An Improvement on Robust H_∞ Control for Uncertain Continuous-Time Descriptor Systems

Hung-Jen Lee, Shih-Wei Kau, Yung-Sheng Liu, Chun-Hsiung Fang*, Jian-Liung Chen, Ming-Hung Tsai, and Li Lee*

Abstract: This paper proposes a new approach to solve robust H_∞ control problems for uncertain continuous-time descriptor systems. Necessary and sufficient conditions for robust H_∞ control analysis and design are derived and expressed in terms of a set of LMIs. In the proposed approach, the uncertainties are allowed to appear in all system matrices. Furthermore, a couple of assumptions that are required in earlier design methods are not needed anymore in the present one. The derived conditions also include several interesting results existing in the literature as special cases.

Keywords: Descriptor systems, H_∞ control, LMI, robust control, uncertainties.

1. INTRODUCTION

It is well known that the descriptor system (also referred to singular systems, or generalized state-space systems, or implicit systems, or semistate systems in the literature) described by the following model

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{1}$$

has higher capability in describing a physical system. In (1), the matrix $E \in \mathbb{R}^{n\times n}$ may be singular. Assume *rank(E)=r* and denote by p the degree of the characteristic polynomial $|sE - A|$. For descriptor systems, it is interesting to note that $0 \le p \le r \le n$. The system (1) is termed to be regular and impulse-free if $p = r$ and termed to be admissible if it is $p = r$ and all roots of $|sE - A| = 0$ are Hurwitz stable.

Manuscript received June 29, 2005; revised January 25, 2006; accepted February 20, 2006. Recommended by Editorial Board member Seung-Bok Choi under the direction of past Editor-in-Chief Myung Jin Chung. This work was supported by National Science Council of Taiwan under Grant No. NSC-92-2213-E-110-024 and NSC-93-2745- E-151-001.

Hung-Jen Lee, Shih-Wei Kau, Chun-Hsiung Fang, and Yung-Sheng Liu are with the Department of Electrical and Electronics Engineering, National Kaohsiung University of Applied Sciences, 415 Chien-Kung Road, Kaohsiung 807, Taiwan (e-mails: {hjlee, shiewkau, chfang}@cc.kuas.edu.tw, sam.ys.liu@foxconn.com).

Jian-Liung Chen, Ming-Hung Tsai, and Li Lee are with the Department of Electrical Engineering, National Sun Yat-Sen University, Kaohsiung 804, Taiwan (e-mails: {jlchen, mhtsai, leeli}@mail.ee.nsysu.edu.tw).

* Corresponding authors.

Descriptor-system models are often more convenient and natural than standard state-space models in the description of interconnected large-scalar systems [3], economic systems [12], electrical network [14], power systems [1], chemical processes [9], and so on [10]. This is the reason why descriptor systems have attracted much interest in recent years [4-13].

The H_∞ control problem of descriptor systems has been addressed by several researchers. For instance, to solve H_∞ control problem, the concept of J-spectral factorization and*(J,J')*-spectral factorization had been extended to descriptor systems in [7] and [15]. Based on the generalized algebraic Riccati equation, necessary and sufficient conditions for H_∞ control of continuous-time and discrete-time descriptor systems were given in [8] and [18], respectively. Recently, because of the numerical efficiency of LMI, the H_∞ control problem of descriptor systems was resolved by using LMI approaches [13,5-20]. When descriptor systems contain uncertainties, the robust H_∞ control result currently available in the literature is very limited. Reference [6] proposed a necessary and sufficient LMI-based condition for robust H_∞ control of uncertain descriptor systems. Based on it and under some assumptions including the admissibility of nominal system, necessary and sufficient GARI-based conditions are developed to solve the state feedback and the dynamic output feedback synthesis problems. However, as indicated in [16], all results of [6] are only *sufficient* due to an incorrect proof of the necessary statement. Differently, an LMI-based approach is proposed in [16] to tackle exactly the same problem as [6]. However, all results obtained in [16] are still *sufficient* only.

In this paper, a new LMI approach is proposed for solving the same problem mentioned above. There are four major contributions in this paper. (I) Necessary and sufficient conditions for robust H_∞ control are derived. Before this presentation, only sufficient conditions for the same problem were obtained. (II) No assumption as needed in [6] is required. (III) The system model considered in this paper is more general since all system matrices are allowed to have uncertainties. In [6,16], only the state matrix contains uncertainties. (IV) The present result includes the major result of [13,18] as special cases.

2. PROBLEM FOMULATION

Consider an uncertain continuous-time descriptor system

$$\begin{aligned} E\dot{x}(t) &= A_\Delta x(t) + B_{w\Delta} w(t) + B_{u\Delta} u(t), \\ z(t) &= C_{z\Delta} x(t) + D_{zw\Delta} w(t) + D_{zu\Delta} u(t), \\ y(t) &= C_{y\Delta} x(t) + D_{yw\Delta} w(t) + D_{yu\Delta} u(t), \end{aligned} \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $w(t) \in \mathbb{R}^{m_w}$ the exogenous input, $u(t) \in \mathbb{R}^{m_u}$ the control input, $z(t) \in \mathbb{R}^{q_z}$ the controlled output, and $y(t) \in \mathbb{R}^{q_y}$ the measured output. Assume the system matrices A_Δ, $B_{w\Delta}$, $B_{u\Delta}$, $C_{z\Delta}$, $D_{zw\Delta}$, $D_{zu\Delta}$, $C_{y\Delta}$, $D_{yw\Delta}$, and $D_{yu\Delta}$ are described as

$$\begin{bmatrix} A_\Delta & B_{w\Delta} & B_{u\Delta} \\ C_{z\Delta} & D_{zw\Delta} & D_{zu\Delta} \\ C_{y\Delta} & D_{yw\Delta} & D_{yu\Delta} \end{bmatrix} \triangleq \begin{bmatrix} A & B_w & B_u \\ C_z & D_{zw} & D_{zu} \\ C_y & D_{yw} & D_{yu} \end{bmatrix} + \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} \Delta \begin{bmatrix} J_1 & J_2 & J_3 \end{bmatrix}, \tag{3}$$

where $A \in \mathbb{R}^{n\times n}$, $B_w \in \mathbb{R}^{n\times m_w}$, $B_u \in \mathbb{R}^{n\times m_u}$, $C_z \in \mathbb{R}^{q_z\times n}$, $D_{zw} \in \mathbb{R}^{q_z\times m_w}$, $D_{zu} \in \mathbb{R}^{q_z\times m_u}$, $C_y \in \mathbb{R}^{q_y\times n}$, $D_{yw} \in \mathbb{R}^{q_y\times m_w}$ and $D_{yu} \in \mathbb{R}^{q_y\times m_u}$ are constant matrices representing the nominal system. $H_1 \in \mathbb{R}^{n\times s}$, $H_2 \in \mathbb{R}^{q_z\times s}$, $H_3 \in \mathbb{R}^{q_y\times s}$, $J_1 \in \mathbb{R}^{s\times n}$, $J_2 \in \mathbb{R}^{s\times m_w}$, and $J_3 \in \mathbb{R}^{s\times m_u}$ provide structure information of uncertainties. $\Delta \in \mathbb{R}^{s\times s}$ is a norm-bounded uncertain matrix satisfying

$$\Delta^T \Delta \le I_s. \tag{4}$$

References [6,16] considered the robust H_∞ control problem of the following special system

$$\begin{aligned} E\dot{x}(t) &= \left(A + H_1 \Delta J_1\right) x(t) + B_w w(t) + B_u u(t), \\ z(t) &= C_z x(t) + D_{zu} u(t), \\ y(t) &= C_y x(t) + D_{yw} w(t), \end{aligned} \tag{5}$$

in which uncertainties appear only on the state matrix. Next definition and lemma are directly quoted from [6].

