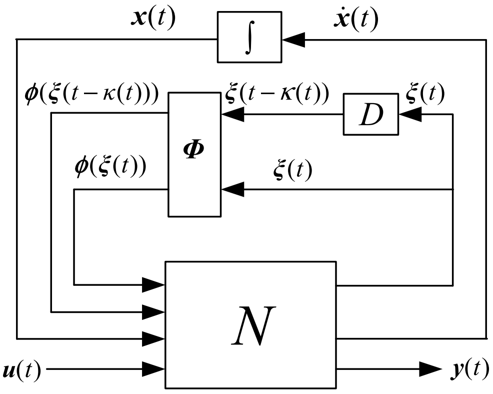

Fig. 1. Continuous-time standard neural network model (SNNM) with inputs and outputs.

operator $\boldsymbol{\Phi}$ satisfying $\phi(0)=0$, $u\in\Re^m$ is the input vector, $y\in\Re^l$ is the output vector, and $L\in\Re$ is the number of nonlinear activation functions (that is, the total number of neurons in the hidden layers and output layer of the neural network).

Remark 1: The novel delayed neural network model (2) unifies several well-known dynamic neural networks with or without delays such as Hopfield neural networks, CNNs, BAM networks, RMLP etc. On the other hand, the system (2) can also describe some neural-network control systems. Ref. [13-16] illustrate that these neural network models and neural-network control systems are special examples of (2).

Firstly, we will analyze the stability of the SNNM (2) at the equilibrium point, on which the inputs and outputs can be set to the zero vectors of appropriate dimensions. Autonomous SNNM can be described by

$$\begin{cases}\dot{x}(t)=Ax(t)+B_p\phi(\xi(t))+B_{pd}\phi(\xi(t-\kappa(t))),\\ \xi(t)=C_qx(t)+D_p\phi(\xi(t))+D_{pd}\phi(\xi(t-\kappa(t))).\end{cases}\tag{4}$$

Since $x=0$, $\xi=0$ is a solution of (4), there exist at least one equilibrium point located at the origin i.e., $x_{eq}=0$, $\xi_{eq}=0$.

In this paper, we assume that the activation functions in the SNNM satisfy the sector conditions $\phi_i(\xi_i(t))/\xi_i(t)\in[q_i,\ u_i]$, i.e., $[\phi_i(\xi_i(t))-q_i\xi_i(t)]\cdot[\phi_i(\xi_i(t))-u_i\xi_i(t)]\le 0$, $u_i>q_i\ge 0$, and the delays in SNNM are constant, i.e., $\kappa(\cdot)=h$. We will discuss the global asymptotic stability for the SNNM in the following text.

Theorem 1: If there exist symmetric positive definite matrices P and Γ, and diagonal semi-positive definite matrix T, such that the following LMI holds:

$$G=\begin{bmatrix}G_{11} & G_{12} & G_{13}\\ * & G_{22} & G_{23}\\ * & * & G_{33}\end{bmatrix}<0,\tag{5}$$

then the origin of SNNM (4) is globally asymptotically stable. The submatrices of G are

$$G_{11}=A^TP+PA-2C_q^TTQUC_q,$$
$$G_{12}=PB_p-2C_q^TTQUD_p+C_q^T(Q+U)T,$$
$$G_{13}=PB_{pd}-2C_q^TTQUD_{pd},$$
$$G_{22}=\Gamma-2D_q^TTQUD_p-2T+D_q^T(Q+U)T+T(Q+U)D_p,$$
$$G_{23}=-2D_q^TTQUD_{pd}+T(Q+U)D_{pd},$$
$$G_{33}=-\Gamma-2D_{pd}^TTQUD_{pd},$$

where $Q=\mathrm{diag}(q_1,\ q_2,\ \ldots,\ q_L)\ge 0$, $U=\mathrm{diag}(u_1,\ u_2,\ \ldots,\ u_L)>0$.

Proof: For simplicity, we denote $x(t)$ as x, $\xi(t)$ as ξ, $\xi_i(t)$ as ξ_i, $\phi_i(\xi_i(t))$ as ϕ_i, $\phi(\xi(t))$ as ϕ, $\phi(\xi(t-h))$ as ϕ_h, and $\phi_i(\xi_i(t-h))$ as $\phi_{h,i}$. For the SNNM (4), we construct Lyapunov-Krasovskii functional:

$$V(x)=x^TPx+\int_{-h}^{0}\phi^T(\xi(t+\theta))\Gamma\phi(\xi(t+\theta))d\theta,$$

where $P>0$, $\Gamma>0$, thus, $\forall x\neq 0$, $V(x)>0$ and $V(x)=0$ if $x=0$. The derivative of $V(x)$ along the solution of the SNNM (4) is

$$\begin{aligned}\frac{dV(x)}{dt}&=2x^TP(Ax+B_p\phi+B_{pd}\phi_h)+\phi^T\Gamma\phi-\phi_h^T\Gamma\phi_h\\&=x^T(A^TP+PA)x+x^TPB_p\phi+x^TPB_{pd}\phi_h\\&\quad+\phi^TB_p^TPx+\phi^T\Gamma\phi+\phi_h^TB_{pd}^TPx-\phi_h^T\Gamma\phi_h\\&=\begin{bmatrix}x\\ \phi\\ \phi_h\end{bmatrix}^T\underbrace{\begin{bmatrix}A^TP+PA & PB_p & PB_{pd}\\ B_p^TP & \Gamma & 0\\ B_{pd}^TP & 0 & -\Gamma\end{bmatrix}}_{T_0}\begin{bmatrix}x\\ \phi\\ \phi_h\end{bmatrix}.\end{aligned}$$

