

# Direct and Indirect Robust Adaptive Fuzzy Controllers for a Class of Nonlinear Systems

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**Abstract:** In this paper, we propose direct and indirect adaptive fuzzy sliding mode control approaches for a class of nonaffine nonlinear systems. In the direct case, we use the implicit function theory to prove the existence of an ideal implicit feedback linearization controller, and hence approximate it to attain the desired performances. In the indirect case, we exploit the linear structure of a Takagi-Sugeno fuzzy system with constant conclusion to establish an affine-in-control model, and therefore design an indirect adaptive fuzzy controller. In both cases, the adaptation laws of the adjustable parameters are deduced from the stability analysis, in the sense of Lyapunov, to get a more accurate approximation level. In addition to their robustness, the design of the proposed approaches does not require the upper bounds of both external disturbances and approximation errors. To show the efficiency of the proposed controllers, a simulation example is presented.

**Keywords:** Adaptive fuzzy control, nonaffine systems, nonlinear systems, sliding mode control.

## 1. INTRODUCTION

Adaptive control schemes for nonlinear systems via feedback linearization concept have been widely employed for decades. The idea of feedback linearization approaches is to transform a nonlinear dynamic system into a linear system through state feedback mechanisms. With such transformations, those well-explored linear controllers can then be applied to meet desired control specifications. Several results and parameter adaptive control schemes have been reported in [1-3]. However, the performances of these approaches are directly relied to the exact cancellation of nonlinear terms. If these nonlinear terms are uncertain or unknown, the performances can be deteriorated due to non-exact cancellation.

As a model free design method, fuzzy systems have been successfully applied to control complex or ill-defined processes whose mathematical models are difficult to obtain [4-5]. The ability of converting linguistic descriptions into automatic control strategy makes it a practical and promising alternative to the classical control scheme for achieving control of complex nonlinear systems. A major drawback of fuzzy control systems is that the fuzzy rules must be previously tuned by trial and error procedures. To

overcome this problem, some research has been focused on the Lyapunov synthesis approach to construct stable adaptive fuzzy controllers [6-10]. The basic idea of most of these works is that with the universal approximation ability of fuzzy systems [11], the plant model is approximated by two adaptive fuzzy systems to construct the control law. To make more accurate the approximation level and hence to improve the tracking performances, the adaptation laws of the adjustable parameters are synthesised from the stability analysis in the sense of Lyapunov. To maintain the performance of fuzzy adaptive control in the presence of external disturbances, some robust schemes based on sliding mode control or  $H_\infty$  technique are presented in the literature [12-24]. However, these approaches are restricted to affine in control plants. To overcome this restriction, some works treating the extension of adaptive control to nonaffine systems have been developed in the literature. Concerning the indirect adaptive control scheme, there are two techniques where the main idea is to synthesise an affine-in-control model of the plant to design the controller. Indeed in [25] and [26], the authors exploit the linear structure of the Takagi-Sugeno systems with triangular membership functions for inputs and constant conclusion, to establish an affine-in-control fuzzy model to describe the dynamic behaviour of the plant. In [27], the Taylor series expansion is used to obtain an affine in a control model of the plant. Concerning the direct control scheme presented in [28,29], the authors used the implicit theorem and variable structure control to prove the existence of feedback control, which has

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been approximated using neural networks. To improve the approximation level and hence the tracking performances, an adaptation law is derived from the stability analysis. However, to ensure the stability and the robustness of the closed loop system in these works, an additional control signal is needed to compensate the approximation errors and the external disturbances. The design of this signal depends on the well-known upper bounds of both the approximation errors and the external disturbances, which is a restrictive assumption due to the fact that these bounds are generally unknown.

In this work, we propose direct and indirect adaptive fuzzy controllers for a class of non-affine systems subject to external disturbances. In the direct case, we use an adaptive fuzzy system to approximate the implicit desired feedback control whose existence is proven by the implicit theorem. In the indirect case, we exploit the linear structure of a Takagi-Sugeno fuzzy system to generate a fuzzy affine-in-control model to approximate the dynamic behaviour of the plant. In both cases, we utilize the modified sliding mode control to ensure the robustness of the closed loop system. Indeed, the approaching phase is assured by an attenuation term that allows the chattering phenomenon to be eliminated and the constraint on the knowledge of the upper bounds of both external disturbances and approximation errors to be overcome. To improve the approximation level, the adaptation laws are derived from the stability analysis in the sense of Lyapunov. To show the efficiency of the proposed approaches, an illustration example is presented.