Definition 1 [6, Definition 2.5]: Given $\gamma > 0$, the unforced system (5) (i.e. $u(t) = 0$) is stated to be quadratically admissible with disturbance attenuation γ for all uncertainties Δ if there exists a nonsingular matrix X such that for all Δ

$$\begin{aligned} & E^T X = X^T E \ge 0, \\ & \left(A + H_1 \Delta J_1\right)^T X + X^T \left(A + H_1 \Delta J_1\right) \\ & \quad + \frac{1}{\gamma^2} X^T B_w B_w^T X + C_z^T C_z < 0. \end{aligned} \tag{6}$$

Lemma 1 [6, Lemma 2.6]: Consider the system in (5) and a prescribed scalar $\gamma > 0$. Assume $\Delta^T \Delta \le \rho^2 I_s$ where ρ is a given real number. Then (6) holds for all Δ if and only if there exists a nonsingular matrix Y, independent of Δ, such that

$$\begin{aligned} & E^T Y = Y^T E \ge 0, \\ & A^T Y + Y^T A + \frac{1}{\gamma^2} Y^T \begin{bmatrix} B_w & \gamma H_1 \end{bmatrix} \begin{bmatrix} B_w^T \\ \gamma H_1^T \end{bmatrix} Y \\ & \quad + \begin{bmatrix} C_z^T & \rho J_1^T \end{bmatrix} \begin{bmatrix} C_z \\ \rho J_1 \end{bmatrix} < 0. \end{aligned} \tag{7}$$

As mentioned in [16] that, actually, Lemma 1 is only sufficient because an obvious argument error appears in the proof of necessity. More precisely, the inequality (20) of [6] can't be as claimed to be derived from substituting (19) into (17a) in [6]. The following simple example shows a contradiction between Definition 1 and Lemma 1. Let $\gamma = 1$, $\rho = 1$ and $E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $A = \begin{bmatrix} -1.2 & 0 \\ 0 & 1 \end{bmatrix}$, $B_w = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $C_z = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $H_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $J_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $\Delta \in \begin{bmatrix} -1 & 1 \end{bmatrix}$. Note that $X = \begin{bmatrix} 1.2 & 0 \\ 0 & -0.2 \end{bmatrix}$ satisfies (6) for all $\Delta \in \left[-1,\ 1\right]$. Hence, by Definition 1, the corresponding system is quadratically admissible with disturbance attenuation 1. However, to check feasibility of (7), by letting $Y = \begin{bmatrix} y_1 & y_2 \\ y_3 & y_4 \end{bmatrix}$ in (7). The first condition of (7) implies $y_1 \ge 0$ and $y_2 = 0$ and the second condition of (7) gives

$$\begin{bmatrix} y_1^2 - 2.4y_1 + y_3^2 + 2 & y_3(1+y_4) \\ y_3(1+y_4) & y_4^2 + 2y_4 \end{bmatrix} < 0,$$

which, by Schur complement and some simple algebra, is equivalent to

$$y_4^2 + 2y_4 < 0 \text{ and } y_1^2 - 2.4y_1 + 2 < \frac{y_3^2}{y_4^2 + 2y_4}$$

or

$$y_4^2 + 2y_4 < 0 \text{ and } (y_1 - 1.2)^2 + 0.56 < \frac{y_3^2}{y_4^2 + 2y_4}.$$

Since it is impossible to find three real numbers y_1, y_3, and y_4 to satisfy the above two inequalities simultaneously, the inequality (7) has no solution at all. This obviously indicates the result of [6] is incorrect. Since all the other results in [6] are based on Lemma 1, they are only sufficient, too.

The goal of this paper is to derive necessary and sufficient LMI-based conditions for robust H_∞ control of (2), which is more general than (5). The new conditions are applied to design two types of controllers so that the closed-loop system is quadratically admissible with disturbance attenuation γ. For solving the robust H_∞ control problem of (2), Definition 1 is extended to a more general case as follows.

Definition 2: Given $\gamma > 0$, the unforced uncertain descriptor system (2) (i.e. $u(t) = 0$) is said to be quadratically admissible with disturbance attenuation γ for all uncertainties Δ satisfying (4) if there exists a nonsingular matrix P such that for all Δ

$$E^T P = P^T E \geq 0, \tag{8}$$

$$A_\Delta^T P + P^T A_\Delta + \left(P^T B_{w\Delta} + C_{z\Delta}^T D_{zw\Delta}\right) \tag{9}$$

$$\left(\gamma^2 I_{m_w} - D_{zw\Delta}^T D_{zw\Delta}\right)^{-1}\left(B_{w\Delta}^T P + D_{zw\Delta}^T C_{z\Delta}\right) + C_{z\Delta}^T C_{z\Delta} < 0.$$

Next Lemma plays a key role in the development of next section.

Lemma 2 [19]: Given appropriate dimensional matrices X, Y, and a symmetric matrix Z, then

$$Z + X\Delta Y + Y^T \Delta^T X^T < 0$$

for all Δ satisfying $\Delta^T \Delta \leq I$ if and only if there exists a scalar $\varepsilon > 0$ such that

$$Z + \varepsilon X X^T + \varepsilon^{-1} Y^T Y < 0.$$

3. MAIN RESULTS

In this section, two necessary and sufficient LMI-based conditions for robust H_∞ analysis and design of system (2) are derived, respectively.

3.1. Robust H_∞ analysis

First, the result of robust H_∞ control analysis of (2) is presented.

Theorem 1: The unforced uncertain continuous-time descriptor system (2) is quadratically admissible with disturbance attenuation γ for all Δ if and only if there exists a nonsingular matrix P and a scalar $\varepsilon > 0$ satisfying

$$X^T E^T = EX \geq 0, \tag{10}$$

$$\begin{bmatrix} X^T A^T + AX + \varepsilon H_1 H_1^T & B_w & X^T C_z^T + \varepsilon H_1 H_2^T & X^T J_1^T \\ B_w^T & -\gamma^2 I_{m_w} & D_{zw}^T & J_2^T \\ C_z X + \varepsilon H_2 H_1^T & D_{zw} & -I_{q_z} + \varepsilon H_2 H_2^T & 0 \\ J_1 X & J_2 & 0 & -\varepsilon I_s \end{bmatrix} < 0. \tag{11}$$

Proof: By congruence and setting $X^{-1} =: P$, (10) becomes (8) and (11) is equivalent to

$$\begin{bmatrix} A^T P + P^T A + \varepsilon P^T H_1^T H_1 P & P^T B_w & C_z^T + \varepsilon P^T H_1 H_2^T & J_1^T \\ B_w^T P & -\gamma^2 I_{m_w} & D_{zw}^T & J_2^T \\ C_z + \varepsilon H_2 H_1^T P & D_{zw} & -I_{q_z} + \varepsilon H_2 H_2^T & 0 \\ J_1 & J_2 & 0 & -\varepsilon I_s \end{bmatrix} < 0,$$

which can be represented further into $\tilde{A} + \varepsilon \tilde{H}\tilde{H}^T + \varepsilon^{-1}\tilde{J}^T\tilde{J} < 0$, where

$$\tilde{A} = \begin{bmatrix} A^T P + P^T A & P^T B_w & C_z^T \\ B_w^T P & -\gamma^2 I_{m_w} & D_{zw}^T \\ C_z & D_{zw} & -I_{q_z} \end{bmatrix},$$

$$\tilde{H} = \begin{bmatrix} P^T H_1 \\ 0 \\ H_2 \end{bmatrix}, \quad \tilde{J} = \begin{bmatrix} J_1 & J_2 & 0 \end{bmatrix}.$$