The sector conditions, $(\phi_i-q_i\xi_i)(\phi_i-u_i\xi_i)\le 0$, can be rewritten as follows:

$$(\phi_i-q_iC_{q,i}x-q_iD_{p,i}\phi-q_iD_{pd,i}\phi_h)\times(\phi_i-u_iC_{q,i}x-u_iD_{p,i}\phi-u_iD_{pd,i}\phi_h)\le 0,$$

which is equivalent to:

$$\begin{aligned}&2\phi_i^2-2\phi_i(q_i+u_i)C_{q,i}x-2\phi_i(q_i+u_i)D_{p,i}\phi\\&-2\phi_i(q_i+u_i)D_{pd,i}\phi_h+2x^TC_{q,i}^Tq_iu_iC_{q,i}x\\&+2\phi^TD_{p,i}^Tq_iu_iD_{p,i}\phi+2\phi_h^TD_{pd,i}^Tq_iu_iD_{pd,i}\phi_h\\&+2\phi^TD_{p,i}^Tq_iu_iD_{pd,i}\phi_h+2\phi_h^TD_{pd,i}^Tq_iu_iD_{p,i}\phi\\&+2x^TC_{q,i}^Tq_iu_iD_{p,i}\phi+2x^TC_{q,i}^Tq_iu_iD_{pd,i}\phi_h\\&+2\phi^TD_{p,i}^Tq_iu_iC_{q,i}x+2\phi_h^TD_{pd,i}^Tq_iu_iC_{q,i}x\le 0,\end{aligned}\tag{6}$$

where $C_{q,i}$ denotes the i^{th} row of C_q, $D_{p,i}$ denotes the i^{th} row of D_p, $D_{pd,i}$ denotes the i^{th} row of D_{pd}. Rewrite (6) in the matrix form as follows:

$$\begin{bmatrix} x \\ \phi_1 \\ \vdots \\ \phi_i \\ \vdots \\ \phi_L \\ \phi_{h,1} \\ \vdots \\ \phi_{h,L} \end{bmatrix}^T T_i^1 \begin{bmatrix} x \\ \phi_1 \\ \vdots \\ \phi_i \\ \vdots \\ \phi_L \\ \phi_{h,1} \\ \vdots \\ \phi_{h,L} \end{bmatrix} + \begin{bmatrix} x \\ \phi \\ \phi_h \end{bmatrix}^T T_i^2 \begin{bmatrix} x \\ \phi \\ \phi_h \end{bmatrix} \le 0,$$

where

$$T_i^1 = \left[\begin{matrix} 0 & 0 & \cdots & -C_{q,i}^T \Xi_i & \cdots \\ 0 & 0 & \cdots & -d_{p,i,1}\Xi_i & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -\Xi_i C_{q,i} & -\Xi_i d_{p,i,1} & \cdots & 2-2\Xi_i d_{p,i,i} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & -d_{p,i,L}\Xi_i & \cdots \\ 0 & 0 & \cdots & -d_{pd,i,1}\Xi_i & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & -d_{pd,i,L}\Xi_i & \cdots \end{matrix}\right.$$

$$\left.\begin{matrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ -\Xi_i d_{p,i,L} & -\Xi_i d_{pd,i,1} & \cdots & -\Xi_i d_{pd,i,L} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 \end{matrix}\right],$$

$$T_i^2 = \begin{bmatrix} 2C_{q,i}^T q_i u_i C_{q,i} & 2C_{q,i}^T q_i u_i D_{p,i} & 2C_{q,i}^T q_i u_i D_{pd,i} \\ 2D_{p,i}^T q_i u_i C_{q,i} & 2D_{p,i}^T q_i u_i D_{p,i} & 2D_{p,i}^T q_i u_i D_{pd,i} \\ 2D_{pd,i}^T q_i u_i C_{q,i} & 2D_{pd,i}^T q_i u_i D_{p,i} & 2D_{pd,i}^T q_i u_i D_{pd,i} \end{bmatrix},$$

$$\Xi_i = (q_i + u_i),$$

$d_{p,i,j}$ is the entry of the matrix D_p at the i^{th} row and j^{th} column, $d_{pd,i,j}$ is the entry of the matrix D_{pd} at the i^{th} row and j^{th} column. By the S-procedure [17], if there exist $\tau_i \ge 0$ (i=1, …, L), such that the following inequality holds

$$T_0 - \sum_{i=1}^{L} \tau_i (T_i^1 + T_i^2) = \begin{bmatrix} A^T P + PA & PB_p & PB_{pd} \\ B_p^T P & \Gamma & 0 \\ B_{pd}^T P & 0 & -\Gamma \end{bmatrix} -$$

$$\begin{bmatrix} 2C_q^T TQUC_q & 2C_q^T TQUD_p - C_q^T \Xi T & 2C_q^T TQUD_{pd} \\ * & 2D_p^T TQUD_p + 2T - D_p^T \Xi T - T\Xi D_p & 2D_p^T TQUD_{pd} - T\Xi D_{pd} \\ * & * & 2D_{pd}^T TQUD_{pd} \end{bmatrix} = G < 0,$$

where Ξ=Q+U, T=diag(τ_1, τ_2, …, τ_L), and $T \ge 0$, then $T_0<0$, that is, $\forall x \ne 0$, d$V(x)$/dt<0 and d$V(x)$/dt=0 iff x=0. According to (5), we conclude that the origin of SNNM (4) is globally asymptotically stable. This completes the proof. □

If B_{pd}=0 and D_{pd}=0, the autonomous SNNM (4) is a non-delayed standard neural network model, which is represented as:

$$\begin{cases} \dot{x}(t) = Ax(t) + B_p \phi(\xi(t)), \\ \xi(t) = C_q x(t) + D_p \phi(\xi(t)). \end{cases} \tag{7}$$

There exist at least one equilibrium point located at the origin i.e., x_{eq}=0, ξ_{eq}=0. For (7), we have following corollary.