## 2. PROBLEM STATEMENT

Consider a single-input single-output (SISO) nonlinear system described by the following differential equation:

$$y^{(n)} = f\left(y, \dot{y}, \dots, y^{(n-1)}, u\right) + d, \quad (1)$$

where  $y \in \mathfrak{R}$  is the measured output,  $u \in \mathfrak{R}$  the control input,  $y^{(i)}$ ,  $i = 1, \dots, n$ , is the  $i^{\text{th}}$  time derivative of  $y$ ,  $f(\cdot)$  is an unknown nonlinear continuous function, and  $d$  is the external disturbances assumed to be unknown but bounded. Without loss of generality, we assume that all the state variables  $\underline{x} = [x_1, \dots, x_n]^T = [y, \dots, y^{(n-1)}]^T$  are available to the measurement.

The objective is to develop a control law using sliding mode control to ensure the tracking performances and the robustness of the closed loop system. So, forcing the system to track a reference

trajectory is equivalent to forcing the plant to attain a sliding surface and to maintain it on it. The Hurwitzian structure of this surface allows the convergence of the plant to the phase plan origin.

## 3. DIRECT ADAPTIVE FUZZY CONTROLLER

In this section our task is to synthesise a direct adaptive fuzzy controller for the system (1). For this, we assume the following:

$$\textbf{Assumption A1: } b_u = \frac{\partial [f(\underline{x}, u)]}{\partial u} > b_0 \neq 0$$

$$\textbf{Assumption A2: } \text{There exists a smooth function } \beta(\underline{x}) > 0 \text{ such that } \left| \frac{\dot{b}_u}{b_u} \right| = \left| \frac{\partial b_u / \partial t}{b_u} \right| b_u \leq \beta(\underline{x}).$$

To attain the desired tracking performances, let's consider the sliding surface  $S$  or the filtered error given as follows [3]:

$$\begin{aligned} S &= -e^{(n-1)} - k_{n-1}e^{(n-2)} - \dots - k_1e \\ &= -e^{(n-1)} - \sum_{i=1}^{n-1} k_i e^{(i-1)}, \end{aligned} \quad (2)$$

where  $e = y_r - y$  denotes the tracking error and  $y_r$  is a bounded reference trajectory. The gains  $k_i$ ,  $i = 1, \dots, n$ , are chosen such that the corresponding polynomial is Hurwitzian.

Using (1) and (2), the time derivative of the switching surface  $S$  can be written as:

$$\begin{aligned} \dot{S} &= -e^{(n)} - \sum_{i=1}^{n-1} k_i e^{(i)} \\ &= f(\underline{x}, u) + d - y_r^{(n)} - \sum_{i=1}^{n-1} k_i e^{(i)}. \end{aligned} \quad (3)$$

**Lemma 1** [29]: For the system (1) free of external disturbances ( $d=0$ ) satisfying A1 and A2, there exists a compact set  $\Phi_u$  and an unique ideal input  $u^*$  such that all  $\underline{x}(0) \in \Phi_u$ , the equation (3) can be expressed as the following form:

$$\dot{S} = -\beta(\underline{x})S, \quad (4)$$

which allows to obtain  $\lim_{x \rightarrow \infty} |y_r - y| = 0$ .  $\square$

The previous lemma guarantees only the existence of a control law guaranteeing the convergence of the tracking error toward zero and does not provide the method of constructing it [29]. Based on the fact that a fuzzy system is an universal approximator [11], we

use a fuzzy Takagi-Sugeno system to approximate the ideal law  $u^*$ . To guarantee the stability of the closed loop system, we add a supplementary signal  $u_s$ . Hence, the proposed control law is given by:

$$u = u_{fuzzy} + u_s, \quad (5)$$

$$u_{fuzzy} = \theta^T \Psi(\underline{x}), \quad (6)$$

$$u_s = -\frac{S}{\rho^2}, \quad (7)$$

where  $\theta$  is the vector of the adjustable parameters,  $\Psi(\underline{x})$  the regressive vector, and  $\rho$  is a positive constant representing the attenuation level of the effects of both the approximation error and the external disturbances. Note that  $u^*$  is written as  $u^* = \theta^{*T} \Psi(\underline{x}) + \delta$ , where  $\theta^*$  is the optimal value of  $\theta$  and  $\delta$  is the approximation error.

Using the Mean Value Theory [30], there exists a positive constant  $\lambda \in ]0, 1[$  such that:

$$f(\underline{x}, u) = f(\underline{x}, u^*) + b_{u\lambda}(u - u^*), \quad (8)$$

$$\text{where } b_{u\lambda} = \left. \frac{\partial [f(\underline{x}, u)]}{\partial u} \right|_{u=u_\lambda} \quad \text{and } u_\lambda = \lambda u + (1 - \lambda)u^*.$$

According to the implicit theorem [31], there exists a  $u^*$  such that  $v + f(\underline{x}, u^*) = 0$  where  $v = \beta(\underline{x})S - y_r^{(n)} - \sum_{i=1}^{n-1} k_i e^{(i)}$ . So, (8) can be written as:

$$\begin{aligned} f(\underline{x}, u) &= -v + b_{u\lambda}(u - u^*) \\ &= -\beta(\underline{x})S + y_r^{(n)} + \sum_{i=1}^{n-1} k_i e^{(i)} + b_{u\lambda}(u - u^*). \end{aligned} \quad (9)$$