By Lemma 2, we obtain

$$\tilde{A} + \tilde{H}\Delta\tilde{J} + \tilde{J}^T \Delta^T \tilde{H}^T < 0,$$

which can be equivalently represented as

$$\begin{bmatrix} (A+H_1\Delta J_1)^T P + P^T (A+H_1\Delta J_1) & P^T (B_w + H_1\Delta J_2) & (C_z + H_2\Delta J_1)^T \\ (B_w + H_1\Delta J_2)^T P & -\gamma^2 I_{m_w} & (D_{zw} + H_2\Delta J_2)^T \\ (C_z + H_2\Delta J_1) & (D_{zw} + H_2\Delta J_2) & -I_{q_z} \end{bmatrix} < 0 \tag{12}$$

for all Δ satisfying (4). Applying Schur complement to (12), then (9) is obtained. □

Remark 1: If the system (2) is uncertainty-free, the result of Theorem 1 reduces to the major results of [13,18]. Thus they can be viewed as special cases of ours.

3.2. Robust H_∞ control design-state feedback cases

In this subsection, the result of Theorem 1 is applied to design state feedback robust H_∞ controllers. Suppose all descriptor variables are measurable. Herein, we are concerned with designing a constant gain matrix K, $u(t) = Kx(t)$, such that the closed-loop system

$$\begin{aligned} E\dot{x}(t) &= \big((A + B_u K) + H_1\Delta(J_1 + J_3 K)\big)x(t) \\ &\quad + (B_w + H_1\Delta J_2)w(t), \\ z(t) &= \big((C_z + D_{zu}K) + H_2\Delta(J_1 + J_3 K)\big)x(t) \\ &\quad + (D_{zw} + H_2\Delta J_2)w(t) \end{aligned} \tag{13}$$

is quadratically admissible with disturbance attenuation γ for all Δ satisfying (4).

Theorem 2: Let $\gamma > 0$ be given. Then there exists a state feedback controller, $u(t) = Kx(t)$, such that (13) is quadratically admissible with disturbance attenuation γ for all Δ if and only if there exist a matrix F, a nonsingular matrix P, and a scalar $\varepsilon > 0$ such that

$$Q^T E^T = EQ \geq 0, \tag{14}$$

$$\begin{bmatrix} Q^T A^T + F^T B_u^T + AQ + B_u F + \varepsilon H_1 H_1^T & B_w & Q^T C_z^T + F^T D_{zu}^T + \varepsilon H_1 H_2^T & Q^T J_1^T + F^T J_3^T \\ B_w^T & -\gamma^2 I_{m_w} & D_{zw}^T & J_2^T \\ C_z Q + D_{zu} F + \varepsilon H_2 H_1^T & D_{zw} & -I_{q_z} + \varepsilon H_2 H_2^T & 0 \\ J_1 Q + J_3 F & J_2 & 0 & -\varepsilon I_s \end{bmatrix} < 0. \tag{15}$$

Moreover, the controller can be chosen as

$$u(t) = Kx(t) = FQ^{-1}x(t).$$

Proof: Let $K = FQ^{-1}$ and by Theorem 1, the result is straightforward. □

3.3. Robust H_∞ control design-output feedback cases

When the states are not fully accessible, output feedback control becomes important. For designing a dynamic output feedback controller in descriptor form, without loss of generality, we may convert system (2) into an SVD coordinate

$$\begin{aligned} \tilde{E}\dot{\tilde{x}}(t) &= \tilde{A}_\Delta \tilde{x}(t) + \tilde{B}_{w\Delta} w(t) + \tilde{B}_{u\Delta} u(t), \\ z(t) &= \tilde{C}_{z\Delta} \tilde{x}(t) + \tilde{D}_{zw\Delta} w(t) + \tilde{D}_{zu\Delta} u(t), \\ y(t) &= \tilde{C}_{y\Delta} \tilde{x}(t) + \tilde{D}_{yw\Delta} w(t) + \tilde{D}_{yu\Delta} u(t), \end{aligned} \tag{16}$$

where

$$\begin{aligned} &\tilde{x}(t) = V^{-1}x(t),\ \tilde{E} = UEV = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix},\ \tilde{A}_\Delta = UA_\Delta V, \\ &\tilde{B}_{w\Delta} = UB_{w\Delta},\ \tilde{B}_{u\Delta} = UB_{u\Delta}, \tilde{C}_{z\Delta} = C_{z\Delta}V,\ \tilde{C}_{y\Delta} = C_{y\Delta}V, \\ &\tilde{D}_{zw\Delta} = D_{zw\Delta},\ \tilde{D}_{zu\Delta} = D_{zu\Delta}, \\ &\tilde{D}_{yw\Delta} = D_{yw\Delta},\ \tilde{D}_{yu\Delta} = D_{yu\Delta}. \end{aligned} \tag{17}$$

Assume the controller is also in descriptor form

$$\begin{aligned} \tilde{E}\dot{x}_o(t) &= A_o x_o(t) + B_o y(t), \\ u(t) &= C_o x_o(t). \end{aligned} \tag{18}$$

Then the closed-loop system is

$$\begin{aligned} \bar{E}\dot{\bar{x}}(t) &= \bar{A}_\Delta \bar{x}(t) + \bar{B}_{w\Delta} w(t), \\ z(t) &= \bar{C}_{z\Delta} \bar{x}(t) + \bar{D}_{zw\Delta} w(t), \end{aligned} \tag{19}$$

where

$$\begin{aligned} &\dot{\bar{x}}(t) = \begin{bmatrix} \dot{\tilde{x}}(t) \\ \dot{x}_o(t) \end{bmatrix},\ \bar{E} = \begin{bmatrix} \tilde{E} & 0 \\ 0 & \tilde{E} \end{bmatrix}, \\ &\bar{A}_\Delta = \begin{bmatrix} \tilde{A}_\Delta & \tilde{B}_{u\Delta} C_o \\ B_o \tilde{C}_{y\Delta} & A_o + B_o \tilde{D}_{yu\Delta} C_o \end{bmatrix},\ \bar{B}_{w\Delta} = \begin{bmatrix} \tilde{B}_{w\Delta} \\ B_o \tilde{D}_{yw\Delta} \end{bmatrix}, \\ &\bar{C}_{z\Delta} = \begin{bmatrix} \tilde{C}_{z\Delta} & \tilde{D}_{zu\Delta} C_o \end{bmatrix},\ \bar{D}_{zw\Delta} = \tilde{D}_{zw\Delta}. \end{aligned} \tag{20}$$

According to Definition 2, the closed-loop system (19) is quadratically admissible with disturbance attenuation γ for all Δ if there exists a nonsingular matrix $\bar{X}$ satisfying

$$\bar{E}\bar{X} = \bar{X}^T \bar{E} \geq 0, \tag{21}$$

$$\begin{bmatrix} \bar{A}_\Delta^T \bar{X} + \bar{X}^T \bar{A}_\Delta & \bar{X}^T \bar{B}_{w\Delta} & \bar{C}_{z\Delta}^T \\ \bar{B}_{w\Delta}^T \bar{X} & -\gamma^2 I_{m_w} & \bar{D}_{zw\Delta}^T \\ \bar{C}_{z\Delta} & \bar{D}_{zw\Delta} & -I_{q_z} \end{bmatrix} < 0, \tag{22}$$

for all possible uncertainties. The main result of the subsection is presented as follows.