Corollary 1: If there exist symmetrical and positive definite matrix P, and diagonal semi-positive definite matrix T, such that the following LMI holds:

$$\begin{bmatrix} \begin{pmatrix} A^T P + PA \\ -2C_q^T TQUC_q \end{pmatrix} & \begin{pmatrix} PB_p - 2C_q^T TQUD_p \\ +C_q^T \Xi T \end{pmatrix} \\ * & \begin{pmatrix} D_p^T \Xi T + T\Xi D_p \\ -2D_p^T TQUD_p - 2T \end{pmatrix} \end{bmatrix} < 0, \tag{8}$$

where $\Xi = Q$+U, then the origin of system (7) is globally asymptotically stable.

The proof of Corollary 1 follows the same idea as that in the proof of Theorem 1, and is omitted here. For Corollary 1, we define the following Lyapunov functional:

$$V(x) = x^T Px.$$

3. FEEDBACK STABILIZATION OF THE SNNM

Firstly, we consider state-feedback control design problem for the SNNM (2) with inputs and outputs.

Based on the stability analysis of the SNNM, we design state-feedback controllers so that the overall closed-loop systems are globally asymptotically stable. The controller is of the form

$$u(t) = Kx(t), \tag{9}$$

where $K \in \Re^{m\times n}$ is the feedback gain. The overall closed-loop system of the SNNM (2) and the feedback controller (9) is described by

$$\begin{cases} \dot{x}(t) = \tilde{A}x(t) + B_p\phi(\xi(t)) + B_{pd}\phi(\xi(t-h)), \\ \xi(t) = \tilde{C}_q x(t) + D_p\phi(\xi(t)) + D_{pd}\phi(\xi(t-h)), \\ y(t) = \tilde{C}_y x(t) + D_{yp}\phi(\xi(t)) + D_{ypd}\phi(\xi(t-h)), \end{cases} \tag{10}$$

where $\tilde{A} = A + B_u K$, $\tilde{C}_q = C_q + D_{qu}K$, $\tilde{C}_y = C_y + D_u K$.

Now, we are in a position to give the main result on the solvability of the state feedback control problem.

Theorem 2: Consider the input-output SNNM (2). There exists a state-feedback control law (9) such that the closed-loop system (10) is globally asymptotically stable provided that there exist symmetric positive definite matrices X and S, a matrix Y, and diagonal positive definite matrix Σ that satisfy the following LMI

$$\begin{bmatrix} \begin{pmatrix} (AX+B_uY)^T \\ +AX+B_uY \end{pmatrix} & \begin{pmatrix} (C_qX+D_{qu}Y)^T\,\Xi \\ +B_p\Sigma \end{pmatrix} & B_{pd}\Sigma \\ * & \begin{pmatrix} \Sigma D_p^{\mathrm{T}}\Xi + \Xi D_p\Sigma \\ +S-2\Sigma \end{pmatrix} & \Xi D_{pd}\Sigma \\ * & * & -S \end{bmatrix} < 0, \tag{11}$$

where $\Xi = Q+U$. Furthermore, the feedback gain K is obtained as $K=YX^{-1}$.

Proof: Using Theorem 1, it is straightforward to obtain conditions of global asymptotic stability for the closed-loop system (10). We note that Inequality (5) is rewritten as

$$\begin{bmatrix} \tilde{A}^T P + P\tilde{A} - 2\tilde{C}_q^T TQU\tilde{C}_q & \begin{pmatrix} PB_p - 2\tilde{C}_q^T TQUD_p \\ +\tilde{C}_q^T \Xi T \end{pmatrix} & \begin{matrix} PB_{pd} - 2\tilde{C}_q^T TQUD_{pd} \end{matrix} \\ * & \begin{pmatrix} \Gamma - 2D_p^T TQUD_p - 2T \\ +D_p^T\Xi T + T\Xi D_p \end{pmatrix} & -2D_p^T TQUD_{pd} + T\Xi D_{pd} \\ * & * & -\Gamma - 2D_{pd}^T TQUD_{pd} \end{bmatrix} < 0. \tag{12}$$

Observing the structure of the parameters in (12), Inequality (12) is nonlinear matrix inequality over P, Γ, T, and K. Next, we will convert (12) into LMI over unknown parameters.

