New Algorithm for Recursive Estimation in Linear Discrete-Time Systems with Unknown Parameters

Vladimir Shin, Jun Il Ahn, and Du Yong Kim*

Abstract: The problem of recursive filtering for linear discrete-time systems with uncertainties is considered. A new suboptimal filtering algorithm is herein proposed. It is based on the fusion formula, which represents an optimal mean-square linear combination of local Kalman estimates with weights depending on cross-covariances between local filtering errors. In contrast to the optimal weights, the suboptimal weights do not depend on current measurements, and thus the proposed algorithm can easily be implemented in real-time. High accuracy and efficiency of the suboptimal filtering algorithm are demonstrated on the following examples: damper harmonic oscillator motion and vehicle motion constrained to a plane.

Keywords: Adaptive filtering, discrete-time system, Kalman filtering, mean-square error, partitioning approach.

1. INTRODUCTION

The problem of estimating the system state in linear dynamic systems with unknown parameters is considered. Suppose that a linear system with state-space description

$$\begin{aligned} x_{k+1} &= F_k(\theta)x_k + G_k(\theta)v_k, \\ y_k &= H_k(\theta)x_k + w_k, \quad k = 0,1,2,... \end{aligned} \tag{1}$$

is being considered, where, as usual, $x_k \in \mathbf{R}^n$ is the state, $y_k \in \mathbf{R}^m$ is the measurement, $v_k \in \mathbf{R}^r \sim N$ $Q_k(\theta))$ and $w_k \in \mathbf{R}^m \sim N(0, R_k(\theta))$ are uncorrelated Gaussian white noise sequences, and distribution of initial state x_0 is Gaussian, $x_0 \sim N(\bar{x}_0(\theta), P_0(\theta))$. In this paper, we assume that the matrices F_k, G_k, Q_k, H_k, R_k, P_0, and the initial mean $\bar{x}_0$ include unknown parameter vector $\theta \in \mathbf{R}^p$, which takes only a finite set of values

$$\theta \in \{\theta_1, \ldots, \theta_N\}. \tag{2}$$

This finite set might be the result of discretizing a continuous parameter space [1,2].

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A fundamental problem associated with such systems is estimation of state x_k from the noisy measurements $y_0^k = \{y_0, y_1, ..., y_k\}$.

Many methods are available for the adaptation of systems [1-6]. In structure adaptation, two filters are primarily used for the system (1) [1,4,6-11]. Both of these filters are based on the Bayesian approach in which the unknown parameter θ is assumed to be random with a *prior* known probability $p(\theta)$. In the first filter, θ is treated as a random constant vector such as $\theta_{k+1} = \theta_k$, or more efficiently,

$$\theta_{k+1} = \theta_k + \xi_k, \tag{3}$$

where ξ_k is any zero-mean Gaussian white sequence.

Then system (1) together with assumption (3) can be reformulated as the nonlinear filtering model:

$$\begin{cases} \begin{bmatrix} x_{k+1} \\ \theta_{k+1} \end{bmatrix} = \begin{bmatrix} F_k(\theta_k)x_k \\ \theta_k \end{bmatrix} + \begin{bmatrix} G_k(\theta_k)v_k \\ \xi_k \end{bmatrix}, \\ y_k = H_k(\theta_k)x_k + w_k \end{cases} \tag{4}$$

for the composite state vector $[x_k \quad \theta_k]^T$, and the suboptimal nonlinear filtering procedures (for example, extended Kalman filter) can be applied to estimate the composite state $[x_k \quad \theta_k]^T$, which contains θ_k as its component. However, it is difficult to estimate the effect of approximations made in the suboptimal realization of nonlinear filters [8,10-12]. The second filter represents the adaptive Kalman filter (AKF), which separates the filtering process x_k from identification of the unknown parameter θ [1-

5]. In this paper we are interested in such an AKF that constitutes a partitioning of the original nonlinear filter into the bank of much simpler N local Kalman filters where each local filter uses its own system model (1), (2) matched to each possible parameter value $\theta = \theta_i$, $i = 1,..., N$, i.e.,

$$x_{k+1} = F_k(\theta_i) x_k + G_k(\theta_i) v_k, \quad v_k \sim N(0, Q_k(\theta_i)),$$
$$y_k = H_k(\theta_i) x_k + w_k, \quad w_k \sim N(0, R_k(\theta_i)). \quad (5)$$

This AKF is also referred to as Bayesian multiple model adaptive estimation (MMAE) [13-18]. The overall estimate of the state of this AKF is given by a weighted sum of local Kalman filters (estimates), thus it can be implemented on a set of parallel processors due to its inherent parallel structure. However, the AKF weights represent the conditional probabilities of the specific parameter values $p(\theta_i | y_0^k)$ which depend on current measurements y_0^k and it is rather difficult to implement the AKF in real-time for the high-dimension of state vector and large number of local Kalman filters.

We propose an alternative suboptimal filter (SF). Similarly to the optimal AKF, the SF represents the state estimate as a weighted sum of parameter-conditional estimates (local Kalman estimates) with the weights depending only on time instants and being independent of current measurements y_0^k. In this case all complex computations connected with the determination of weights are based only on a priori data about the system model (noise statistics, system matrices, and parameter values $\theta = \theta_i$) and do not use the results of measurements y_0^k. This gives an opportunity to design a low-complexity SF that can be easily implemented in real-time, especially in high-dimension problems.

This paper is organized as follows. In Section 2, we describe the optimal AKF for discrete-time dynamic systems. In Section 3, we derive general equations for the SF that represent a linear combination of local Kalman filters. Each local Kalman filter is fused by the minimum mean-square criterion. In Section 4, the suboptimal filtering algorithm implementation steps are given. In Section 5, the SF is applied for a special class of linear systems with measurement uncertainties. A solution of the joint detection-estimation problem based on the SF is provided. In Section 6, the SF is numerically tested in real-life system models. In Section 7, conclusions are made.