Theorem 3: Let $\gamma > 0$ be given. The following two statements are equivalent.

(I) There exists a controller (18) such that the closed-loop system (19) is quadratically admissible with disturbance attenuation γ for all Δ.

(II) (a) There exist A_k, B_k, C_k, a scalar $\varepsilon > 0$, and two nonsingular matrices X_1 and Y_1 satisfying

$$\begin{bmatrix} \tilde{E}X_1 & \tilde{E} \\ \tilde{E} & Y_1^T\tilde{E} \end{bmatrix} = \begin{bmatrix} X_1^T\tilde{E} & \tilde{E} \\ \tilde{E} & \tilde{E}Y_1 \end{bmatrix} \geq 0, \tag{23}$$

$$\begin{bmatrix}
\begin{pmatrix} X_1^TUAV \\ +B_kC_yV \\ +V^TA^TU^TX_1 \\ +V^TC_y^TB_k^T \end{pmatrix} & V^TA^TU^T + A_k & X_1^TUB_w + B_kD_{yw} & V^TC_z^T & \begin{pmatrix} X_1^TUH_1 \\ +B_kH_3 \end{pmatrix} & V^TJ_1^T \\
UAV + A_k^T & \begin{pmatrix} UAVY_1 \\ +UB_uC_k \\ +Y_1^TV^TA^TU^T \\ +C_k^TB_u^TU^T \end{pmatrix} & UB_w & \begin{pmatrix} Y_1^TV^TC_z^T \\ +C_k^TD_{zu}^T \end{pmatrix} & UH_1 & \begin{pmatrix} Y_1^TV^TJ_1^T \\ +C_k^TJ_3^T \end{pmatrix} \\
B_w^TU^TX_1 + D_{yw}^TB_k^T & B_w^TU^T & -\gamma^2 I_{m_w} & D_{zw}^T & 0 & J_2^T \\
C_zV & C_zVY_1 + D_{zu}C_k & D_{zw} & -I_{q_z} & H_2 & 0 \\
H_1^TU^TX_1 + H_3^TB_k^T & H_1^TU^T & 0 & H_2^T & -\varepsilon^{-1}I_s & 0 \\
J_1V & J_1VY_1 + J_3C_k & J_2 & 0 & 0 & -\varepsilon I_s
\end{bmatrix} < 0. \tag{24}$$

(b) There exist X_2 and two nonsingular matrices X_3 and Y_3 satisfying

$$I - X_1Y_1 = X_2Y_3, \tag{25}$$
$$\tilde{E}X_3 = X_2^T\tilde{E}, \tag{26}$$

where X_1 and Y_1 are obtained from part (a). Moreover, the controller (18) can be chosen as

$$\begin{aligned} C_o &= C_kY_3^{-1}, \\ B_o &= X_3^{-T}B_k, \\ A_o &= X_3^{-T}\big(A_k - X_1^TUAVY_1 - B_kC_yVY_1 \\ &\qquad - X_1^TUB_uC_k - B_kD_{yu}C_k\big)Y_3^{-1} \end{aligned} \tag{27}$$

Proof: (I)$\Rightarrow$(II) Assume $\bar{X}$ satisfies (21) and (22) for all possible uncertainties. Partition $\bar{X}$ in accordance with the block structure of $\bar{E}$ as

$$\bar{X} = \begin{bmatrix} X_1 & X_2 \\ X_3 & X_4 \end{bmatrix}, \tag{28}$$

where $X_i \in \mathbb{R}^{n\times n}$, $i = 1,2,3,4$. Since $\bar{X}$ is nonsingular. Define

$$\bar{Y} \triangleq \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_4 \end{bmatrix} = \bar{X}^{-1}. \tag{29}$$

According to Propositions 1 and 2 (see Appendix), we know that all X_i's and Y_i's are invertible. In the following, we will show that the above X_i and Y_i, i = 1,2,3,4, satisfy (23)-(26). From the (1,1) and (2,1) blocks of $\bar{X}\bar{Y} = I$, it gives

$$X_1Y_1 + X_2Y_3 = I, \tag{30}$$
$$X_3Y_1 + X_4Y_3 = 0. \tag{31}$$

(30) implies (25). Using (30) and (31), rewrite $\bar{X}$ as

$$\begin{aligned} \bar{X} &= \begin{bmatrix} X_1 & Y_3^{-1} - X_1Y_1Y_3^{-1} \\ X_3 & -X_3Y_1Y_3^{-1} \end{bmatrix} = \begin{bmatrix} X_1 & I \\ X_3 & 0 \end{bmatrix}\begin{bmatrix} I & -Y_1Y_3^{-1} \\ 0 & Y_3^{-1} \end{bmatrix} \\ &= \begin{bmatrix} X_1 & I \\ X_3 & 0 \end{bmatrix}\begin{bmatrix} I & Y_1 \\ 0 & Y_3 \end{bmatrix}^{-1} \triangleq \Psi_1\Psi_2^{-1}, \end{aligned} \tag{32}$$

where

$$\Psi_1 \triangleq \begin{bmatrix} X_1 & I \\ X_3 & 0 \end{bmatrix}, \quad \Psi_2 \triangleq \begin{bmatrix} I & Y_1 \\ 0 & Y_3 \end{bmatrix}. \tag{33}$$

By (32), we have

$$(21) \Leftrightarrow \Psi_2^T\left(\bar{E}\Psi_1\Psi_2^{-1}\right)\Psi_2 = \Psi_2^T\left(\Psi_2^{-T}\Psi_1^T\bar{E}\right)\Psi_2 \geq 0 \tag{34}$$

$$\begin{aligned} &\Leftrightarrow \Psi_2^T\bar{E}\Psi_1 = \Psi_1^T\bar{E}\Psi_2 \geq 0 \\ &\Leftrightarrow \begin{bmatrix} I & 0 \\ Y_1^T & Y_3^T \end{bmatrix}\begin{bmatrix} \tilde{E} & 0 \\ 0 & \tilde{E} \end{bmatrix}\begin{bmatrix} X_1 & I \\ X_3 & 0 \end{bmatrix} = \begin{bmatrix} X_1^T & X_3^T \\ I & 0 \end{bmatrix}\begin{bmatrix} \tilde{E} & 0 \\ 0 & \tilde{E} \end{bmatrix}\begin{bmatrix} I & Y_1 \\ 0 & Y_3 \end{bmatrix} \geq 0 \\ &\Leftrightarrow \begin{bmatrix} \tilde{E}X_1 & \tilde{E} \\ Y_1^T\tilde{E}X_1 + Y_3^T\tilde{E}X_3 & Y_1^T\tilde{E} \end{bmatrix} = \begin{bmatrix} X_1^T\tilde{E} & X_1^T\tilde{E}Y_1 + X_3^T\tilde{E}Y_3 \\ \tilde{E} & \tilde{E}Y_1 \end{bmatrix} \geq 0 \end{aligned} \tag{35}$$

$\Rightarrow$ (23).

