Tracking Control of Robotic Manipulators based on the All-Coefficient Adaptive Control Method

Yongjun Lei and Hongxin Wu

Abstract: A multi-variable Golden-Section adaptive controller is proposed for the tracking control of robotic manipulators with unknown dynamics. With a small sample time, the unknown dynamics of the robotic manipulator are denoted equivalently by a characteristic model of a 2-order multivariable time-varying difference equation. The coefficients of the characteristic model change slowly with time and some of their valuable characteristic relationships emerge. Based on the characteristic model, an adaptive algorithm with a simple form for the control of robotic manipulators is presented, which combines the multi-variable Golden-Section adaptive control law with the weighted least squares estimation method. Moreover, a compensation neural network law is incorporated into the designed controller to reduce the influence of the coefficients estimation error on the control performance. The results of the simulations indicate that the developed control scheme is effective in robotic manipulator control.

Keywords: Adaptive control, all-coefficient adaptive control, characteristic models, manipulators, multivariable control, golden-section control.

1. INTRODUCTION

Manipulator dynamics are highly coupled, nonlinear, and often have parametric and nonparametric uncertainty [1]. Therefore, over the past decade, many researchers have been engaged in the exploration of the adaptive control of manipulators. However, most of the related works [1-3] were based on the manipulator continuous-time linear parametrization model, which is a linear equation in terms of the unknown or un-precisely known parameter vector. As a result, the knowledge of the system dynamics structure had to be known in advance. In addition, most continuous-time control laws require a large amount of computation. Whereas, the digital control algorithm is usually very simple and can be easily implemented on-line using microcomputers, so it is predominantly more effective in actual application [4,5]. But only a few literatures [4-7] have studied the manipulator discrete-time model and discrete-time control so far, among which few researches have actively pursued manipulator adaptive control.

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The All-Coefficient Adaptive Control (ACAC) method [8], which was proposed creatively by Professor Wu in the 1980s, has found extensive application [8-10] in actual systems for its simple algorithm, excellent control performance, good adaptability and strong robustness. Golden Section Adaptive Control (GSAC), as a production of ACAC theory, is a new kind of adaptive control method that combines the Golden-Section control law with a parameter estimation algorithm, such as the least square estimation algorithm [8,11,12]. The advantage of this control method is that it can guarantee the stability of the closed-loop system and achieve good control performance even before the estimates of the parameters converge to the true values [11], which is very difficult to guarantee for other adaptive control methods. In this paper, based on the ACAC theory, when the sample time is small enough, the manipulator dynamic model is equivalent to a 2 order discrete-time equation whose coefficients are slowly time-varying and also possess some valuable characteristic relationships. We define the discrete-time equation as the characteristic model of manipulators, and then a multi-variable GSAC law is presented based on the manipulator characteristic model. With this control scheme, the problems resulting from the parametric and nonparametric uncertainty can be overcome effectively. Moreover, a neural network (NN) compensation law is incorporated into the system controller to eliminate the negative influence resulted from the estimation errors of the characteristic model coefficients to some extent.

This paper is organized as follows: In Section 2, the dynamics formulation and the manipulator characteristic model is introduced. In Section 3, a multi-variable GSAC law with NN compensation law is presented. A simulation example is given in Section 4. Finally, some conclusions are made in Section 5.

2. MANIPULATOR DYNAMIC MODEL AND CHARACTERISTIC MODEL

Consider a robotic manipulator with n degrees of freedom. The continuous Lagrangian dynamic model [1-3] is given by

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}+\boldsymbol{G}(\boldsymbol{q})=\boldsymbol{u}, \tag{1}$$

where $\boldsymbol{q}\in R^n$ and $\dot{\boldsymbol{q}}\in R^n$ are the vector of generalized joint coordinates and velocity coordinates, respectively. The inertia matrix $\boldsymbol{M}(\boldsymbol{q})=\boldsymbol{M}^T(\boldsymbol{q})>0$, and there exist two constant positive scalars $M_{\min}$ and $M_{\max}$ such that $M_{\min}\le\|\boldsymbol{M}\|\le M_{\max}$, $\boldsymbol{u}\in R^n$ is the vector of commanded generalized force, and $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$ and $\boldsymbol{G}(\boldsymbol{q})$ are the terms due to Coriolis, Centripetal and gravity forces. In actual application, the uncertain parameters and un-modeled dynamics usually exist in the established dynamic model in (1).