2. OPTIMAL ADAPTIVE KALMAN FILTER

Consider the discrete-time linear dynamic system with unknown parameters given by (1) and (2). According to the Bayesian approach, it is assumed that a *prior* probability for θ, $p(\theta)$, is available,

$$p_i = p(\theta_i) \geq 0, \quad i = 1,..., N, \quad p_1 + \cdots + p_N = 1. \quad (6)$$

If the unknown parameter θ belongs to the discrete space (2), then optimal mean-square state estimate $\hat{x}_k^{opt}$ represents weighted sum

$$\hat{x}_k^{opt} = \sum_{i=1}^{N} \tilde{c}_k^{(i)}, \quad (7)$$

where N denotes the total number of values θ_i resulting from discretizing the continuous space of θ, and

$$\hat{x}_k^{(i)} \overset{\text{def}}{=} \hat{x}_k(\theta_i) \quad (8)$$

is local Kalman estimate matched to the linear system (5), and

$$\tilde{c}_k^{(i)} = \mathrm{p}(\theta_i | y_0^k) \quad (9)$$

represents conditional probability of θ_i given y_0^k. The local Kalman estimates $\hat{x}_k^{(i)}$, $i = 1, ..., N$ are determined by the standard discrete Kalman filter equations:

$$\hat{x}_k^{(i)} = F_k^{(i)} \hat{x}_{k-1}^{(i)} + K_k^{(i)} \left[y_k - H_k^{(i)} F_k^{(i)} \hat{x}_{k-1}^{(i)} \right], \ \hat{x}_0^{(i)} = \overline{x}_0^{(i)},$$
$$M_k^{(i)} = F_{k-1}^{(i)} P_{k-1}^{(i)} F_{k-1}^{(i)T} + G_{k-1}^{(i)} Q_{k-1}^{(i)} G_{k-1}^{(i)T},$$
$$K_k^{(i)} = M_k^{(i)} H_k^{(i)T} \left[H_k^{(i)} M_k^{(i)} H_k^{(i)T} + R_k^{(i)} \right]^{-1}, \quad (10)$$
$$P_k^{(i)} = \left[I_{\mathrm{n}} - K_k^{(i)} H_k^{(i)} \right] M_k^{(i)}, \quad i = 1,..., N,$$

where

$$F_k^{(i)} = F_k(\theta_i), \ G_k^{(i)} = G_k(\theta_i), \ Q_k^{(i)} = Q_k(\theta_i),$$
$$H_k^{(i)} = H_k(\theta_i), \ R_k^{(i)} = R_k(\theta_i), \quad (11)$$
$$\overline{x}_0^{(i)} = \overline{x}_0(\theta_i), \ P_0^{(i)} = P_0(\theta_i), \ i = 1,..., N.$$

And the conditional probabilities $p(\theta_i | y_0^k)$, $i =$ $1,..., N$ are provided by the recursive Bayesian formula [1,2,5]:

$$p(\theta_i | y_0^k) = \frac{L_k^{(i)} p(\theta_i | y_0^{k-1})}{\sum_{j=1}^{N} L_k^{(j)} p(\theta_j | y_0^{k-1})},$$

$$k = 1,2,..., \quad p\left(\theta_i \middle| y_0^0\right) = p_i, \quad i = 1,...,N,$$
$$L_k^{(i)} = \left|\tilde{P}_k^{(i)}\right|^{-1/2} \exp\left[-\left(\tilde{y}_k^{(i)}\right)^T \left(\tilde{P}_k^{(i)}\right)^{-1} \tilde{y}_k^{(i)}\right],$$
$$\tilde{P}_k^{(i)} = H_k^{(i)} M_k^{(i)} \left(H_k^{(i)}\right)^T + R_k^{(i)},$$
$$\tilde{y}_k^{(i)} = y_k - H_k^{(i)} F_k^{(i)} x_{k-1}^{(i)}, \quad i = 1,...,N. \tag{12}$$

As already mentioned above, the AKF (7)-(12) can be a very costly algorithm to implement, since it requires complex calculations of the conditional probabilities $p\left(\theta_i \middle| y_0^k\right)$ at each time instant $k > 0$.

In this paper we develop an alternative SF for systems (1) and (2). This filter does not require calculations of the conditional probabilities $p\left(\theta_i \middle| y_0^k\right)$ at each time instant. And as a consequence, the obtained filtering algorithm reduces the computational burden and on-line computational requirements.

3. SUBOPTIMAL FILTER

As well as the optimal AKF (7)-(12) the SF filter represents a weighted sum of the local Kalman estimates (8),

$$\hat{x}_k^{\text{sub}} = \sum_{i=1}^N c_k^{(i)} \hat{x}_k^{(i)}, \qquad \sum_{i=1}^N c_k^{(i)} = I_n, \tag{13}$$

where I_n is the $n \times n$ identity matrix, and $c_k^{(1)}, \ldots, c_k^{(N)}$ are $n \times n$ weights depending only on time instance k and being determined from the mean-square criterion

$$J_k\left(c_k^{(1)},...,c_k^{(N)}\right) = E\left\|x_k - \hat{x}_k^{\text{sub}}\right\|^2$$
$$= \sum_{i=1}^N p\left(\theta_i\right) E\left\|x_k\left(\theta_i\right) - \hat{x}_k^{\text{sub}}\right\|^2 \to \min_{c_k^{(i)}}. \tag{14}$$

Remark 1: The weighted sums (7) and (13) are different, since the AKF's weights $\tilde{c}_k^{(i)}$ depend on current measurements $\tilde{c}_k^{(i)} = \tilde{c}_k^{(i)}\left(y_0^k\right)$ and thus they can be computed only during the experiment, whereas the SF's weights $c_k^{(i)}$ can be pre-computed, as they do not depend on measurements.

Theorem 1: (i) The weights $c_k^{(1)}, \ldots, c_k^{(N)}$ are given by the following linear algebraic equations

$$\sum_{i=1}^N c_k^{(i)} \left[\tilde{P}_k^{(ij)} - \tilde{P}_k^{(iN)}\right] = 0, \quad j = 1,...,N-1, \tag{15}$$

$$\sum_{i=1}^N c_k^{(i)} = I_n. \tag{15}$$

(ii) The overall error covariance

$$P_k^{\text{sub}} = E\left[e_k^{\text{sub}}\left(e_k^{\text{sub}}\right)^T\right], \quad e_k^{\text{sub}} = x_k - \hat{x}_k^{\text{sub}} \tag{16}$$

is given by

$$P_k^{\text{sub}} = \sum_{i,j=1}^N c_k^{(i)} \tilde{P}_k^{(ij)} c\left(\begin{smallmatrix} j \\ k \end{smallmatrix}\right)^T, \quad \tilde{P}_k^{(ij)} = \sum_{h=1}^N p_h P_k^{(hij)}, \tag{17}$$

where

$$P_k^{(hij)} = E\left[e_k^{(hi)}\left(e_k^{(hj)}\right)^T\right], \quad h,\ i,\ j = 1,\ldots,N \tag{18}$$

are the local error cross-covariances of the filtering errors

$$e_k^{(hi)} = x_k\left(\theta_h\right) - \hat{x}_k^{(i)}. \tag{19}$$

Proof of Theorem 1 is given in Appendix A.