Inequality (12) can be expressed as

$$\begin{bmatrix} \tilde{A}^T P + P\tilde{A} & PB_p + \tilde{C}_q^T\Xi T & PB_{pd} \\ * & \Gamma - 2T + D_p^T\Xi T + T\Xi D_p & T\Xi D_{pd} \\ * & * & -\Gamma \end{bmatrix} - 2\times\begin{bmatrix} \tilde{C}_q^T \\ D_p^T \\ D_{pd}^T \end{bmatrix} TQU\begin{bmatrix} \tilde{C}_q & D_p & D_{pd} \end{bmatrix} < 0. \tag{13}$$

Since $TQU \geq 0$, if

$$\begin{bmatrix} \tilde{A}^T P + P\tilde{A} & PB_p + \tilde{C}_q^T\Xi T & PB_{pd} \\ * & \Gamma - 2T + D_p^T\Xi T + T\Xi D_p & T\Xi D_{pd} \\ * & * & -\Gamma \end{bmatrix} < 0 \tag{14}$$

holds, inequalities (12) and (13) also hold. Pre-and post-multiplying the left-hand side matrix of (14) by diag (P^{-1}, T^{-1}, T^{-1}), the inequality (14) is equivalent to

$$\begin{bmatrix} P^{-1}\tilde{A}^T + \tilde{A}P^{-1} & B_pT^{-1} + P^{-1}\tilde{C}_q^T\Xi & B_{pd}T^{-1} \\ * & \begin{pmatrix} T^{-1}\Gamma T^{-1} - 2T^{-1} \\ +T^{-1}D_p^T\Xi + \Xi D_pT^{-1} \end{pmatrix} & \Xi D_{pd}T^{-1} \\ * & * & -T^{-1}\Gamma T^{-1} \end{bmatrix} < 0. \tag{15}$$

Let $X=P^{-1}$, $Y=KX$, $\Sigma=T^{-1}$, and $S=T^{-1}\Gamma T^{-1}$, the inequality (15) is rewritten as (11). Therefore, if inequality (15) have feasible solution, the feedback gain is $K=YX^{-1}$. The proof of Theorem 2 is thus completed. □

If $B_{pd}=0$, $D_{pd}=0$, and $D_{ypd}=0$, the SNNM (2) with inputs and outputs is a non-delayed system, which is described by

$$\begin{cases} \dot{x}(t) = Ax(t) + B_p\phi(\xi(t)) + B_u u(t), \\ \xi(t) = C_q x(t) + D_p\phi(\xi(t)) + D_{qu}u(t), \\ y(t) = C_y x(t) + D_{yp}\phi(\xi(t)) + D_u u(t). \end{cases} \tag{16}$$

The following corollary provides a design approach of a state-feedback controller for system (16) without delays.

Corollary 2: Consider the input-output SNNM (16). There exists a state-feedback control law (9) such that the closed-loop system is globally asymptotically stable provided that there exist symmetric positive definite matrices X, a matrix Y, and diagonal positive definite matrix Σ that satisfy the following LMI

$$\begin{bmatrix} \begin{pmatrix} (AX+B_uY)^T \\ +AX+B_uY \end{pmatrix} & B_p\Sigma+(C_qX+D_{qu}Y)^T\Xi \\ * & -2\Sigma+\Sigma D_p^T\Xi+\Xi D_p\Sigma \end{bmatrix} < 0, \quad (17)$$

where $\Xi=Q+U$. Furthermore, the feedback gain K is obtained as $K=YX^{-1}$.

Next, we will provide the design procedure of the full-order dynamic output feedback controller for the SNNM (2). The output feedback controller is of the form

$$\begin{cases} \dot{x}_c(t)=A_cx_c(t)+B_cy(t), \\ u(t)=C_cx_c(t), \end{cases} \quad (18)$$

where $x_c\in\Re^n$ is the controller state, $A_c\in\Re^{n\times n}$, $B_c\in\Re^{n\times l}$, and $C_c\in\Re^{m\times n}$ are matrices of appropriate size. The overall closed-loop system of the SNNM (2) and the feedback controller (18) is described by

$$\begin{cases} \dot{\tilde{x}}(t)=\tilde{A}\tilde{x}(t)+\tilde{B}_p\phi(\xi(t))+\tilde{B}_{pd}\phi(\xi(t-h)), \\ \xi(t)=\tilde{C}_q\tilde{x}(t)+D_p\phi(\xi(t))+D_{pd}\phi(\xi(t-h)), \\ y(t)=\tilde{C}_y\tilde{x}(t)+D_{yp}\phi(\xi(t))+D_{ypd}\phi(\xi(t-h)), \end{cases} \quad (19)$$

where

$$\tilde{x}=\begin{bmatrix} x & x_c \end{bmatrix}^T, \quad \tilde{A}=\begin{bmatrix} A & B_uC_c \\ B_cC_y & A_c+B_cD_uC_c \end{bmatrix},$$

$$\tilde{B}_p=\begin{bmatrix} B_p \\ B_cD_{yp} \end{bmatrix}, \quad \tilde{B}_{pd}=\begin{bmatrix} B_{pd} \\ B_cD_{ypd} \end{bmatrix},$$

$$\tilde{C}_q=\begin{bmatrix} C_q & D_{qu}C_c \end{bmatrix}, \quad \tilde{C}_y=\begin{bmatrix} C_y & D_uC_c \end{bmatrix}.$$

It is obvious that we have the following theorem to design output feedback controllers for the SNNM (2) from Theorem 1.

Theorem 3: There exists a full-order dynamic output feedback control law (18) such that the closed-loop system (19) is globally asymptotically stable if there exist symmetric positive definite matrices P and Γ, diagonal semi-positive definite matrix T, and matrices A_c, B_c and C_c that satisfy the following nonlinear matrix inequality

$$\begin{bmatrix} \tilde{A}^TP+P\tilde{A} & P\tilde{B}_p+\tilde{C}_q^T\Xi T & P\tilde{B}_{pd} \\ * & \Gamma-2T+D_p^T\Xi T+T\Xi D_p & T\Xi D_{pd} \\ * & * & -\Gamma \end{bmatrix} < 0, \quad (20)$$

where $\Xi=Q+U$. Furthermore, the controller parameters A_c, B_c, and C_c can be obtained by solving the non-convex optimization problem (20).