From (3) and (9), we can obtain:

$$\dot{S} = -\beta(\underline{x})S + b_{u\lambda}(u - u^*) + d. \quad (10)$$

Using (5), equation (10) becomes:

$$\begin{aligned} \dot{S} &= -\beta(\underline{x})S + b_{u\lambda}u_{fuzzy} + b_{u\lambda}u_s \\ &\quad - b_{u\lambda}\theta^{*T}\Psi(\underline{x}) - b_{u\lambda}\delta + d \end{aligned} \quad (11)$$

or

$$\begin{aligned} \dot{S} &= -\beta(\underline{x})S + b_{u\lambda}\theta^T\Psi(\underline{x}) - b_{u\lambda}\theta^{*T}\Psi(\underline{x}) \\ &\quad - b_{u\lambda}\frac{S}{\rho^2} - b_{u\lambda}\delta + d, \end{aligned} \quad (12)$$

$$\dot{S} = -\beta(\underline{x})S + b_{u\lambda}\tilde{\theta}^T\Psi(\underline{x}) - b_{u\lambda}\frac{S}{\rho^2} - b_{u\lambda}\delta + d, \quad (13)$$

which gives

$$b_{u\lambda}^{-1}\dot{S} = -b_{u\lambda}^{-1}\beta(\underline{x})S + \tilde{\theta}^T\Psi(\underline{x}) - \frac{S}{\rho^2} - \delta + b_{u\lambda}^{-1}d, \quad (14)$$

where  $\tilde{\theta} = \theta - \theta^*$ .

Consider the following Lyapunov function:

$$V = \frac{1}{2}b_{u\lambda}^{-1}S^2 + \frac{1}{2\eta}\tilde{\theta}^T\tilde{\theta}. \quad (15)$$

Differentiating (15) along (12) yields:

$$\dot{V} = \frac{1}{2}\dot{S}b_{u\lambda}^{-1}S + \frac{1}{2}Sb_{u\lambda}^{-1}\dot{S} + \frac{1}{2\eta}\tilde{\theta}^T\dot{\tilde{\theta}} + \frac{1}{2\eta}\tilde{\theta}^T\dot{\tilde{\theta}}. \quad (16)$$

Using the fact that  $\dot{\tilde{\theta}} = \dot{\theta} - \dot{\theta}^* = \dot{\theta}$  and since the elements of  $\dot{V}$  are scalars, the equation (16) can be rewritten as:

$$\begin{aligned} \dot{V} &= Sb_{u\lambda}^{-1}\dot{S} + \frac{1}{\eta}\tilde{\theta}^T\dot{\theta} \\ &= S\left(-b_{u\lambda}^{-1}\beta(\underline{x})S + \tilde{\theta}^T\Psi(\underline{x}) - \frac{S}{\rho^2} - b_{u\lambda}\delta + b_{u\lambda}^{-1}d\right) + \frac{1}{\eta}\tilde{\theta}^T\dot{\theta} \\ &= -b_{u\lambda}^{-1}\beta(\underline{x})S^2 + \tilde{\theta}^T S\Psi(\underline{x}) - \frac{S^2}{\rho^2} - S(\delta - b_{u\lambda}^{-1}d) + \frac{1}{\eta}\tilde{\theta}^T\dot{\theta} \\ &= -b_{u\lambda}^{-1}\beta(\underline{x})S^2 - \frac{S^2}{\rho^2} - S(\delta - b_{u\lambda}^{-1}d) + \frac{1}{\eta}\tilde{\theta}^T(\dot{\theta} + \eta S\Psi(\underline{x})) \\ &\leq -\frac{S^2}{\rho^2} - S(\delta - b_{u\lambda}^{-1}d) + \frac{1}{\eta}\tilde{\theta}^T(\dot{\theta} + \eta S\Psi(\underline{x})). \end{aligned} \quad (17)$$

Choosing the following adaptation law:

$$\dot{\theta} = -\eta S\Psi(\underline{x}) \quad (18)$$

gives:

$$\begin{aligned} \dot{V} &\leq -\frac{S^2}{\rho^2} - S(\delta - b_{u\lambda}^{-1}d) \\ &\leq -\frac{S^2}{2\rho^2} - \frac{S^2}{2\rho^2} - S(\delta - b_{u\lambda}^{-1}d) \\ &\leq -\frac{S^2}{2\rho^2} - \frac{1}{2}\left[\frac{S^2}{\rho^2} + 2S(\delta - b_{u\lambda}^{-1}d) + \rho^2(\delta - b_{u\lambda}^{-1}d)^2\right] \\ &\quad + \frac{\rho^2}{2}(\delta - b_{u\lambda}^{-1}d)^2 \\ &\leq -\frac{S^2}{2\rho^2} + \frac{\rho^2}{2}(\delta - b_{u\lambda}^{-1}d)^2. \end{aligned} \quad (19)$$