Theorem 2: The local error cross-covariances $P_k^{(hij)}$ in (18) can be represented as

$$P_k^{(hij)} = L_{xx,k}^{(hh)} - L_{x\hat{x},k}^{(hj)} - \left(L_{x\hat{x},k}^{(hi)}\right)^T + L_{\hat{x}\hat{x},k}^{(ij)}, \tag{20}$$

where

$$L_{xx,k}^{(ij)} = E\left[x_k^{(i)} x_k^{(j)T}\right],$$
$$L_{x\hat{x},k}^{(hj)} = E\left[x_k^{(h)} \hat{x}_k^{(j)T}\right],$$
$$L_{\hat{x}\hat{x},k}^{(ij)} = E\left[\hat{x}_k^{(i)} \hat{x}_k^{(j)T}\right] \tag{21}$$

are determined by the following recursive:

$$L_{xx,k+1}^{(ij)} = F_k^{(i)} L_{xx,k}^{(ij)} F_k^{(j)^T} + G_k^{(i)} Q_k^{(i)} G_k^{(j)^T},$$
$$L_{x\hat{x},k+1}^{(hj)} = F_k^{(h)} L_{x\hat{x},k}^{(hj)} A_{k+1}^{(j)^T} + F_k^{(h)} L_{xx,k}^{(hj)} B_{k+1}^{(j)^T}$$
$$+ G_k^{(h)} Q_k^{(h)} G_k^{(j)^T} H_{k+1}^{(j)^T} k_{k+1}^{(j)^T},$$
$$L_{\hat{x}\hat{x},k+1}^{(ij)} = A_{k+1}^{(i)} L_{\hat{x}\hat{x},k}^{(ij)} A_{k+1}^{(j)^T} + A_{k+1}^{(i)} L_{x\hat{x},k}^{(ji)^T} B_{k+1}^{(j)^T}$$
$$+ B_{k+1}^{(i)} L_{x\hat{x},k}^{(ij)} A_{k+1}^{(j)^T} + B_{k+1}^{(i)} L_{xx,k}^{(ij)} B_{k+1}^{(j)^T}$$
$$+ C_{k+1}^{(i)} C_{k+1}^{(j)^T} + D_{k+1}^{(i)} D_{k+1}^{(j)^T}, \tag{22}$$

where $K_{k+1}^{(j)}$ stands for the local Kalman gains (10), and

$$
\begin{aligned}
&A_{k+1}^{(j)} = \left(I_n - K_{k+1}^{(j)} H_{k+1}^{(j)}\right) F_{k+1}^{(j)}, \\
&B_{k+1}^{(j)} = K_{k+1}^{(j)} H_{k+1}^{(j)} F_k^{(j)}, \\
&C_{k+1}^{(j)} = K_{k+1}^{(j)} H_{k+1}^{(j)} G_k^{(j)} Q_k^{(j)^{1/2}}, \\
&D_{k+1}^{(j)} = K_{k+1}^{(j)} R_{k+1}^{(j)^{1/2}}, \\
&L_{xx,0}^{(ij)} = \bar{x}_0^{(i)} \bar{x}_0^{(j)^T}, \quad L_{x\hat{x},0}^{(hj)} = L_{\hat{x}\hat{x},0}^{(hj)} = \bar{x}_0^{(h)} \bar{x}_0^{(j)^T}, \\
&i,\ j,\ h = 1,...,N.
\end{aligned} \tag{23}
$$

Proof of Theorem 2 is given in Appendix B.

Thus, the suboptimal filter is specified by the set of recursive relations (13), (15), (20)-(23).

4. REAL-TIME IMPLEMENTATION OF THE SUBOPTIMAL FILTER

The SF equations (10), (13)-(20) can be divided into two parts:

Part 1 ("Off-line equations")**:**

$$
\begin{aligned}
&M_k^{(i)} = F_{k-1}^{(i)} P_{k-1}^{(i)} F_{k-1}^{(i)^T} + G_{k-1}^{(i)} Q_{k-1}^{(i)} G_{k-1}^{(i)^T}, \\
&K_k^{(i)} = M_k^{(i)} H_k^{(i)^T} \left[H_k^{(i)} M_k^{(i)} H_k^{(i)^T} + R_k^{(i)} \right]^{-1}, \\
&P_k^{(i)} = \left[I_n - K_k^{(i)} H_k^{(i)} \right] M_k^{(i)}, \quad i = 1,...,N,
\end{aligned} \tag{24}
$$

$$
\begin{aligned}
&L_{xx,k}^{(ij)} = F_{k\text{-}1}^{(i)} L_{xx,k\text{-}1}^{(ij)} F_{k\text{-}1}^{(j)^T} + G_{k\text{-}1}^{(i)} Q_{k\text{-}1}^{(i)} G_{k\text{-}1}^{(j)^T}, \\
&L_{x\hat{x},k}^{(hj)} = F_{k\text{-}1}^{(h)} L_{xx,k\text{-}1}^{(hj)} A_k^{(j)^T} + F_{k\text{-}1}^{(h)} L_{xx,k\text{-}1}^{(hj)} B_k^{(j)^T} \\
&\qquad + G_{k\text{-}1}^{(h)} Q_{k\text{-}1}^{(h)} G_{k\text{-}1}^{(j)^T} H_k^{(j)^T} K_k^{(j)^T}, \\
&L_{\hat{x}\hat{x},k}^{(ij)} = A_k^{(i)} L_{\hat{x}\hat{x},k\text{-}1}^{(ij)} A_k^{(j)^T} + A_k^{(i)} L_{x\hat{x},k\text{-}1}^{(ji)^T} B_k^{(j)^T} \\
&\qquad + B_k^{(i)} L_{x\hat{x},k\text{-}1}^{(ij)} A_k^{(j)^T} + B_k^{(i)} L_{xx,k\text{-}1}^{(ij)} B_k^{(j)^T} \\
&\qquad + C_k^{(i)} C_k^{(j)^T} + D_k^{(i)} D_k^{(j)^T}, \\
&A_k^{(i)} = \left[I_n - K_k^{(i)} H_k^{(i)} \right] F_k^{(i)}, \quad B_k^{(i)} = K_k^{(i)} H_k^{(i)} F_{k\text{-}1}^{(i)}, \\
&C_k^{(i)} = K_k^{(i)} H_k^{(i)} G_{k\text{-}1}^{(i)} Q_{k\text{-}1}^{(i)^{1/2}}, \quad D_k^{(i)} = K_k^{(i)} R_k^{(i)^{1/2}}. \\
&\qquad h,\ i,\ j = 1,...,N
\end{aligned} \tag{25}
$$