The problem described by (20) is bilinear matrix inequality (BMI) feasible problem over the unknown controller parameters (A_c, B_c, and C_c) and Lyapunov parameters (P, Γ, and T). BMI problems, in general, are proven to be NP-hard [18]. Local search algorithms typically either use an iterative search method by solving alternated LMIs, or approximate the BMI problem by an LMI problem based on linearization techniques [19,20]. Most of the global BMI algorithms are variations of the branch and bound method [21,22]. In what follows, we transform (20) into the LMI problems under the assumption that $B_u=0$ in the SNNM (2). If $u(t)$ is an input of neural network, $u(t)$ only exists in the nonlinear terms, i.e., $B_u=0$. In most case, the assumption is satisfied, so we have the following corollary.

Corollary 3: Consider the input-output SNNM (2) where $B_u=0$. There exists a full-order dynamic output feedback control law (18) such that the closed-loop system (19) is globally asymptotically stable provided that there exist symmetric positive definite matrices X, Y and Γ, diagonal semi-positive definite matrix T, matrices $\hat{A}$ and $\hat{B}$, and special structure matrix $\hat{C}$ that satisfy the following LMI

$$\begin{bmatrix} AX+XA^T & A+\hat{A}^T & B_p+\hat{C}^T\Xi & B_{pd} \\ * & A^TY+YA+\hat{B}C_y+(\hat{B}C_y)^T & YB_p+\hat{B}D_{yp}+C_q^TT\Xi & YB_{pd}+\hat{B}D_{ypd} \\ * & * & \Gamma-2T+\Xi TD_p+D_p^TT\Xi & \Xi TD_{pd} \\ * & * & * & -\Gamma \end{bmatrix} < 0, \quad (21)$$

$$\begin{bmatrix} Y & I \\ I & X \end{bmatrix} > 0, \quad (22)$$

where $\Xi=Q+U$. Furthermore, the desired dynamic output feedback controller is given in the form of (18) with parameters as follow:

$$\begin{aligned} D_{qu}C_c &= T^{-1}(\hat{C}-TC_qX)(M^{\mathrm{T}})^{-1}, \\ B_c &= N^{-1}\hat{B}, \end{aligned} \quad (23)$$

$$A_c = N^{-1}(\hat{A} - YAX)(M^{\mathrm{T}})^{-1} - B_c C_y X (M^{\mathrm{T}})^{-1} - B_c D_u C_c,$$

where M and N are any nonsingular matrices satisfying

$$MN^{\mathrm{T}} = I - XY. \tag{24}$$

Proof: According to Theorem 3, if Inequality (20) holds, the closed-loop system (19) is globally asymptotically stable. To proceed, we partition P and its inverse P^{-1} as

$$P = \begin{bmatrix} Y & N \\ N^T & W \end{bmatrix}, \ P^{-1} = \begin{bmatrix} X & M \\ M^T & Z \end{bmatrix},$$

where $X\in\Re^{n\times n}$ and $Y\in\Re^{n\times n}$ are symmetric positive definite matrices. The condition $P\cdot P^{-1}=I$ implies

$$P\begin{bmatrix} X & I \\ M^T & 0 \end{bmatrix} = \begin{bmatrix} I & Y \\ 0 & N^T \end{bmatrix}.$$

Defining

$$F_1 = \begin{bmatrix} X & I \\ M^T & 0 \end{bmatrix}, \ F_2 = \begin{bmatrix} I & Y \\ 0 & N^T \end{bmatrix}.$$

and under the condition $B_u=0$, we obtain

$$F_1^T P\tilde{A}F_1 = F_2^T \tilde{A}F_1 = \begin{bmatrix} AX & A \\ \begin{pmatrix} YAX + NB_cC_yX \\ +N(A_c + B_cD_uC_c)M^T \end{pmatrix} & YA + NB_cC_y \end{bmatrix}, \tag{25}$$

$$F_1^T P\tilde{B}_p = F_2^T \tilde{B}_p = \begin{bmatrix} B_p \\ YB_p + NB_cD_{yp} \end{bmatrix}, \tag{26}$$

$$F_1^T P\tilde{B}_{pd} = F_2^T \tilde{B}_{pd} = \begin{bmatrix} B_{pd} \\ YB_{pd} + NB_cD_{ypd} \end{bmatrix}, \tag{27}$$

$$\tilde{C}_q F_1 = \begin{bmatrix} C_qX + D_{qu}C_cM^T & C_q \end{bmatrix}. \tag{28}$$

Pre- and post-multiplying the left-hand side matrix of (20) by diag (F_1^{T}, I, I) and diag (F_1, I, I), respectively, the inequality (20) is equivalent to

$$\begin{bmatrix} F_1^T\tilde{A}^TPF_1 + F_1^TP\tilde{A}F_1 & F_1^TP\tilde{B}_p + F_1^T\tilde{C}_q^T\Xi T & F_1^TP\tilde{B}_{pd} \\ * & \Gamma - 2T + D_p^T\Xi T + T\Xi D_p & T\Xi D_{pd} \\ * & * & -\Gamma \end{bmatrix} < 0. \tag{29}$$

Substituting (29) by (25)-(28) and defining

$$\begin{cases} \hat{A} = YAX + NB_cC_yX + N(A_c + B_cD_uC_c)M^{\mathrm{T}}, \\ \hat{B} = NB_c, \\ \hat{C} = TC_qX + TD_{qu}C_cM^T, \end{cases} \tag{30}$$

we obtain Ieq.(21). The controller parameters A_c, B_c, and C_c can be deduced from (30). It is worth noting that the structure of matrix $\hat{C}$ is determined by the form of D_{qu}.