Integrating the above inequality from  $t=0$  and  $T$ , we have:

$$V(T) - V(0) \leq -\int_0^T \frac{S^2}{2\rho^2} dt + \frac{\rho^2}{2} \int_0^T (\delta - b_{u\lambda}^{-1}d)^2 dt, \quad (20)$$

$$\int_0^T \frac{S^2}{2\rho^2} dt \leq V(0) - V(T) + \frac{\rho^2}{2} \int_0^T (\delta - b_{u\lambda}^{-1}d)^2 dt. \quad (21)$$

Using the fact that  $V(T) \geq 0$ , the above inequality can be simplified as:

$$\int_0^T \frac{S^2}{2\rho^2} dt \leq V(0) + \frac{\rho^2}{2} \int_0^T (\delta - b_{u\lambda}^{-1}d)^2 dt, \quad (22)$$

(22) guarantees that  $S \in L_\infty$ . Because all the variables in the right-hand side of (14) are bounded, i.e.,  $\dot{S} \in L_\infty$ . Since the right side of (22) are also bounded,  $S \in L_2$  [28,29]. Using Barbalat Lemma, we have  $S \rightarrow 0$  when  $t \rightarrow \infty$  [29]. Therefore, the tracking error converges to the origin, i.e.,  $\lim_{x \rightarrow \infty} e = 0$  [3].

To ensure the convergence of the adaptive algorithm, we introduce the projection algorithm defined by (23).

$$\dot{\theta} = \begin{cases} -\eta S \Psi(\underline{x}) & \text{if } [\|\theta\| \leq u_{\max}] \\ & \text{or } [\|\theta\| = u_{\max} \\ & \text{and } S \Psi(\underline{x}) \geq 0] \\ -\eta \left[ S \Psi(\underline{x}) + S \frac{\theta \theta^T}{\|\theta\|^2} \Psi(\underline{x}) \right] & \text{if } [\|\theta\| = u_{\max} \\ & \text{and } S \Psi(\underline{x}) < 0] \end{cases} \quad (23)$$

#### 4. INDIRECT ADAPTIVE FUZZY CONTROLLER

In the case where Assumptions A1 and A2 are restrictive, and we cannot satisfy them, we can use an indirect adaptive fuzzy controller. The main idea is to construct an affine model of the plant using a Takagi-Sugeno system. Then we use it to synthesise a robust controller allowing it to ensure the tracking performances and the robustness of the closed loop system.

The plant is constructed from a Takagi-Sugeno system whose inputs are the state variables  $x_1$ ,  $i=1, \dots, n$ , and  $u$ . For each variable  $x_1$ ,  $i=1, \dots, n$ , and  $u$ , we define  $p_i$  and  $M$  fuzzy sets. Hence, the  $j^{\text{th}}$  rule can be written in the form:

$$\begin{aligned} \text{Rule } j \text{ IF } [x_1 \text{ is } A_1^{j1} \text{ And } \dots x_n \text{ is } A_n^{jn} \text{ And } u \text{ is } B^m] \\ \text{THEN } [\dot{x}_n = \theta^{(j1, \dots, jn, m)}], \end{aligned} \quad (24)$$

where  $j_i \in \{1, \dots, p_i\}$  for  $i=1, \dots, n$  and  $m \in \{1, \dots, M\}$ .  $\theta^{(j1, \dots, jn, m)}$  is a constant corresponding to the  $j^{\text{th}}$  rule where the fuzzy sets  $A_1^{j1}, \dots, A_n^{jn}, B^m$  are used.

Using the singleton fuzzifier, the centre average defuzzification and the product inference engine, the output of the Takagi-Sugeno system can be given by [8]:

$$\dot{x}_n = \frac{\sum_{j1}^{p1} \sum_{jn}^{pn} \sum_1^M \theta^{(j1, \dots, jn, m)} \left[ \prod_{i=1}^n \mu_{A_i^{ji}}(x_i) \right] \mu_{B^m}(u)}{\sum_{j1}^{p1} \sum_{jn}^{pn} \sum_1^M \left[ \prod_{i=1}^n \mu_{A_i^{ji}}(x_i) \right] \mu_{B^m}(u)}, \quad (25)$$

or on the following vectorial form:

$$\dot{x}_n = \Theta^T \Psi(\underline{x}, u), \quad (26)$$

where  $\Psi(\underline{x}, u)$  is a  $p_1 \times \dots \times p_n \times M$  dimensional vector with its  $(j1, \dots, jn, m)^{\text{th}}$  element given by:

$$\Psi^{(j1, \dots, jn, m)} = \frac{\left[ \prod_{i=1}^n \mu_{A_i^{ji}}(x_i) \right] \mu_{B^m}(u)}{\sum_{j1}^{p1} \sum_{jn}^{pn} \sum_1^M \left[ \prod_{i=1}^n \mu_{A_i^{ji}}(x_i) \right] \mu_{B^m}(u)}, \quad (27)$$

and  $\Theta = [\theta^{(1, \dots, 1, 1)}, \dots, \theta^{(p1, \dots, pn, M)}]^T$  denotes the vector of the adjustable parameters.