$$
\begin{aligned}
&\tilde{P}_k^{(ij)} = \sum_{h=1}^{N} p_h P_k^{(hij)} \\
&P_k^{(jij)} = L_{xx,k}^{(hh)} - L_{x\hat{x},k}^{(hj)} - L_{xx,k}^{(hj)^T} + L_{\hat{x}\hat{x},k}^{(ij)}, \\
&\qquad h,\ i,\ j = 1,...,N
\end{aligned} \tag{26}
$$

$$
\sum_{i=1}^{N} c_k^{(i)} \left[\tilde{P}_k^{(ij)} - \tilde{P}_k^{(iN)} \right] = 0,
$$

$$
j = 1,...,N-1, \quad \sum_{i=1}^{N} c_k^{(i)} = I_n.
$$

We may note that the local Kalman gains $K_k^{(i)}$ in (24), the local error cross-covariances $P_k^{(hij)}$ and $\tilde{P}_k^{(ij)}$ in (25) and (26), and finally the SF's weights $c_k^{(i)}$ in (26) can be *pre-computed*, since they do not depend on the current measurements y_0^k, but only on the system matrices $F_k^{(i)}$, $G_k^{(i)}$, $H_k^{(i)}$, noise statistics $Q_k^{(i)}$, $R_k^{(i)}$, initial conditions $\bar{x}_0^{(i)}$, $P_0^{(i)}$, and also on the values of the parameter $\theta = \theta_1, \ldots, \theta_N$, which are part of system models (1) and (2).

Thus, once the measurement schedule has been settled, the real-time implementation of the SF requires only the computation of the local Kalman estimates $\hat{x}_k^{(1)}, \ldots, \hat{x}_k^{(N)}$ and the final fusion of the suboptimal estimate $\hat{x}_k^{\text{sub}}$ using only current measurements y_k.

Part 2 ("On-line equations")**:**

$$
\begin{aligned}
&\hat{x}_k^{(\mathrm{i})} = F_k^{(i)} \hat{x}_{k-1}^{(i)} + K_k^{(i)} \left[y_k - H_k^{(i)} F_k^{(i)} \hat{x}_{k-1}^{(i)} \right], \\
&\hat{x}_0^{(i)} = \bar{x}_0^{(i)}, \\
&i = 1,..., N, \\
&\hat{x}_k^{\text{sub}} = c_k^{(1)} \hat{x}_k^{(1)} + \cdots + c_k^{(N)} \hat{x}_k^{(N)}.
\end{aligned} \tag{27}
$$

Remark 2: Since θ takes a finite number of values $\theta = \theta_1, \ldots, \theta_N$, the local Kalman estimates (27) are separated for values of $i = 1, \ldots, N.$ Each estimate $\hat{x}_k^{(\mathrm{i})}$ is found independently of other estimates $\hat{x}_k^{(1)}, \ldots, \hat{x}_k^{(i\text{-}1)}$, $\hat{x}_k^{(i+1)}, \ldots, \hat{x}_k^{(N)}$. Therefore, it can be evaluated in parallel. The SF is also robust, since it can be corrected even if one of the parallel local Kalman estimate $\hat{x}_k^{(\mathrm{i})}$ diverges. In this case, the corresponding weight $c_k^{(i)}$ in the weighted sum (27) will tend to be zero $\left(c_k^{(i)} \to 0\right)$, thereby indicating that the diverging estimate $\hat{x}_k^{(\mathrm{i})}$ will be discarded in the weighted sum (27).

In several applications, there may be a nonzero probability that the measurement model takes N sensor modes. This kind of problem is called a joint classification-estimation problem [1]. One such application is a joint detection-estimation when the tracking of a target trajectory in space is considered where the target may or may not be present, so that the target must be detected as well as the trajectory tracked [1,12]. The SF can be used for these applications.

5. A JOINT CLASSIFICATION-ESTIMATION PROBLEM

In this section, we consider special linear dynamic systems only with measurement uncertainties

$$\begin{aligned} x_{k+1} &= F_k x_k + G_k v_k, \quad v_k \sim N\left(0, Q_k\right), \\ x_0 &\sim N\left(\bar{x}_0, P_0\right), \\ y_k &= H_k\left(\theta\right) + w_k, \quad w_k \sim N\left(0, R_k\left(\theta\right)\right), \end{aligned} \tag{28}$$

where F_k, G_k, Q_k, P_0, and $\bar{x}_0$ are known matrices and initial mean vector, respectively; $H_k\left(\theta\right)$ and $R_k\left(\theta\right)$ are known matrices time-functions, but including the unknown parameter vector θ, which takes a finite set of values (sensor modes) (2).

In this case the state x_k does not depend onthe unknown parameter θ, hence the mean-square criterion (14) takes the following simple form

$$\begin{aligned} J_k\left(c_k^{(1)},\dots,c_k^{(N)}\right) &= E\left\| x_k - \hat{x}_k^{sub} \right\|^2 \\ &= tr\left\{ E\left[x_k - \hat{x}_k^{sub} \right] \left[x_k - \hat{x}_k^{sub} \right]^T \right\} \\ &= tr\left\{ E\left[\sum_{i=1}^{N} c_k^{(i)} \left(x_k - \hat{x}_k^{(i)} \right) \right] \left[\sum_{j=1}^{N} c_k^{(j)} \left(x_k - \hat{x}_k^{(j)} \right) \right]^T \right\} \\ &= tr\left\{ \sum_{i,j=1}^{N} c_k^{(i)} P_k^{(ij)} c_k^{(j)T} \right\} \to \min_{c_k^{(i)}}, \end{aligned} \tag{29}$$

where

$$P_k^{(ij)} = E\left[e_k^{(i)} \left(e_k^{(j)} \right)^T \right], \quad i, j = 1, \dots, N \tag{30}$$

are the local error cross-covariances of the filtering errors $e_k^{(i)} = x_k - \hat{x}_k^{(i)}$.

For system (28) and optimization criterion (29), the SF is specified by Theorem 3.