By (35) and (30), we have

$$\begin{aligned} \tilde{E} &= X_1^T\tilde{E}Y_1 + X_3^T\tilde{E}Y_3 \\ &= \tilde{E}X_1Y_1 + X_3^T\tilde{E}Y_3 \\ &\Rightarrow \tilde{E}\left(I - X_1Y_1\right) = X_3^T\tilde{E}Y_3 \\ &\Rightarrow \tilde{E}X_2Y_3 = X_3^T\tilde{E}Y_3 \\ &\Rightarrow (26). \end{aligned}$$

Substituting $\bar{X} = \Psi_1\Psi_2^{-1}$ into (22) yields

$$\begin{bmatrix} \bar{A}_\Delta^T \psi_1 \psi_2^{-1} + \psi_2^{-T}\psi_1^T \bar{A}_\Delta & \psi_2^{-T}\psi_1^T \bar{B}_{w\Delta} & \bar{C}_{z\Delta}^T \\ \bar{B}_{w\Delta}^T \psi_1\psi_2^{-1} & -\gamma^2 I_{m_w} & \bar{D}_{zw\Delta}^T \\ \bar{C}_{z\Delta} & \bar{D}_{zw\Delta} & -I_{q_z}\end{bmatrix} < 0 \quad \forall\Delta, \tag{36}$$

or equivalently,

$$\begin{bmatrix} \psi_2^T \bar{A}_\Delta^T \psi_1 + \psi_1^T \bar{A}_\Delta \psi_2 & \psi_1^T \bar{B}_{w\Delta} & \psi_2^T \bar{C}_{z\Delta}^T \\ \bar{B}_{w\Delta}^T \psi_1 & -\gamma^2 I_{m_w} & \bar{D}_{zw\Delta}^T \\ \bar{C}_{z\Delta}\psi_2 & \bar{D}_{zw\Delta} & -I_{q_z}\end{bmatrix} < 0 \quad \forall\Delta. \tag{37}$$

Using (20) and (33), (37) is equivalent to

$$\begin{bmatrix} \begin{matrix} X_1^T \tilde{A}_\Delta + X_3^T B_o \tilde{C}_{y\Delta} \\ +\tilde{A}_\Delta^T X_1 + \tilde{C}_{y\Delta}^T B_o^T X_3 \end{matrix} & \begin{pmatrix} \tilde{A}_\Delta^T + X_1^T \tilde{A}_\Delta Y_1 \\ +X_3^T B_o \tilde{C}_{y\Delta} Y_1 + X_1^T \tilde{B}_{u\Delta} C_o Y_3 \\ +X_3^T A_o Y_3 + X_3^T B_o \tilde{D}_{yu\Delta} C_o Y_3 \end{pmatrix} & X_1^T \tilde{B}_{w\Delta} + X_3^T B_o \tilde{D}_{yw\Delta} & \tilde{C}_{z\Delta}^T \\ \begin{pmatrix} \tilde{A}_\Delta + Y_1^T \tilde{A}_\Delta^T X_1 + Y_1^T \tilde{C}_{y\Delta}^T B_o^T X_3 \\ +Y_3^T C_o^T \tilde{B}_{u\Delta}^T X_1 + Y_3^T A_o^T X_3 \\ +Y_3^T C_o^T \tilde{D}_{yu\Delta}^T B_o^T X_3 \end{pmatrix} & \begin{matrix} \tilde{A}_\Delta Y_1 + \tilde{B}_{u\Delta} C_o Y_3 \\ +Y_1^T \tilde{A}_\Delta^T + Y_1^T \tilde{A}_\Delta^T \\ +Y_3^T C_o^T \tilde{B}_{u\Delta}^T \end{matrix} & \tilde{B}_{w\Delta} & Y_1^T \tilde{C}_{z\Delta}^T + Y_3^T C_o^T \tilde{D}_{zu\Delta}^T \\ \tilde{B}_{w\Delta}^T X_1 + \tilde{D}_{yw\Delta}^T B_o^T X_3 & \tilde{B}_{w\Delta}^T & -\gamma^2 I_{m_w} & \tilde{D}_{zw\Delta}^T \\ \tilde{C}_{z\Delta} & \tilde{C}_{z\Delta} Y_1 + \tilde{D}_{zu\Delta} C_o Y_3 & \tilde{D}_{zw\Delta} & -I_{q_z} \end{bmatrix} < 0. \tag{38}$$

In view of (17) and (3), the inequality (38) can be reformulated as, $\forall\ \Delta$,

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \begin{pmatrix} X_1^T U(B_w + H_1\Delta J_2) \\ +X_3^T B_o (D_{yw} + H_3 \Delta J_2) \end{pmatrix} & V^T (C_z + H_2\Delta J_1)^T \\ \Omega_{12}^T & \Omega_{22} & U(B_w + H_1 \Delta J_2) & \begin{pmatrix} Y_1^T V^T (C_z + H_2 \Delta J_1)^T \\ +Y_3^T C_o^T (D_{zu} + H_2\Delta J_3)^T \end{pmatrix} \\ \begin{pmatrix} (B_w + H_1\Delta J_2)^T U^T X_1 \\ +(D_{yw} + H_3\Delta J_2)^T B_o^T X_3 \end{pmatrix} & (B_w + H_1 \Delta J_2)^T U^T & -\gamma^2 I_{m_w} & (D_{zw} + H_2 \Delta J_2)^T \\ (C_z + H_2\Delta J_1) V & \begin{pmatrix} (C_z + H_2 \Delta J_1) V Y_1 \\ +(D_{zu} + H_2 \Delta J_3) C_o Y_3 \end{pmatrix} & (D_{zw} + H_2 \Delta J_2) & -I_{q_z} \end{bmatrix} < 0, \tag{39}$$

where

$$\begin{aligned} \Omega_{11} &= X_1^T U (A + H_1 \Delta J_1) V + X_3^T B_o (C_y + H_3 \Delta J_1) V \\ &\quad + V^T (A + H_1\Delta J_1)^T U^T X_1 + V^T (C_y + H_3 \Delta J_1)^T B_o^T X_3, \\ \Omega_{12} &= V^T (A + H_1 \Delta J_1)^T + X_1^T U (A + H_1 \Delta J_1) V Y_1 \\ &\quad + X_3^T B_o (C_y + H_3 \Delta J_1) V Y_1 \\ &\quad + X_1^T U (B_u + H_1 \Delta J_3) C_o Y_3 + X_3^T A_o Y_3 \\ &\quad + X_3^T B_o (D_{yu} + H_3 \Delta J_3) C_o Y_3, \\ \Omega_{22} &= U (A + H_1 \Delta J_1) V Y_1 + U (B_u + H_1 \Delta J_3) C_o Y_3 \\ &\quad + Y_1^T V^T (A + H_1 \Delta J_1)^T U^T + Y_3^T C_o^T (B_u + H_1 \Delta J_3)^T U^T. \end{aligned}$$