Let's consider that the membership functions of linguistic variables of  $u$  have the form of a triangle and are placed evenly throughout the whole defined space  $U_u$  as illustrated in Fig. 1. The space  $U_u$  can

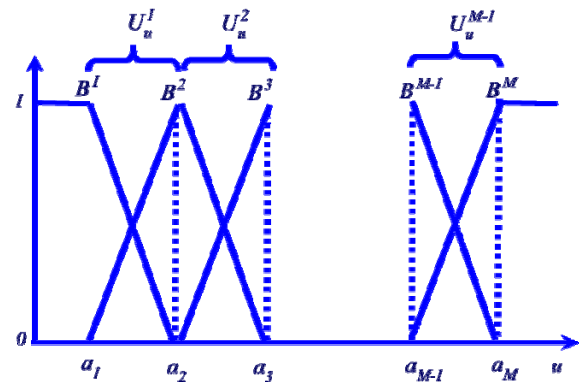


Fig. 1. The membership functions of  $u$ .

be decomposed into several subspaces  $U_u^\alpha$ ,  $\alpha=1,2,\dots,M-1$  [25]. If  $u$  exists in the subspace  $U_u^\alpha$ , all membership function values are given by:

$$\mu_{B^m}(u) = \begin{cases} \frac{u - a_{m+1}}{a_m - a_{m+1}} & m = \alpha \\ \frac{a_{m-1} - u}{a_{m-1} - a_m} & m = \alpha + 1 \\ 0 & \text{otherwise,} \end{cases} \quad (28)$$

where  $a_m$  is a constant satisfying  $\mu_{B^m}(a_m) = 1$ .

For a given value of  $\alpha = \{1, 2, \dots, M-1\}$ , the control input  $u$  exists in the subspace  $U_u^\alpha$ . So, substituting (28) in (25) gives:

$$\begin{aligned} \hat{f}(\underline{x}, u, \Theta) &= \sum_{j_1}^{p_1} \dots \sum_{j_n}^{p_n} \Psi_s \left[ a_\alpha \theta^{(j_1, \dots, j_n, \alpha)} - a_{\alpha+1} \theta^{(j_1, \dots, j_n, \alpha+1)} \right] \\ &\quad + \sum_{j_1}^{p_1} \dots \sum_{j_n}^{p_n} \Psi_s \left[ \theta^{(j_1, \dots, j_n, \alpha)} - \theta^{(j_1, \dots, j_n, \alpha+1)} \right] u \\ &= \Phi_1^\alpha(\underline{x}) + \Phi_2^\alpha(\underline{x})u, \end{aligned} \quad (29)$$

$$\text{where } \Psi_s = \frac{1}{a_\alpha - a_{\alpha+1}} \frac{\left[ \prod_{i=1}^n \mu_{A_i^{j_i}}(x_i) \right]}{\sum_{j_1}^{p_1} \dots \sum_{j_n}^{p_n} \sum_{l=1}^M \left[ \prod_{i=1}^n \mu_{A_i^{j_i}}(x_i) \right] \mu_{B^l}(u)}.$$

Therefore, the fuzzy system can be decomposed into  $M-1$  subsystems, which allows to obtain an affine-in-control model of the plant [25,26].

After synthesising the fuzzy model, our next task is to develop a robust controller to ensure the global stability and the robustness of the closed loop system. So, the proposed control law is given by:

$$u = \left[ \Phi_2^\alpha(\underline{x}) \right]^{-1} \left[ -\Phi_1^\alpha(\underline{x}) + y_r^{(n)} + \sum_{i=1}^{n-1} k_i e^{(i)} + u_s \right], \quad (30)$$

where  $u_s$  denotes an additional term guaranteeing the robustness of the closed loop system by attenuating the effects of both the external disturbances and the approximation errors to a prescribed level  $\rho$ . To attain this objective, we

choose  $u_s$  as given in (7):  $u_s = -\frac{S}{\rho^2}$ .