Theorem 3: Let $\hat{x}_k^{(i)} = \hat{x}_k\left(\theta_i\right)$ be the local Kalman estimates in (8). Then

(i) The optimal weights $c_k^{(1)},\dots,c_k^{(N)}$ minimizing criterion (29) are given by the equations

$$\sum_{i=1}^{N} c_k^{(i)} \left[P_k^{(ij)} - P_k^{(iN)} \right] = 0, \quad j = 1, \dots, N-1, \quad \sum_{i=1}^{N} c_k^{(i)} = I_n \tag{31}$$

(ii) The overall error covariance P_k^{sub} is given by

$$P_k^{sub} = \sum_{i,j=1}^{N} c_k^{(i)} P_k^{(ij)} \left(c_k^{(j)} \right)^T, \tag{32}$$

where $P_k^{(ii)} = P_k^{(i)}$ is the local error covariance determined by the standard Kalman filter equations (24), and $P_k^{(ij)}$, $i \neq j$ is the local cross-covariance satisfying the following recursive:

$$\begin{aligned} P_k^{(ij)} &= \left[I_n - K_k^{(i)} H_k^{(i)} \right] \\ &\times \left[F_{k-1}^{(i)} P_{k-1}^{(ij)} F_{k-1}^{(j)T} + G_{k-1}^{(i)} Q_{k-1}^{(i)} G_{k-1}^{(j)T} \right] \\ &\times \left[I_n - K_k^{(j)} H_k^{(j)} \right]^T, \\ P_0^{(ij)} &= P_0, \quad i, j = 1, \dots, N, \quad i \neq j. \end{aligned} \tag{33}$$

6. EXAMPLES

6.1. Joint detection-estimation: damper harmonic oscillator motion

The system model of the harmonic oscillator is considered in [10, p. 104]:

$$\dot{x}_t = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\alpha \end{bmatrix} x_t + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_t, \quad 0 \le t \le 1, \tag{34}$$

where $x_t = \begin{bmatrix} x_{1,t} & x_{2,t} \end{bmatrix}^T$ and $x_{1,t}$ is position, and $x_{2,t}$ is velocity, v_t is a scalar zero-mean white Gaussian noise with known intensity q, $E\left(v_t v_s\right) = q\delta\left(t-s\right)$, $x_0 \sim N\left(\bar{x}_0, P_0\right)$.

In this paper, we consider discrete time systems. So, we change the continuous system model into a discrete version.

$$\begin{aligned} x_{k+1} &= \begin{bmatrix} 1 & \Delta t \\ -\omega_n^2 \Delta t & 1 - 2\alpha\Delta t \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_k, \\ k &= 1, 2, \dots 100, \end{aligned} \tag{35}$$

where $v_k \sim N(0, q\Delta t)$.

The position is observed with uncertainty. Then the measurement model is written as

$$y_k = \theta x_{1,k} + w_k, \quad k = 1, 2, \dots 100, \tag{36}$$

where $\{w_k\}$ is zero-mean white Gaussian sequence, $w_k \sim N\left(0, r\right)$, and the unknown parameter θ takes only two values, i.e.,

$$\theta = \begin{cases} \theta_1 = 1, & p(\theta_1) = 0.5 \\ \theta_2 = 0, & p(\theta_2) = 0.5. \end{cases} \tag{37}$$

This represents the measurement model which takes two sensor modes with $\theta_1 = 1$ (signal-present) and $\theta_2 = 0$ (signal-absent).

We compare two adaptive filters: the optimal AKF

$$\hat{x}_k^{\text{opt}} = \tilde{c}_k^{(1)} \hat{x}_k(\theta_1) + \tilde{c}_k^{(2)} \hat{x}_k(\theta_2), \quad \tilde{c}_k^{(i)} = p\left(\theta_i \middle| y_0^k\right), \quad i = 1,2 \tag{38}$$

and the SF

$$\hat{x}_k^{\text{sub}} = c_k^{(1)} \hat{x}_k(\theta_1) + c_k^{(2)} \hat{x}_k(\theta_2). \tag{39}$$

In this case $N = 2$, and the solution of the linear algebraic equations (31) coincides with the Bar-Shalom-Campo formulas for the optimal combination of two correlated estimates [19],

$$c_k^{(1)} = \left(P_k^{(22)} - P_k^{(21)}\right)\left(P_k^{(11)} + P_k^{(22)} - P_k^{(12)} - P_k^{(21)}\right)^{-1}, \quad c_k^{(2)} = \left(P_k^{(11)} - P_k^{(12)}\right)\left(P_k^{(11)} + P_k^{(22)} - P_k^{(12)} - P_k^{(21)}\right)^{-1}. \tag{40}$$

The performance of the SF is expressed in terms of computation load and loss in estimation accuracy with respect to the AKF. The model parameters, noises statistics, and initial conditions are subject to

$$\omega_n^2 = 0.64, \quad \alpha = 0.16, \quad q = 1, \quad r = 0.1, \quad \bar{x}_0 = [0.0 \quad 0.0]^T, \quad P_0 = \text{diag}[2.0 \quad 1.0], \quad \Delta t = 0.01. \tag{41}$$

Two cases were considered: in the first case, $\theta_1 = 1$ is the true parameter value in (36); in the second case, $\theta_2 = 0$ is the true parameter value. Figs. 1-4 present the time histories of the filter characteristics for the first case. Such time histories are a perfect analogy to the second case. In Figs. 1 and 2 we show the mean-square errors (MSEs) for the

AKF $P_{ii,k}^{\text{opt}} = E\left[\left(x_{i,k} - \hat{x}_{i,k}^{\text{opt}}\right)^2\right]$ and

SF $P_{ii,k}^{\text{sub}} = E\left[\left(x_{i,k} - \hat{x}_{i,k}^{\text{sub}}\right)^2\right]$, respectively.

As we see from Fig. 1, the difference between the optimal and suboptimal MSEs for position is negligible for the steady-state regime. Also the suboptimal MSE for velocity in Fig. 2 is within a few percentages of the optimum one. The numerical simulations were performed using a computer with the following specifications: Intel® Pentium® 4 CPU 2.8GHz 512Mb RAM. The computation time for evaluation of the suboptimal state estimate $\hat{x}_k^{\text{sub}}$ is 3.8 times less than for optimal estimate $\hat{x}_k^{\text{opt}}$. This is

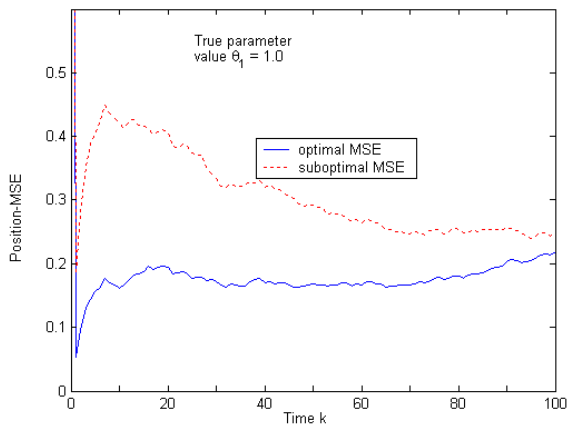

Fig. 1. Comparison of MSEs of position estimates for AKF and SF: $P_{11,k}^{\text{opt}}$ (solid line); $P_{11,k}^{\text{sub}}$ (dashed line).