Using the condition $P^{-1}\cdot P=I$, we have $MN^{\mathrm{T}}=I-XY$. By the Schur complement formula [17], the inequality (22) can be expressed as $Y-X^{-1}>0$, therefore $I-XY$ is nonsingular. This ensures that there always exist nonsingular matrices M and N such that (24) is satisfied. We thus complete the proof. □

4. APPLICATION EXAMPLES

To apply Theorems 2 and 3 (or Corollaries 2 and 3) to synthesize feedback controllers to stabilize neural-network-based nonlinear systems, it is necessary to transform them into the SNNMs (2) (or SNNMs (16)). The following examples, synthesis of feedback stabilizing controllers for SISO continuous-time nonlinear system in [23] and [24] modeled by a dynamic recurrent neural network (DRNN), and for chaotic neural network, illustrate that the SNNM can be widely applied to the synthesis of nonlinear systems.

4.1. State-feedback controller synthesis for the single-input/single-output nonlinear system

Consider a continuous-time, single-input/single-output (SISO), nonlinear-control affine system described as follows [23,24]:

$$\begin{cases} \dot{\chi}(t) = f(\dot{\chi}(t)) + g(t)u(t), \\ y(t) = h(\chi(t)), \end{cases} \tag{31}$$

where $\chi(t)\in\Re^n$ is the state vector, $u(t)\in\Re$ is the control input, $y(t)\in\Re$ is the output, $g(t)\in\Re^n$ is the parametric vector, $f(\chi(t))\in C(\Re^n;\ \Re^n)$ and $h(\chi(t))\in C(\Re^n;\ \Re)$ are continuous nonlinear mapping. Delgado *et al.* employed the following DRNN to approximate the nonlinear system (31) in [23,24].

$$\begin{cases} \dot{\eta}(t) = -\eta(t) + W\sigma(\eta(t)) + Vu(t), \\ y_{NN}(t) = C\eta(t), \end{cases} \tag{32}$$

where $y_{NN}(t)\in\Re$ is the output of the DRNN, $\eta(t)\in\Re^N$ is the state vector of the approximating DRNN, and N is the number of neurons in the DRNN. Delgado *et al.* provided the approximation conditions in [23], that is,

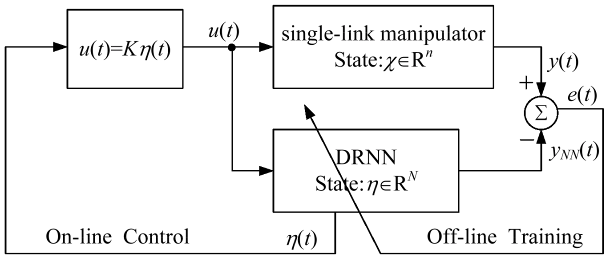

Fig. 2. The close-loop system of the single-link manipulator.

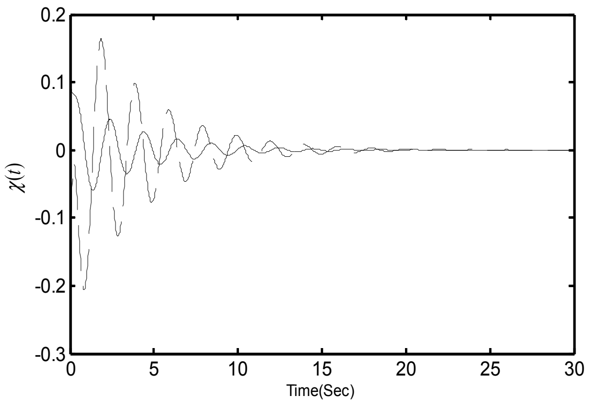

Fig. 3. The state responses $\chi_1(t)$ (solid line) and $\chi_2(t)$ (dashed line) for the closed-loop system where the initial states of the system (34) are $\chi_1(0)=0.1$, $\chi_2(0)=-0.2$, and the initial states of the DRNN (32) are initialized arbitrarily.

if $N \geq n$, the DRNN (32) can approximate the nonlinear system (31). When the approximation error can be guaranteed to be small in a neighborhood of the origin, a controller designed for a neural-network approximation model can maintain its performance while applied to the actual nonlinear system [5].

We can transform (32) into the SNNM (16), where $x(t)=\eta(t)$, $A=-I$, $B_p=W$, $B_u=V$, $C_q=I$, $D_p=0$, $D_{qu}=0$, $C_y=C$, $D_{yp}=0$, $D_u=0$, and $\phi(\xi(t))=\sigma(\xi(t))$, and design the state-feedback control law by Corollary 2.