Using (30), the time derivative of the sliding surface is given by:

$$\dot{S} = f(\underline{x}, u) - \hat{f}(\underline{x}, u, \Theta) + u_s + d. \quad (31)$$

If we note by  $\hat{f}^*(\underline{x}, u, \Theta^*) = \Theta^{*T} \Psi(\underline{x}, u)$  the optimal value of  $\hat{f}(\underline{x}, u, \Theta) = \Theta^T \Psi(\underline{x}, u)$  and by  $w = f(\underline{x}, u) - \hat{f}^*(\underline{x}, u, \Theta^*)$  the minimal approximation error, (31) can be rewritten as:

$$\dot{S} = w + \tilde{\Theta}^T \Psi(\underline{x}, u) + d - \frac{S}{\rho^2}, \quad (32)$$

To determine the adaptation law of the adjustable parameter vector  $\Theta$ , we consider the following Lyapunov function:

$$V = \frac{1}{2} S^2 + \frac{1}{2\sigma} \tilde{\Theta}^T \tilde{\Theta}, \quad (33)$$

where  $\sigma$  is a positive constant given by the designer.

Using (32) and the fact that  $\dot{\tilde{\Theta}} = -\dot{\Theta}$ , the time derivative of (33) can be written as:

$$\dot{V} = S \left[ w + \tilde{\Theta}^T \Psi(\underline{x}, u) + d - \frac{S}{\rho^2} \right] - \frac{1}{\sigma} \tilde{\Theta}^T \dot{\Theta}, \quad (34)$$

$$\dot{V} = S[w + d] - \frac{S^2}{\rho^2} - \frac{1}{\sigma} \tilde{\Theta}^T [\dot{\Theta} - \sigma S \Psi(\underline{x}, u)], \quad (35)$$

$$\dot{V} = S[w + d] - \frac{S^2}{2\rho^2} - \frac{S^2}{2\rho^2} - \frac{1}{\sigma} \tilde{\Theta}^T [\dot{\Theta} - \sigma S \Psi(\underline{x}, u)], \quad (36)$$

$$\begin{aligned} \dot{V} &= -\frac{1}{2} \left[ \rho^2 [w + d]^2 - 2S[w + d] - \frac{S^2}{\rho^2} \right] \\ &\quad + \frac{1}{2} \rho^2 [w + d]^2 - \frac{S^2}{2\rho^2} - \frac{1}{\sigma} \tilde{\Theta}^T [\dot{\Theta} - \sigma S \Psi(\underline{x}, u)], \end{aligned} \quad (37)$$

$$\begin{aligned} \dot{V} &= -\frac{1}{2} \left[ \rho [w + d] - \frac{S}{\rho} \right]^2 + \frac{\rho^2}{2} [w + d]^2 \\ &\quad - \frac{S^2}{2\rho^2} - \frac{1}{\sigma} \tilde{\Theta}^T [\dot{\Theta} - \sigma S \Psi(\underline{x}, u)], \end{aligned} \quad (38)$$

$$\dot{V} \leq \frac{1}{2} [w + d]^2 - \frac{S^2}{2\rho^2} - \frac{1}{\sigma} \tilde{\Theta}^T [\dot{\Theta} - \sigma S \Psi(\underline{x}, u)]. \quad (39)$$

If we choose the following adaptation law:

$$\dot{\Theta} = \sigma S \Psi(\underline{x}, u) \quad (40)$$

and using the same mathematical tool development presented in the previous section, we can obtain:

$$\int_0^T \frac{S^2}{2\rho^2} dt \leq V(0) + \frac{\rho^2}{2} \int_0^T [w + d]^2 dt. \quad (41)$$

As proven in the previous section, we have  $S \in L_2$

and  $\dot{S} \in L_\infty$ . Hence, we have  $S \rightarrow 0$  when  $t \rightarrow \infty$ . According to the Hurwitzian structure of the sliding surface, the system converges to the origin of the phase plan [3].

**Remark 1:** To overcome the singularity problem when  $\Phi_2^\alpha(\underline{x}) = 0$  without complicating the controller structure, we propose to substitute the term  $[\Phi_2^\alpha(\underline{x})]^{-1}$  by  $\frac{[\Phi_2^\alpha(\underline{x})]}{\varepsilon + [\Phi_2^\alpha(\underline{x})]^2}$  [33] which yields

the new control law:

$$u = \frac{[\Phi_2^\alpha(\underline{x})]}{\varepsilon + [\Phi_2^\alpha(\underline{x})]^2} \left[ -\Phi_1^\alpha(\underline{x}) + y_r^{(n)} + \sum_{i=1}^{n-1} k_i e^{(i)} + u_s \right], \quad (42)$$

where  $\varepsilon$  is a small positive constant.