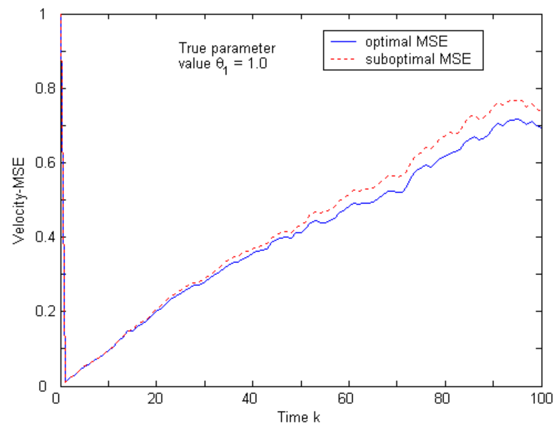

Fig. 2. Comparison of MSEs of velocity estimate for AKF and SF: $P_{22,k}^{\text{opt}}$ (solid line); $P_{22,k}^{\text{sub}}$ (dashed line).

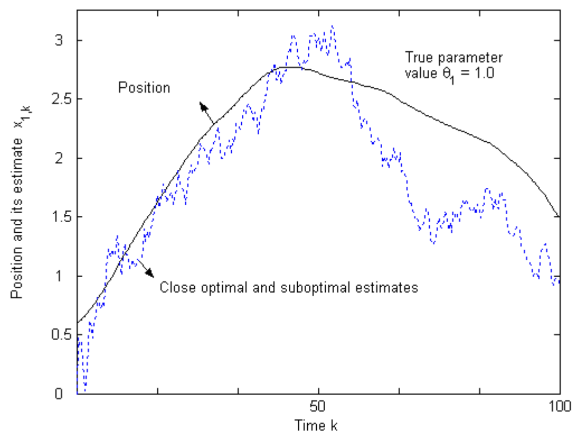

Fig. 3. Optimal and suboptimal position estimates: Real position $x_{1,k}$ (solid line); $\hat{x}_{1,k}^{\text{opt}}$ (dotted line); $\hat{x}_{1,k}^{\text{sub}}$ (dashed line).

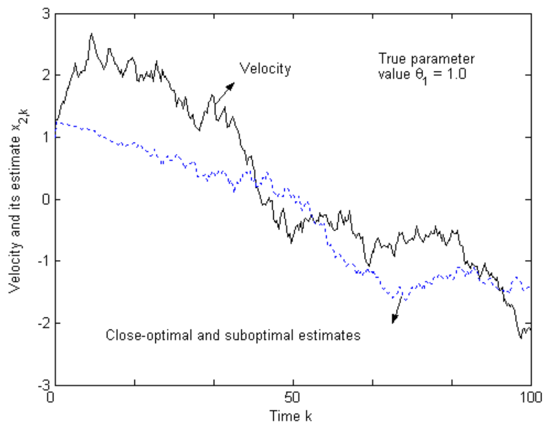

Fig. 4. Optimal and suboptimal velocity estimates: Real velocity $x_{2,k}$ (solid line); $\hat{x}_{2,k}^{\text{opt}}$ (dotted line); $\hat{x}_{2,k}^{\text{sub}}$ (dashed line).

due to the fact that the suboptimal weights $c_k^{(i)}$ in (40) are pre-computed. It provides the best balance between the computational efficiency and the desired estimation accuracy. Figs. 3 and 4 present the result of simulations of the optimal and suboptimal positions and velocity estimates. Through Figs. 3 and 4 we can also check the accuracy of SF from seeing that the optimal and suboptimal estimates are very close to each other.

6.2. Vehicle motion model with unknown initial statistics

Consider a four-dimensional system with state x_k,

$$x_k = \begin{bmatrix} r_x \\ r_y \\ V_x \\ V_y \end{bmatrix} = \begin{bmatrix} x-\text{position} \\ y-\text{position} \\ x-\text{velocity} \\ y-\text{velocity} \end{bmatrix}, \quad (42)$$

which represents a vehicle motion constrained to a plane according to the following equation [1, p. 89]

$$x_{k+1} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} x_k + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} v_k, \quad v_k = \begin{bmatrix} \xi_k \\ \eta_k \end{bmatrix}, \quad k = 1, 2, \ldots, T, \quad (43)$$

where $\xi_k \sim N(0, \quad q_k)$, $\eta_k \sim N(0, \quad q_\eta)$.

Assume that the initial state x_0 is zero-mean white Gaussian vector,

$$x_0 \sim N(\bar{x}_0, P_0). \quad (44)$$

The measurement equation is

$$y_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x_k + w_k, \quad k = 1, 2, \ldots, T, \quad (45)$$

where the measurement error $\{w_k \in \mathbf{R}^2\}$ is zero-mean white Gaussian sequences with covariance $R_k = \text{diag}(r_1 \quad r_2)$.

We now wish to apply Theorems 1 and 2 to a case where the initial mean $\bar{x}_0 \in \mathbf{R}^4$ is unknown. Let the prior information on $\bar{x}_0$ be given by four hypotheses H_i, i = 1, 2, 3, 4:

$$\begin{aligned} \bar{x}_0(H_1) &= \bar{x}(\theta_1) = [-14 \quad 6 \quad 0.014 \quad -0.006]^T, \\ \bar{x}_0(H_2) &= \bar{x}(\theta_2) = [-15 \quad 7 \quad 0.018 \quad -0.005]^T, \\ \bar{x}_0(H_3) &= \bar{x}(\theta_3) = [-13 \quad 4 \quad 0.020 \quad -0.001]^T, \\ \bar{x}_0(H_4) &= \bar{x}(\theta_4) = [-12 \quad 5 \quad 0.010 \quad -0.005]^T \end{aligned} \quad (46)$$

with the equal prior probabilities $p(\theta_i) = 0.25$ for $i = 1, 2, 3, 4$. Initial covariance P_0 is the same for all hypotheses, i.e., $P_0 = \text{diag}\left[1 \quad 1 \quad 10^{-6} \quad 10^{-6}\right]$, T=40.