(39) can be rewritten in a more compact expression

$$\hat{A} + \hat{H}\Delta\hat{J} + \hat{J}^T \Delta \hat{H}^T < 0 \quad \forall\Delta, \tag{40}$$

where

$$\hat{A} = \begin{bmatrix} X_1^T U A V + X_3^T B_o C_y V + V^T A^T U^T X_1 + V^T C_y^T B_o^T X_3 & \begin{pmatrix} V^T A^T U^T + X_1^T U A V Y_1 + X_3^T B_o C_y V Y_1 \\ + X_1^T U B_u C_o Y_3 + X_3^T A_o Y_3 + X_3^T B_o D_{yu} C_o Y_3 \end{pmatrix} & X_1^T U B_w + X_3^T B_o D_{yw} & V^T C_z^T \\ \begin{pmatrix} U A V + Y_1^T V^T A^T U^T X_1 + Y_1^T V^T C_y^T B_o^T X_3 \\ + Y_3^T C_o^T B_u^T U^T X_1 + Y_3^T A_o^T X_3 + Y_3^T C_o^T D_{yu}^T B_o^T X_3 \end{pmatrix} & U A V Y_1 + U B_u C_o Y_3 + Y_1^T V^T A^T U^T + Y_3^T C_o^T B_u^T U^T & U B_w & Y_1^T V^T C_z^T + Y_3^T C_o^T D_{zu}^T \\ B_w^T U^T X_1 + D_{yw}^T B_o^T X_3 & B_w^T U^T & -\gamma^2 I_{m_w} & D_{zw}^T \\ C_z V & C_z V Y_1 + D_{zu} C_o Y_3 & D_{zw} & -I_{q_z} \end{bmatrix},$$

$$\hat{H} = \begin{bmatrix} X_1^T U H_1 + X_3^T B_o H_3 \\ U H_1 \\ 0 \\ H_2 \end{bmatrix}, \quad \hat{J} = \begin{bmatrix} J_1 V & J_1 V Y_1 + J_3 C_o Y_3 & J_2 & 0 \end{bmatrix}. \tag{41}$$

According to Lemma 2, the inequality (40) holds for all Δ of (4) if and only if there exists a scalar $\varepsilon > 0$ such that

$$\hat{A} + \varepsilon \hat{H}\hat{H}^T + \varepsilon^{-1} \hat{J}^T \hat{J} < 0. \tag{42}$$

In (42), denote

$$\begin{aligned} A_k &= X_1^T UAVY_1 + X_3^T B_o C_y VY_1 \\ &\quad + X_1^T UB_u C_o Y_3 + X_3^T A_o Y_3 + X_3^T B_o D_{yu} C_o Y_3, \\ B_k &= X_3^T B_o, \\ C_k &= C_o Y_3, \end{aligned} \tag{43}$$

and use Schur complement, we have (24).

(II)$\Rightarrow$(I) If X_1, Y_1, X_3, and Y_3 are solved from (23)-(26), construct $\bar{X}$ by

$$\bar{X} \triangleq \begin{bmatrix} X_1 & Y_3^{-1} - X_1 Y_1 Y_3^{-1} \\ X_3 & -X_3 Y_1 Y_3^{-1} \end{bmatrix},$$

we will show that such $\bar{X}$ is nonsingular and satisfy (21) and (22) for all Δ of (4). Note that $\bar{X}$ can be factorized as

$$\bar{X} = \begin{bmatrix} X_1 & I \\ X_3 & 0 \end{bmatrix} \begin{bmatrix} I & Y_1 \\ 0 & Y_3 \end{bmatrix}^{-1} \triangleq \psi_1 \psi_2^{-1}.$$

The nonsingularity of X_3 and Y_3 implies that of $\bar{X}$. From (23), (25), and (26)

$$\begin{aligned} \tilde{E} = \tilde{E}\left(X_1 Y_1 + X_2 Y_3\right) &= \tilde{E} X_1 Y_1 + \tilde{E} X_2 Y_3 \\ &= X_1^T \tilde{E} Y_1 + X_3^T \tilde{E} Y_3. \end{aligned} \tag{44}$$

Using (44) and (23), with the help of derivative between (34) and (35), we have $\bar{E}\bar{X} = \bar{X}^T \bar{E} \geq 0$. Furthermore, denote C_o, B_o, and A_o by (27) if (23)-(26) is feasible. In view of the derivatives between (36) and (42), it is easy to verify that such $\bar{X}$ also satisfies (22) $\forall \Delta$. □

Remark 2: Based on the result of Theorem 3, the H_∞ minimization design via dynamic output feedback for uncertain descriptor system (2) can be formulated as the following constrained optimization problem

minimize γ
subject to (23-26).

This problem can be solved efficiently by using LMI software, e.g. Scilab 2.6.

Remark 3: Using the proposed LMI-based approach to design dynamic output feedback controllers, the assumptions (A1)-(A4) needed in [6] are no more required. Thus our approach relaxes the design constraints.

4. A NUMERICAL EXAMPLE

Consider an uncertain continuous-time descriptor system

$$\begin{aligned} E\dot{x}(t) &= A_\Delta x(t) + B_{w\Delta} w(t) + B_{u\Delta} u(t), \\ z(t) &= C_{z\Delta} x(t) + D_{zw\Delta} w(t) + D_{zu\Delta} u(t), \\ y(t) &= C_{y\Delta} x(t) + D_{yw\Delta} w(t) + D_{yu\Delta} u(t), \end{aligned}$$

where $E = \begin{bmatrix} 1 & -0.1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, and uncertain system matrices A_Δ, $B_{w\Delta}$, $B_{u\Delta}$, $C_{z\Delta}$, $D_{zw\Delta}$, $D_{zu\Delta}$, $C_{y\Delta}$, $D_{yw\Delta}$, and $D_{yu\Delta}$ are described as in (3) with

$$A = \begin{bmatrix} -3 & 1 & 4 \\ 1 & 0 & -1 \\ -2 & 0 & 0 \end{bmatrix}, B_w = \begin{bmatrix} -0.3 \\ -0.45 \\ -0.3 \end{bmatrix}, B_u = \begin{bmatrix} 3 & 2 \\ 2 & -1 \\ 0 & -2 \end{bmatrix},$$

$$C_z = \begin{bmatrix} -1 & 1 & 0 \\ -3 & 0.1 & 2 \end{bmatrix}, D_{zw} = \begin{bmatrix} -0.3 \\ 0.45 \end{bmatrix}, D_{zu} = \begin{bmatrix} 0 & 0 \\ 0 & 0.3 \end{bmatrix},$$

$$C_y = \begin{bmatrix} 1 & 1 & -2 \end{bmatrix}, D_{yw} = -0.54, D_{yu} = \begin{bmatrix} 2 & 1 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} -0.1 \\ -0.18 \\ 0.1 \end{bmatrix}, H_2 = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix}, H_3 = 0.4,$$

$$J_1 = \begin{bmatrix} 0.1 & 0.1 & -0.9 \end{bmatrix}, J_2 = 0.6, J_3 = \begin{bmatrix} -0.8 & 0 \end{bmatrix}.$$

In this example, the uncertainty is formulated as

$$\begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} \Delta \begin{bmatrix} J_1 & J_2 & J_3 \end{bmatrix},$$

where Δ is a time-invariant uncertain scalar lying in [-1,1]. By Theorem 1, the given uncertain descriptor system is not admissible for all $\Delta \in [-1, 1]$. In the following, we want to find a dynamic output feedback controller (18) such that the closed-loop uncertain descriptor system is quadratically admissible with disturbance attenuation γ for all Δ. From Remark 2, we obtain $\gamma = 0.7070$, $\varepsilon = 2.34$,

$$X_1 = \begin{bmatrix} 0.3148 & -0.3969 & 0 \\ -0.3969 & 1.2851 & 0 \\ 37.6385 & 3.2018 & -14.9701 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} 3081.3348 & 3167.0251 & 0 \\ 3167.0251 & 3258.5813 & 0 \\ 2520965.8528 & -690755.9541 & -2713279.7539 \end{bmatrix},$$

$$A_k = \begin{bmatrix} -55.9425 & -68.1142 & -5.7252 \\ -150.7793 & -160.3615 & -0.2578 \\ 54.4256 & 68.2844 & 8.0605 \end{bmatrix},$$

$$B_k = \begin{bmatrix} -25.3884 \\ -3.7543 \\ 11.0222 \end{bmatrix},$$

$$C_k = \begin{bmatrix} -2835359.8185 & 777840.9875 & 3052431.7746 \\ -16776555.7025 & 4635752.9509 & 18088522.9763 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad X_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$Y_3 = \begin{bmatrix} 287.7668 & 296.1238 & 0 \\ -2847.0837 & -2929.7354 & 0 \\ 37612908.6785 & -10470297.6060 & -40617976.3845 \end{bmatrix}.$$