**Remark 2:** To ensure the convergence of the adaptive algorithm, we modify the adaptation law by using the projection technique as given by (43).

$$\dot{\Theta} = \begin{cases} \sigma S \Psi(\underline{x}, u) & \text{:if } [\|\Theta\| \leq f_{\max}] \\ & \text{or } [\|\Theta\| = f_{\max} \\ & \text{and } S \Psi(\underline{x}, u) \leq 0] \\ \sigma \left[ S \Psi(\underline{x}, u) - S \frac{\Theta \Theta^T}{\|\Theta\|^2} \Psi(\underline{x}) \right] & \text{:if } [\|\Theta\| = f_{\max} \\ & \text{and } S \Psi(\underline{x}, u) > 0] \end{cases} \quad (43)$$

## 5. AN ILLUSTRATION EXAMPLE

To illustrate the effectiveness of the proposed adaptive controllers, we consider the following system:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_1^2 + 0.1u^3 + 0.1(1 + x_2^2)u + \sin(0.1u) \\ y = x_1. \end{cases} \quad (44)$$

In this simulation example, the control objective is to determine  $u$  so that the output  $y$  follows the desired reference trajectory  $y_r = \sin(t)$ . To show the robustness of the proposed approaches, we consider that the system is subject to external disturbances in the form  $d = 0.2(\sin(t) + \sin(2t))$ .

To synthesise the direct adaptive fuzzy controller, we use a fuzzy system with the state variables  $x_1$  and  $x_2$  as inputs and  $u_{fuzzy}$  as output. For each input variable, we define five fuzzy sets covering uniformly their universes of discourse  $[-1.5, 1.5]$ , whose corresponding membership functions are

triangular. The plant must attain the following surface  $S = \dot{e} + e$ , and slide on it to reach the origin  $\dot{e} = e = 0$ . Figs. 2, 3 and 4 present the simulation results for the initial state  $[x_1 \ x_2]^T = [0.5 \ 0.5]^T$  and the attenuation level  $\rho = 0.05$ . Figs. 2 and 3 demonstrate the good tracking performances and the convergence of the state variables to their reference trajectories. In

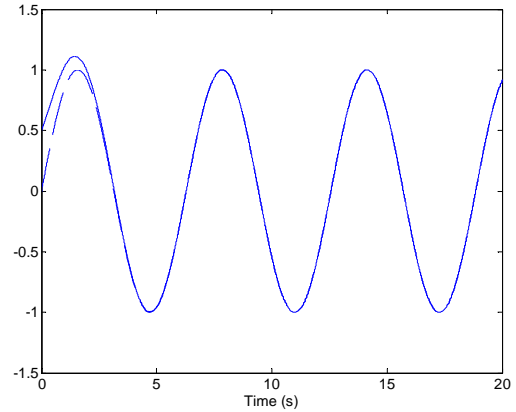


Fig. 2. Evolution of the state variable  $x_1$  (solid line) and its reference trajectory (dashed line).

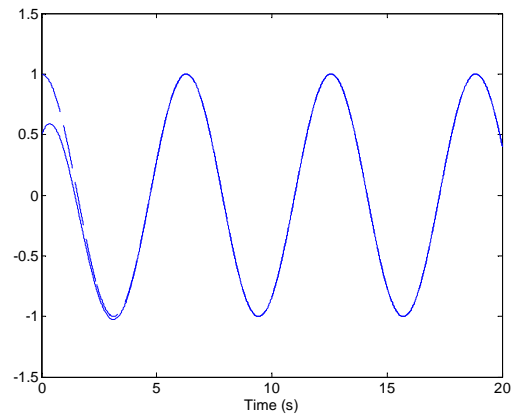


Fig. 3. Evolution of the state variable  $x_2$  (solid line) and its reference trajectory (dashed line).

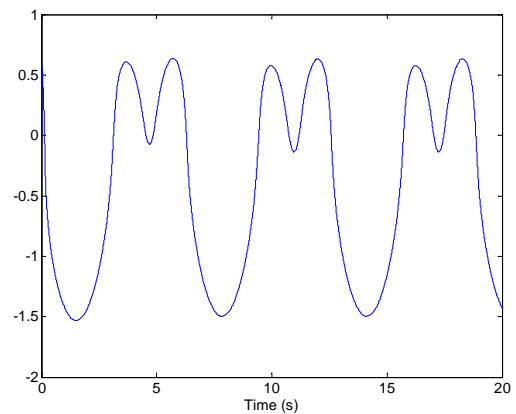


Fig. 4. Applied control signal.

Fig. 4, we remark that the control signal does not contain any abrupt variation despite the knowledge unavailability of the upper bound of the external disturbances.