We describe here the results of simulations of two filters: the optimal AKF and SF. Figs. 5-6 present the time histories of the optimal and suboptimal estimates of the state variables V_x, V_y, whereas Figs. 7 and 8 exhibit the corresponding MSEs for the case when the true initial mean $\bar{x}_0 = \bar{x}_0(H_1)$ is assumed to be of hypothesis H_1. Such time histories are perfectly

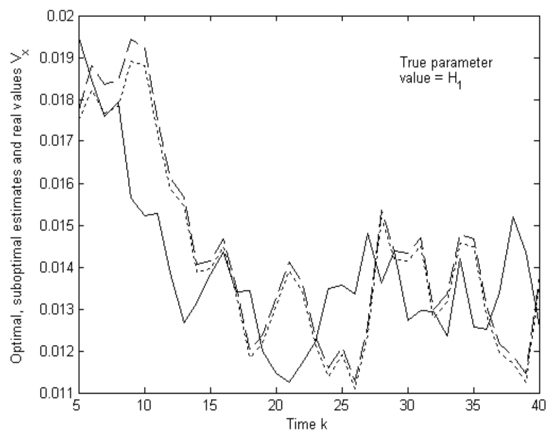

Fig. 5. Optimal and suboptimal estimates of velocity: real velocity V_x (solid line); $\hat{V}_x^{\text{opt}}$ (dotted line); $\hat{V}_x^{\text{sub}}$ (dashed line).

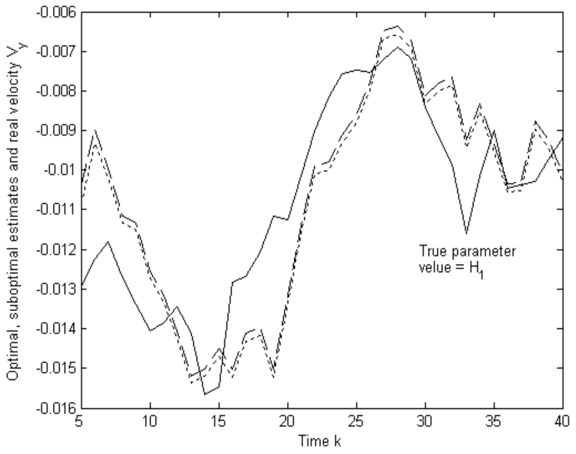

Fig. 6. Optimal and suboptimal estimates of velocity: real velocity V_y (solid line); $\hat{V}_y^{\text{opt}}$ (dotted line); $\hat{V}_y^{\text{sub}}$ (dashed line).

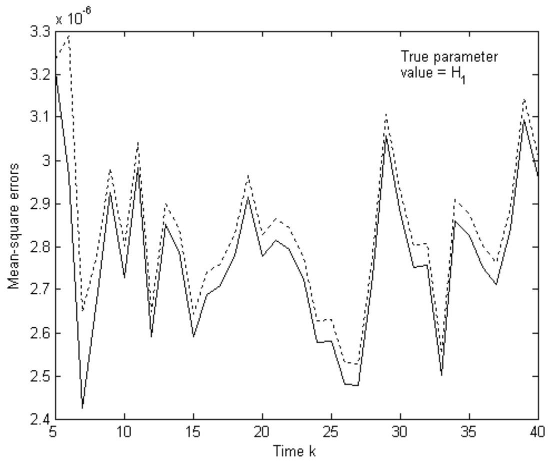

Fig. 7. MSEs of the optimal $\hat{V}_x^{\text{opt}}$ (solid line) and suboptimal $\hat{V}_x^{\text{sub}}$ (dotted line) estimates.

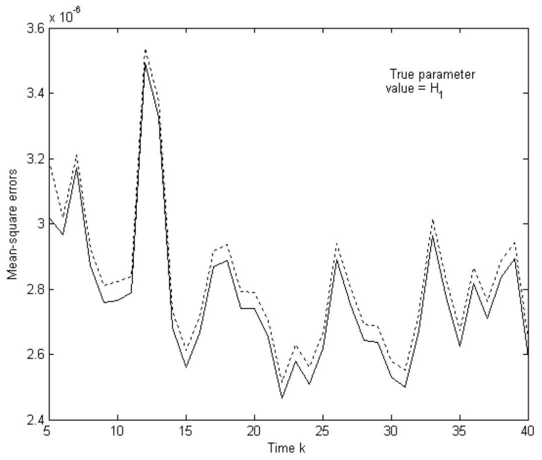

Fig. 8. MSEs of the optimal $\hat{V}_y^{\text{opt}}$ (solid line) and suboptimal $\hat{V}_y^{\text{sub}}$ (dotted line) estimates.

similar to other hypotheses. The computation time for evaluation of $\hat{x}_k^{\text{sub}}$ is 10.75 times less than for $\hat{x}_k^{\text{opt}}$.

7. CONCLUSION

In this paper, we have designed a new suboptimal filter for linear discrete-time dynamic systems with uncertainties. This filter represents a linear combination of local Kalman filters with weights depending only on time instance. Each local Kalman filter is fused by the minimum mean-square criterion. The proposed low-complexity filter has a parallel structure and thus it is suitable for parallel processing. Simulation results demonstrate high accuracy of the designed filter.

APPENXIX A: PROOF OF THE THEOREM 1

Using (13) and (19) the criterion $J_k = E\left\| x_k - \hat{x}_k^{\text{sub}} \right\|^2$ can be rewritten as

$$\begin{aligned} J_k &= \sum_{h=1}^{N} p(\theta_h) E\left\| x_k(\theta_h) - \hat{x}_k^{\text{sub}} \right\|^2 \\ &= \sum_{h=1}^{N} p_h tr\left\{ E\left[x_k(\theta_h) - \hat{x}_k^{\text{sub}} \right] \left[x_k(\theta_h) - \hat{x}_k^{\text{sub}} \right]^T \right\} \\ &= \sum_{h=1}^{N} p_h tr\left\{ \sum_{i,j=1}^{N} c_k^{(i)} E\left[e_k^{(hi)} \left(e_k^{(hj)} \right)^T \right] \left(c_k^{(j)} \right)^T \right\} \\ &= tr\left\{ \sum_{i,j=1}^{N} c_k^{(i)} \tilde{P}_k^{(ij)} \left(c_k^{(j)} \right)^T \right\} \to \min_{c_k^{(i)}}, \end{aligned} \qquad \text{(A1)}$$

where

$$\tilde{P}_k^{(ij)} = \sum_{h=1}^{N} p_h E\left[e_k^{(hi)} \left(e_k^{(hj)} \right)^T \right] = \sum_{h=1}^{N} p_h P_k^{(hij)}. \qquad \text{(A2)}$$

The formulas (A1) and (A2) give the overall error covariance (16).