Therefore, from (27), the parameters of dynamic output feedback controller are

$$C_o = \begin{bmatrix} 4226.0084 & 430.2168 & -0.0751 \\ 11768.0156 & 1198.6767 & -0.4453 \end{bmatrix}, \; B_o = \begin{bmatrix} -25.3884 \\ -3.7543 \\ 11.0222 \end{bmatrix},$$

$$A_o = \begin{bmatrix} 1418882.1907 & 144505.1089 & -51.6867 \\ 172610.5842 & 17579.1161 & -6.2106 \\ -590777.8358 & -60167.1167 & 21.3711 \end{bmatrix}.$$

Let $T_{zw}^{c}(s)$ stand for the transfer matrix of the closed-loop system from w to z. Note that $T_{zw}^{c}(s)$ is also a function of Δ. Fig. 1 shows the curves of $\sigma_{\max}\left(T_{zw}^{c}(j\omega)\right)$, where $\sigma_{\max}$ denotes the largest singular value of $T_{zw}^{c}(j\omega)$. Since the uncertainty Δ lies between [-1,1] in the example, each curve in Fig. 1 represents the function $\sigma_{\max}\left(T_{zw}^{c}(j\omega)\right)$ corresponding to a different Δ in [-1,1]. From Fig. 1, one can see that the maximum of $\sup_{s\in j\omega}\left\{\sigma_{\max}\left(T_{zw}^{c}(s)\right)\right\}$ for all allowable Δ occurs at 0.707. That means the minimal value of γ that all dynamic output feedback controllers can achieve is 0.707.

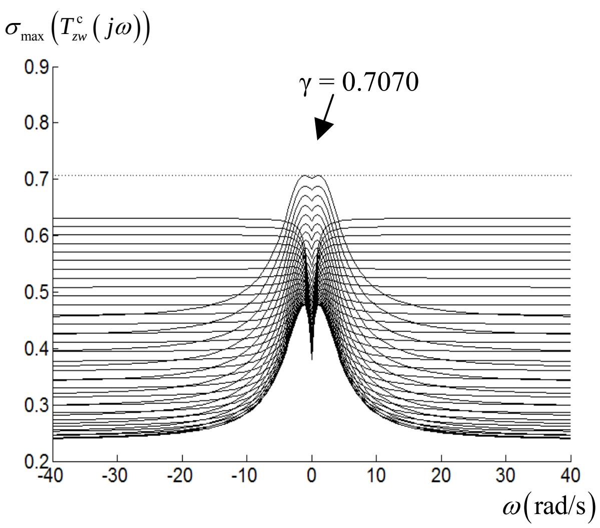

Fig. 1. $\sigma_{\max}\left(T_{zw}^{c}(j\omega)\right)$ of the closed-loop system.

5. CONCLUSION

In this paper, a new LMI approach is proposed to solve the robust H_∞ control problem for uncertain descriptor systems. The state feedback and dynamic output feedback controller design are investigated. Necessary and sufficient conditions for the existence of the robust H_∞ controllers are derived and expressed in the LMI formulation. Although only continuous-time cases are discussed, the presented technique can be applied to the discrete-time cases in a similar way. Four major contributions of the paper are summarized as follows: (I) This paper is the first one to present necessary and sufficient LMI-based conditions for robust H_∞ control analysis and design of the uncertain descriptor systems (2). (II) The requirements of system property while designing output feedback controller have been removed. No assumption as needed in [6] is required by the proposed approach. (III) The uncertain system model considered in this paper is more general than the ones investigated in the previous literature. (IV) Some interesting results [13,18] for H_∞ control of descriptor systems are included as special cases of ours.

APPENDIX

Material of the appendix is a direct adoption from [2].

Proposition 1: Let $\bar{X} \in R^{2n\times 2n}$ be a nonsingular solution to (21) and (22). Suppose it can be partitioned as in (28). Then, without loss of generality, all $X_i^{'}s$ may be assumed to be nonsingular as well.

Proof: Suppose that $X_i^{'}s$ are singular, then there always exists a small $\delta > 0$ such that the matrix $\tilde{X}$ defined below

$$\tilde{X} = \begin{bmatrix} \tilde{X}_1 & \tilde{X}_2 \\ \tilde{X}_3 & \tilde{X}_4 \end{bmatrix} \triangleq \begin{bmatrix} X_1 & X_2 \\ X_3 & X_4 \end{bmatrix} + \delta \begin{bmatrix} I_n & I_n \\ I_n & I_n \end{bmatrix}$$

has its all submatrices $\tilde{X}_i^{'}s$ being nonsingular. Note that δ can be chosen small enough so that the LMI (22) won't be violated when $\bar{X}$ is replaced by $\tilde{X}$. Moreover, since

$$\bar{E}\begin{bmatrix} I_n & I_n \\ I_n & I_n \end{bmatrix} = \begin{bmatrix} I_n & I_n \\ I_n & I_n \end{bmatrix}\bar{E} \geq 0,$$

it is straightforward to show that

$$\bar{E}\tilde{X} = \tilde{X}^T\bar{E} \geq 0.$$

Therefore, starting from any solution to (21) and (22), which does have some singular submatrices, we can always find a very close solution that will meet the nonsingularity requirement on its submatrices. □

Let $\overline{Y} \triangleq \overline{X}^{-1}$ and partition $\overline{Y}$ as in (29). By Proposition 1, we have the following results.

Proposition 2: $\overline{Y}_i$, $i = 1, 2, 3, 4$ are nonsingular.

Proof: Since $\overline{X}$ and X_4 are invertible, by the matrix inversion formulas, we have

$$\begin{bmatrix} X_1 & X_2 \\ X_3 & X_4 \end{bmatrix}^{-1} = \begin{bmatrix} \Upsilon^{-1} & -\Upsilon^{-1}X_2X_4^{-1} \\ -X_4X_3\Upsilon^{-1} & X_4^{-1}+X_4^{-1}X_3\Upsilon^{-1}X_2X_4^{-1} \end{bmatrix} \triangleq \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_4 \end{bmatrix},$$

where $\Upsilon \triangleq X_1 - X_2X_4^{-1}X_3$. Since X_2, X_3, X_4, and Υ are all nonsingular, the above equality implies that Y_1, Y_2, and Y_3 are nonsingular. Finally, since X_1 and X_4 are nonsingular, Y_4 can be rewritten as

$$Y_4 = X_4^{-1} + X_4^{-1}X_3\Upsilon^{-1}X_2X_4^{-1} = \left(X_4 - X_3X_1^{-1}X_2\right)^{-1}.$$

Therefore, Y_4 is nonsingular, too.

REFERENCE

[1] B. Benkatasubramanian, "On a singular transformation for analyzing the global dynamics of a class of singular DAE's," *Proc. of SINS'92*, pp. 364-371, 1992.