To illustrate our SNNM-based feedback control design algorithm and to demonstrate the closed-loop stability guarantee, we consider a single-link manipulator which is described by the second-order nonlinear differential equation:

$$ml^2\ddot{\theta}(t)+v\dot{\theta}(t)+mgl\sin\theta(t)=u(t), \tag{33}$$

where the length, mass and friction coefficients are l=1m, m=2.0kg, and v=1.0kg m^2/s, respectively. Let $\chi_1(t)=\theta(t)$, $\chi_2(t)=\dot{\theta}(t)$, and $y(t)=\theta(t)$, we obtain the following state representation of the system (33):

$$\begin{cases}\dot{\chi}_1(t)=\chi_2(t),\\ \dot{\chi}_2(t)=-9.8\sin\chi_1(t)-0.5\chi_2(t)+0.5u(t),\\ y(t)=\chi_1(t).\end{cases} \tag{34}$$

In [24], the system (34) was identified with a DRNN (32) where $\sigma(\eta)=\tanh(\eta)$ and N=2. The parameters of the DRNN (32) after the training in [24] were

$$W=\begin{bmatrix}0.61 & 2.79\\ -4.04 & 0.29\end{bmatrix},\ V=\begin{bmatrix}0\\ 0.23\end{bmatrix},\ C=\begin{bmatrix}1 & 0\end{bmatrix}.$$

We transform the system (32) into the SNNM (16) where $x(t)=\eta(t)$, $A=-I$, $B_p=W$, $B_u=V$, $C_q=I$, $D_p=0$, $D_{qu}=0$, $C_y=C$, $D_{yp}=0$, and $D_u=0$, $\phi(\xi(t))=\tanh(\xi(t))$, $Q=0$, $U=I$. While we adopt the state feedback controller (9), where $u(t)\in\Re$, $K\in\Re^{1\times 2}$, the close-loop system is shown in Fig. 2. According to Corollary 2, solving the LMI (17) by the convex optimization technique of MATLAB LMI Toolbox [25], we obtain the feedback gain $K=[-4.1432\ \ -27.7556]$. Fig. 3 shows the state response of the close-loop systems with state-feedback controllers. It is obvious that the states asymptotically converge to the zeroes.

4.2. Feedback control of chaotic neural network with time delays

Now, we consider the following chaotic neural network with time delays:

$$\begin{cases}\dot{x}_1(t)=-x_1(t)+3\tanh(x_1(t))+0.02\tanh(1.5x_2(t))\\ \qquad -5.8\tanh(x_1(t-1))+0.785\tanh(2x_3(t-1)),\\ \dot{x}_2(t)=-x_2(t)+3\tanh(1.5x_2(t))\\ \qquad -5.8\tanh(1.5x_2(t-1))+0.785\tanh(2x_3(t-1)),\\ \dot{x}_3(t)=-x_3(t)+\tanh(2x_3(t))-5.8\tanh(2x_3(t-1)),\end{cases} \tag{35}$$

with the initial value $x_1(0)=0.8$, $x_2(0)=5$, $x_3(0)=0.2$. The behaviour of the chaotic neural network is shown in Fig. 4. We convert the system (35) into the autonomous SNNM depicted by

$$\begin{cases}\dot{x}(t)=Ax(t)+B_p\phi(\xi(t))+B_{pd}\phi(\xi(t-1)),\\ \xi(t)=C_qx(t),\end{cases} \tag{36}$$

where

$$x=\begin{bmatrix}x_1(t) & x_2(t) & x_3(t)\end{bmatrix},\ A=\mathrm{diag}(-1,-1,-1),$$

$$B_p=\begin{bmatrix}3 & 0.02 & 0\\ 0 & 3 & 0\\ 0 & 0 & 1\end{bmatrix},\ B_{pd}=\begin{bmatrix}-5.8 & 0 & 0.785\\ 0 & -5.8 & 0.785\\ 0 & 0 & -5.8\end{bmatrix},$$

$$C_q=\begin{bmatrix}1 & 0 & 0\\ 0 & 1.5 & 0\\ 0 & 0 & 2\end{bmatrix}.$$

In order to globally stabilize the SNNM (36) (i.e., system (35)), we adopt the following control laws.

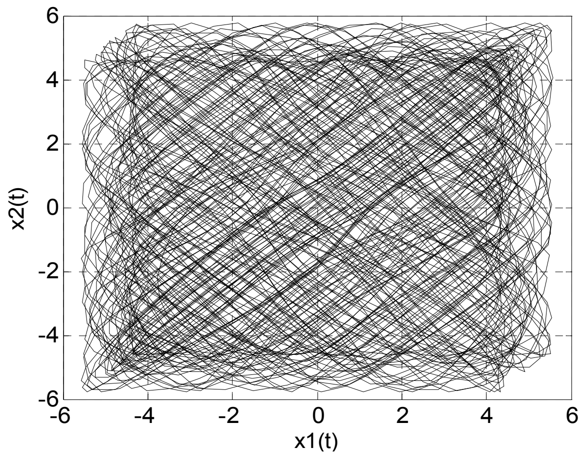

Fig. 4. Phase portrait with the plane ($x_1(t)$, $x_2(t)$).

4.2.1 Static state feedback control

By introducing an external control term $u(t)$ into the SNNM (36), it yields the control system of the form

$$\begin{cases}\dot{x}(t)=Ax(t)+B_p\phi(\xi(t)) \\ \qquad +B_{pd}\phi(\xi(t-1))+B_u u(t), \\ \xi(t)=C_q x(t),\end{cases} \tag{37}$$

where B_u=diag(1,1,1), and the controller (9), where $u(t)\in\Re^3$, $K\in\Re^{3\times3}$. According to Theorem 2, solving the LMI (11), we obtain the feedback gain as

$$K=\begin{bmatrix}-26.7380 & 1.1277 & -19.6468 \\ -0.7708 & -39.9005 & 7.0757 \\ 17.6104 & -1.9526 & -35.4105\end{bmatrix}.$$

The state responses of the close-loop system of open loop system (37) under the state feedback controller (9) are shown in Fig. 5, where the states are initialized arbitrarily.