Next substituting the expression

$$c_k^{(N)} = I_n - \left[c_k^{(1)} + \cdots + c_k^{(N\text{-}1)} \right]$$

into (A1) we obtain

$$\begin{aligned} J_k = tr\Bigg\{ &\sum_{i,j=1}^{N\text{-}1} c_k^{(i)} \tilde{P}_k^{(ij)} \left(c_k^{(j)} \right)^T + \sum_{i=1}^{N\text{-}1} \left[c_k^{(i)} \tilde{P}_k^{(iN)} + \tilde{P}_k^{(Ni)} \left(c_k^{(i)} \right)^T \right] \\ &- \sum_{i,j=1}^{N\text{-}1} \left[c_k^{(i)} \tilde{P}_k^{(iN)} \left(c_k^{(j)} \right)^T + c_k^{(j)} \tilde{P}_k^{(Ni)} \left(c_k^{(i)} \right)^T \right] \\ &+ \tilde{P}_k^{(NN)} - \left[\sum_{i=1}^{N\text{-}1} c_k^{(i)} \right] \tilde{P}_k^{(NN)} \text{-} \tilde{P}_k^{(NN)} \left[\sum_{j=1}^{N\text{-}1} c_k^{(j)} \right]^T \end{aligned}$$

$$\left. - \sum_{i,j=1}^{N-1} c_k^{(i)} \tilde{P}_k^{(NN)} \left(c_k^{(j)} \right)^T \right\} \to \min_{c_k^{(1)},\ldots,c_k^{(N-1)}} \quad \text{(A3)}$$

Differentiating each summand of the criterion J_k in (A3) with respect to $c_k^{(1)},\ldots,c_k^{(N\text{-}1)}$ using the formulas

$$\begin{aligned} &\tilde{P}_k^{(\text{ij})} = \left(\tilde{P}_k^{(ji)} \right)^T, \quad \tilde{P}_k^{(ii)} = \left(\tilde{P}_k^{(ii)} \right)^T, \\ &\frac{\partial}{\partial \text{c}_k^{(i)}} \left[tr\left(c_k^{(i)} \tilde{P}_k^{(ij)} \right) \right] = \left(\tilde{P}_k^{(ij)} \right)^T, \\ &\frac{\partial}{\partial \text{c}_k^{(i)}} \left[tr\left(\tilde{P}_k^{(ij)} c_k^{(i)} \right) \right] = \tilde{P}_k^{(\text{ij})}, \\ &\frac{\partial}{\partial \text{c}_k^{(i)}} \left[tr\left(c_k^{(i)} \tilde{P}_\text{k}^{(ij)} \left(c_k^{(i)} \right)^T \right) \right] = c_k^{(i)} \left[\tilde{P}_k^{(ij)} + \left(\tilde{P}_k^{(ij)} \right)^T \right] \end{aligned} \quad \text{(A4)}$$

and then setting the result to zero,

$$\frac{\partial J_k}{\partial c_k^{(i)}} = 0, \quad i = 1,\ldots,N\text{-}1$$

we obtain the linear algebraic equations (15) for unknown weights $c_k^{(1)},\ldots,c_k^{(N)}$.

This completes the proof of Theorem 1.

APPENDIX B: PROOF OF THE THEOREM 2

Representation (19) immediately follows from (18). The derivation of (22) and (23) is based on the recursive for the state $x_k^{(h)} \overset{\text{def}}{=} x_k\left(\theta_h\right)$ and local estimate $\hat{x}_k^{(j)} \overset{\text{def}}{=} \hat{x}_k\left(\theta_j\right)$. Using (1) and (9) and notations (13) we obtain

$$x_{k+1}^{(h)} = F_k^{(h)} x_k^{(h)} + G_k^{(h)} {Q_k^{(h)}}^{1/2} \tilde{v}_k, \quad \text{(B1)}$$

$$\begin{aligned} \hat{x}_{k+1}^{(j)} &= F_{k+1}^{(j)} \hat{x}_k^{(j)} + K_{k+1}^{(j)} \left[y_{k+1}^{(j)} - H_{k+1}^{(j)} F_{k+1}^{(j)} \hat{x}_k^{(j)} \right] \\ &= \left[F_{k+1}^{(j)} - K_{k+1}^{(j)} H_{k+1}^{(j)} F_{k+1}^{(j)} \right] \hat{x}_{k+1}^{(j)} \\ &\quad + K_{k+1}^{(j)} \left[H_{k+1}^{(j)} x_{k+1}^{(j)} + {R_{k+1}^{(j)}}^{1/2} \tilde{w}_{k+1} \right] \\ &= \left[I_n - K_{k+1}^{(j)} H_{k+1}^{(j)} \right] F_{k+1}^{(j)} \hat{x}_{k+1}^{(j)} \\ &\quad + K_{k+1}^{(j)} \left[H_{k+1}^{(j)} \left(F_k^{(j)} x_k^{(j)} + G_k^{(j)} {Q_k^{(j)}}^{1/2} \tilde{v}_k \right) + R_{k+1}^{(j)} \tilde{w}_{k+1} \right] \\ &= A_{k+1}^{(\text{j})} \hat{x}_k^{(j)} + B_{k+1}^{(j)} x_k^{(j)} + C_{k+1}^{(j)} \tilde{v}_\text{k} + D_{k+1}^{(j)} \tilde{w}_{k+1}, \end{aligned} \quad \text{(B2)}$$

where $K_{k+1}^{(j)}$ is the local Kalman gains (10), $\tilde{v}_k \sim N\left(0, I_n\right)$ and $\tilde{w}_{k+1} \sim N\left(0, I_m\right)$ are Gaussian white noise sequences with identity covariances, and $A_{k+1}^{(j)}$, $B_{k+1}^{(j)}$, $C_{k+1}^{(j)}$, and $D_{k+1}^{(j)}$ are given by (23).

According to the assumption that the white noises $\tilde{v}_k$ and $\tilde{w}_{k+1}^{(j)}$ are mutually uncorrelated, equations (B1) and (B2) yield linear difference equations for the second-order moments (21) and (22).

This completes the proof of Theorem 2.

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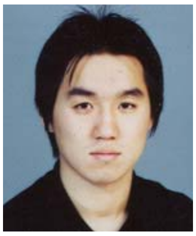

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