[2] J.-L. Chen, *LMI Approach to Positive Real Analysis and Design for Descriptor Systems*, Ph.D. Dissertation, National Sun Yat-Sen University, Taiwan, 2003.

[3] L. Dai, *Singular Control Systems - Lecture Notes in Control and Information Sciences*, Springer-Verlag, Berlin, 1989.

[4] C.-H. Fang and L. Lee, "Stability robustness analysis of uncertain discrete-time descriptor systems," *Proc. of the 15th World Congress of IFAC*, Barcelona, Spain, 2002.

[5] K.-L. Hsiung and L. Lee, "Lyapunov inequality and bounded real lemma for discrete-time descriptor systems," *IEE Proc., Control Theory Appl.*, vol. 146, no. 4, pp. 327-331, 1999.

[6] J.-C. Hung, H.-S. Wang, and F.-R. Chang, "Robust H_∞ control for uncertain linear time-invariant descriptor systems," *IEE Proc. Control Theory Appl.*, vol. 147, no. 6, pp. 648-654, 2000.

[7] T. Katayama, "*(J,J')*-spectral factorization and conjugation for discrete-time descriptor system," *Circ. System Signal Process*, vol. 15, pp. 649-669, 1996.

[8] A. Kawamoto and T. Katayama, "Standard H_∞ control problem for descriptor systems," *Proc. of the 36th CDC*, pp. 4130-4133, 1997.

[9] A. Kumar and P. Daoutids, "Feedback control of nonlinear differential-algebraic equation systems," *AIChE Journal*, vol. 41, pp. 619-636, 1995.

[10] F. L. Lewis, "A survey of linear singular systems," *J. Circuits, Systems, and Signal Processing*, vol. 5, no. 1, pp. 3-36, 1986.

[11] C. Lin, J. Wang, D. Wang, and C. B. Soh, "Robustness of uncertain descriptor systems," *Systems and Control Letters*, vol. 31, pp. 129-138, 1997.

[12] D. G. Luenberger and A. Arbel, "Singular dynamic Leontief systems," *Econometrica*, vol. 45, pp. 991-994, 1977.

[13] I. Masubuchi, Y. Kamitane, A. Ohara, and N. Suda, "H_∞ control for descriptor systems: A matrix inequalities approach," *Automatica*, vol. 33, no. 4, pp. 669-673, 1997.

[14] R. W. Newcomb, "The semistate description of nonlinear time variable circuits," *IEEE Trans. on Circuits Systems*, vol. 28, no. 1, pp. 62-71, 1981.

[15] K. Takaba, N. Morihira, and T. Katayama, "H_∞ control for descriptor systems - a *J*-spectral factorization approach," *Proc. of the 33rd CDC*, pp. 2251-2256, 1994.

[16] M.-H. Tsai, J.-H. Chen, and L. Lee, "Robust H_∞ control for uncertain continuous time descriptor systems," *Proc. of R.O.C. Automatic Control Conference*, pp. 1367-1372, 2002

[17] A. Rehm and F. Allgower, "An LMI approach towards H_∞ control of discrete-time descriptor systems," *Proc. of the ACC*, Anchorage, USA, pp. 614-619, 2002.

[18] H.-S. Wang, C.-F. Yung, and F.-R. Chang, "Bound real lemma and H_∞ control for descriptor systems," *IEE Proc.-Control Theory Appl.*, vol. 145, no. 3, pp. 316-322, 1998.

[19] L. Xie, "Output feedback H_∞ control of systems with parameter uncertainty," *Int. J. Control*, vol. 63, no. 4, pp. 741-750, 1996.

[20] S. Xu and C. Yang, "H_∞ state feedback control for discrete singular systems," *IEEE Trans. on Automatic Control*, vol. 45, no. 7, pp. 1405-1409, 2000.

Hung-Jen Lee was born in Taipei, Taiwan in 1955. He received his Bachelor degree in Electronics Engineering from National Chiao-Tung University in 1977 and his Master degree in Electrical Engineering from National Taiwan University in 1982. Since 1984, he has been an Instructor of Department of Electronic Engineering at National Kaohsiung University of Applied Sciences. His research interests are in the areas of circuit analysis and database web application.

Shih-Wei Kau was born in Taichung, Taiwan, in 1950. He received his diploma from the National Kaohsiung Normal University in 1974. He got research scholarship of DSE for Industry control at the Mamhann University, German in 1980 and 1981. Currently, he is working torward his Ph.D. at the Strathclyde University, UK. From 1974 to 1979 he served as a Teaching Assistant of Electrical Engineering Department at National Kaohsiung Institute of Technology (NKIT). In 1982 he returned to NKIT and served as an Instructor of EE and also the Chair of Computer center at NKIT. Now, he is working at National Kaohsiung University of Applied Sciences. His research interests are in the areas of neural genetic application in industry, control system integrated in remote control using neuron chip, PLC control system etc.

Yung-Sheng Liu was born in Hsin-Chu, Taiwan, in 1980. He received his B.S. degree from Department of Electrical Engineering, Naitonal Yunlin University of Science and Technology, Yunlin, Taiwan, in 2002. He received his M.S. degree in the Electrical Engineering, National Kaohsiung University of Applied Sciences, Kaohsiung, Taiwan, in 2004. Currently, he is working for Ingrasys Technology Inc., Tao-Yuan, Taiwan. His research interests include fuzzy control, linear system control, and singular systems.

Chun-Hsiung Fang was born in Tainan, Taiwan, in 1963. He received his diploma from the National Kaohsiung Institute of Technology in 1983, the M.S. degree from National Taiwan University in 1987, and the Ph.D. degree from National Sun Yat-Sen University in 1997, all in Electrical Engineering. From 1987 to 1990, he served as an Instructor of Electrical Engineering Department at National Kaohsiung Institute of Technology and was promoted to be an Associate Professor in 1990. Since 1993, he has been a Full Professor. Currently, he is a Professor of Electrical Engineering Department at National Kaohsiung University of Applied Sciences. From 2000 to 2001, he was a Visiting Scholar at the University of Maryland, College Park, Maryland. His research interests are in the areas of robust control, singular systems, and fuzzy control.

Jian-Liung Chen received the B.S., M.S. degree in Automatic Control Engineering from the Feng Chia University, Taichung, Taiwan, in 1993, 1996, respectively, and the Ph.D. degree in Electrical Engineering from the National Sun Yat-Sen University in 2003. Currently, he is an Assistant Professor of the Department of Electrical Engineering, Kao-Yuan University, Lu-Chu Hsiang, Kaohsiung, Taiwan, where has been since 2005. His research interests include LMI approach in robust control and descriptor system theory.

Ming-Hung Tsai received the B.S. degree in Electronic Engineering from the Feng Chia University, Taichung, Taiwan, in 2000, and the M.S. degree in Electrical Engineering from the National Sun Yat-Sen University in 2002. Currently, he joined the CNet Technology Inc., Hsin-Chu City, Taiwan, and is presently a Hardware Engineer. His recent research interests are in wireless communication systems, RF circuit design, and networking design.

Li Lee received the B.S. degree in Control Engineering from the National Chiao Tung University, Taiwan, in 1978, the M.S. degree in Electrical Engineering from the National Cheng Kung University, Taiwan, in 1984, and the Ph.D. degree in Electrical Engineering from the University of Maryland, College Park, Maryland, in 1992. After graduation, he joined the Department of Electrical Engineering, National Sun Yat-Sen University. His research interests are in dynamical system theory and robust control design.