4.2.2 Full-order dynamic output feedback control

By introducing an external control term $u(t)$ into the SNNM (36), it yields the control system of the form

$$\begin{cases}\dot{x}(t)=Ax(t)+B_p\phi(\xi(t))+B_{pd}\phi(\xi(t-1)), \\ \xi(t)=C_q x(t)+D_{qu}u(t), \\ y(t)=C_y x(t),\end{cases}$$

where D_{qu}=diag(1,1,1), $C_y=\begin{bmatrix}1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 3\end{bmatrix}$, and the controller (18), where $A_c\in\Re^{3\times3}$, $B_c\in\Re^{3\times3}$, and $C_c\in\Re^{3\times3}$. Then, using the MATLAB LMI Control Toolbox [25] to solve the LMIs in (21) and (22) by Corollary 3, a desired dynamic output feedback controller can be constructed as in (18) with

$$A_c=\begin{bmatrix}-12.1868 & 19.0678 & -147.7005 \\ -21.7185 & -17.4271 & 389.9062 \\ 100.3549 & -330.2285 & -21.6490\end{bmatrix},$$

$$B_c=10^4\times\begin{bmatrix}0.1542 & -0.3928 & 0.1367 \\ -0.8840 & 1.1687 & -0.2430 \\ 2.0012 & -0.4390 & -0.7523\end{bmatrix},$$

$$C_c=\begin{bmatrix}0.0232 & -0.0129 & 0.0029 \\ 0.0030 & 0.0165 & 0.0516 \\ 0.0229 & 0.0530 & -0.0251\end{bmatrix}.$$

With the aforementioned dynamic output feedback controller, the simulation results of the state response of the closed-loop system are given in Fig. 6 where the states are initialized arbitrarily.

From these simulation examples, it can be seen the designed static state feedback controller and dynamic output feedback controller both ensure the global asymptotic stability of the closed-loop system.

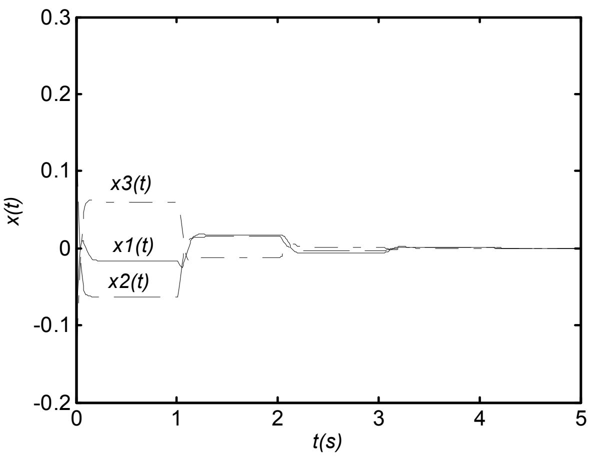

Fig. 5. State trajectories of $x_1(t)$ (solid line), $x_2(t)$ (dashed line) and $x_3(t)$(dot-dashed line).

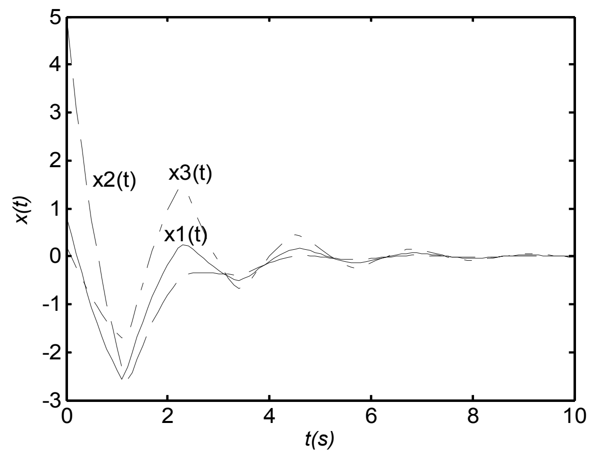

Fig. 6. State response of $x_1(t)$ (solid line), $x_2(t)$(dashed line) and $x_3(t)$(dot-dashed line).

5. CONCLUSIONS

In this paper, we study the design approaches of static state feedback controllers and dynamic output feedback controllers for a class of nonlinear systems composed of neural networks such that the closed-loop systems are globally asymptotically stable. The resulting design equations are a set of linear matrix inequalities which can be solved by the MATLAB LMI Control Toolbox [25] to determine control signals. Central to our design are the introduction of the SNNM, and the transformation of the neural-network-based nonlinear system to the SNNM. The design approaches can be extended to synthesize any nonlinear systems as long as their equations can be transformed into the SNNMs. Simulation results of some examples have showed the effectiveness and the applicability of the proposed design approaches. Here, it is worth noting that there are no unified ways about how to convert the non-SNNM into the SNNM, but generally state-transformation is applied.

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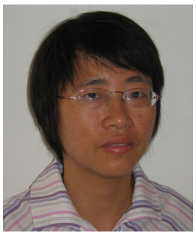

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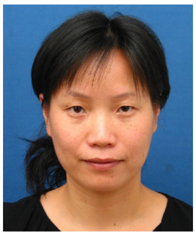

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