To overcome the restrictive assumptions A1 and A2 used to design the control law (5), we propose to use the indirect adaptive fuzzy controller given by (31). For this, we must at first define a fuzzy system, which inputs the state variables  $(x_1 \ x_2)$  and the control input  $u$ , to approximate the dynamic function  $f(\underline{x}, u)$ . The universe of discourse of the inputs  $x_1$ ,  $x_2$ , and  $u$  are respectively  $[-1.5 \ 1.5]$ ,  $[-1.5 \ 1.5]$  and  $[-2.5 \ 2.5]$ . For each input variable, we have defined respectively 5, 5, and 6 linguistic sets. Figs. 5 and 6 give the evolution of the state variables  $x_1$  and  $x_2$  together with the corresponding reference signals. Despite using an affine-in-control fuzzy model to synthesise the control law, we can note the good tracking performances and the convergence of the system to the desired trajectories. This can be justified

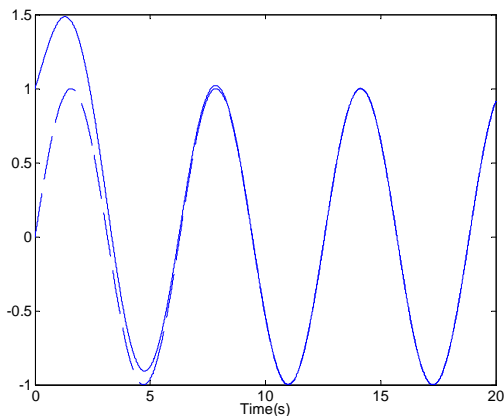


Fig. 5. Evolution of the state variable  $x_1$  (solid line) and its reference trajectory (dashed line).

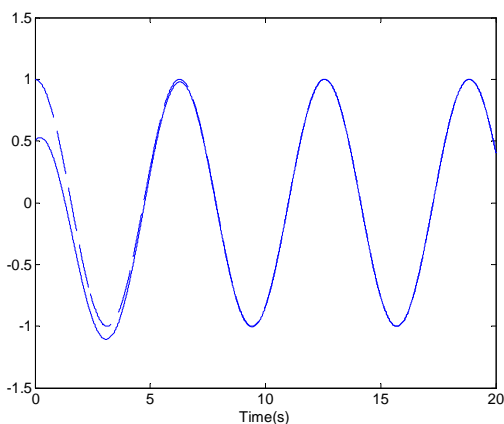


Fig. 6. Evolution of the state variable  $x_2$  (solid line) and its reference trajectory (dashed line).

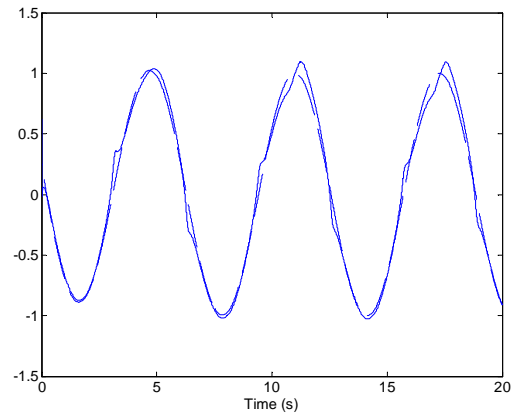


Fig. 7. Evolution of the dynamic function  $f(\underline{x}, u)$  (dashed line) and its approximation (continuous line).

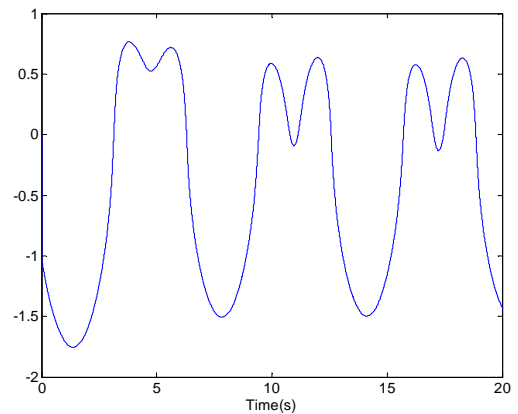


Fig. 8. Applied control signal.

by the good approximation level assured by the proposed approach as illustrated in Fig. 7. The applied control signal to attain our objective is given by Fig. 8.

In summary, we can conclude that the proposed approaches (direct and indirect schemes) allow the desired tracking performances to be attained and also ensure the robustness of the closed loop system, without using classical methods of linearization. Furthermore, the design of these controllers does not require any knowledge regarding the disturbances structure or their bounds.

## 6. CONCLUSION

In this paper direct and indirect robust adaptive fuzzy controllers for a class of nonaffine nonlinear systems are presented. In the direct case, based on the implicit theorem, a fuzzy adaptive system is used to attain the desired performances. In the indirect case, a Takagi-Sugeno system is used to synthesise an affine-in-control model, and hence to design the controller. In the two cases, the robustness of the closed loop system is guaranteed by a modified sliding mode

control where the upper bounds of both the external disturbances and the approximation errors are not required. To improve the approximation accuracy, the adaptation laws of the adjustable parameters are deduced from the stability analysis in the sense of Lyapunov. The simulation results demonstrate both good tracking performances and high efficiency of these approaches. Current works are focussed on the use of a state observer to overcome the assumption on the availability of the state variables to the measurement.

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