When the sample time T_s is small enough, at instant $t=kT_s$ $\dot{\boldsymbol{q}}$ and $\ddot{\boldsymbol{q}}$ can be approximated by $\dot{\boldsymbol{q}}\approx\frac{\boldsymbol{q}(k)-\boldsymbol{q}(k-1)}{T_s}$ and $\ddot{\boldsymbol{q}}\approx\frac{\boldsymbol{q}(k+1)-2\boldsymbol{q}(k)+\boldsymbol{q}(k-1)}{T_s^2}$, respectively. Using the above relationships the discrete-time representation of (1) becomes

$$\begin{aligned}&\frac{1}{T_s^2}\boldsymbol{M}(\boldsymbol{q}(k))\bullet\\&\left(\boldsymbol{q}(k+1)-\boldsymbol{f}_1(k)\boldsymbol{q}(k)-\boldsymbol{f}_2(k)\boldsymbol{q}(k-1)+\boldsymbol{G}'(k)\right)\\&=\boldsymbol{u}(k).\end{aligned} \tag{2a}$$

Premultiplying (2a) by $T_s^2\boldsymbol{M}^{-1}(\boldsymbol{q}(k))$ results in

$$\begin{aligned}&\boldsymbol{q}(k+1)-\boldsymbol{f}_1(k)\boldsymbol{q}(k)-\boldsymbol{f}_2(k)\boldsymbol{q}(k-1)+\boldsymbol{G}'(k)\\&=\boldsymbol{\beta}(k)\boldsymbol{u}(k),\end{aligned} \tag{2b}$$

where

$$\boldsymbol{f}_1(k)=2\boldsymbol{I}-T_s\boldsymbol{M}^{-1}(\boldsymbol{q}(k))\boldsymbol{C}(\boldsymbol{q}(k),\dot{\boldsymbol{q}}(k)),$$
$$\boldsymbol{f}_2(k)=-\boldsymbol{I}+T_s\boldsymbol{M}^{-1}(\boldsymbol{q}(k))\boldsymbol{C}(\boldsymbol{q}(k),\dot{\boldsymbol{q}}(k)),$$
$$\boldsymbol{G}'(k)=T_s^2\boldsymbol{M}^{-1}(\boldsymbol{q}(k))\boldsymbol{G}(\boldsymbol{q}(k)),$$
$$\boldsymbol{\beta}(k)=T_s^2\boldsymbol{M}^{-1}(\boldsymbol{q}(k)),$$

and $\boldsymbol{I}$ denotes the unitary diagonal matrix with an appropriate dimension.

If the designed $\boldsymbol{u}(t,\boldsymbol{q},\dot{\boldsymbol{q}})$ is continuous in t, $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$, then the solution $(\boldsymbol{q},\dot{\boldsymbol{q}})$ of (1) will be continuously differentiable. Let $\boldsymbol{W}(\boldsymbol{q},\dot{\boldsymbol{q}})=\boldsymbol{M}^{-1}(\boldsymbol{q})\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$ and $w_{ij}(\boldsymbol{q},\dot{\boldsymbol{q}})$ be ij-th element of matrix $\boldsymbol{W}(\boldsymbol{q},\dot{\boldsymbol{q}})$; We define $\Delta\boldsymbol{F}_1(k)\triangleq\frac{1}{T_s}(\boldsymbol{f}_1(k)-\boldsymbol{f}_1(k-1))$, and then $\Delta\boldsymbol{F}_1(k)$ can be expressed as

$$\Delta\boldsymbol{F}_1(k)=\boldsymbol{W}(\boldsymbol{q}(k),\dot{\boldsymbol{q}}(k))-\boldsymbol{W}(\boldsymbol{q}(k-1),\dot{\boldsymbol{q}}(k-1)).$$

For the ij-th element $\Delta f_{1,ij}(k)$ of matrix $\Delta\boldsymbol{F}_1(k)$ we can get

$$\begin{aligned}&\Delta f_{1,ij}(k)\\&=-\left(w_{ij}(\boldsymbol{q}(k),\dot{\boldsymbol{q}}(k))-w_{ij}(\boldsymbol{q}(k-1),\dot{\boldsymbol{q}}(k-1))\right)\\&=-\frac{\partial}{\partial\boldsymbol{q}^T}\left(w_{ij}(\varsigma_1\boldsymbol{q},\varsigma_1\dot{\boldsymbol{q}})\right)\Big|_{\substack{\boldsymbol{q}=\boldsymbol{q}(k-1)\\\dot{\boldsymbol{q}}=\dot{\boldsymbol{q}}(k-1)}}(\boldsymbol{q}(k)-\boldsymbol{q}(k-1))\\&\quad-\frac{\partial}{\partial\dot{\boldsymbol{q}}^T}\left(w_{ij}(\varsigma_1\boldsymbol{q},\varsigma_1\dot{\boldsymbol{q}})\right)\Big|_{\substack{\boldsymbol{q}=\boldsymbol{q}(k-1)\\\dot{\boldsymbol{q}}=\dot{\boldsymbol{q}}(k-1)}}(\dot{\boldsymbol{q}}(k)-\dot{\boldsymbol{q}}(k-1))\\&=-\frac{\partial}{\partial\boldsymbol{q}^T}\left(w_{ij}(\varsigma_1\boldsymbol{q},\varsigma_1\dot{\boldsymbol{q}})\right)\Big|_{\substack{\boldsymbol{q}=\boldsymbol{q}(k-1)\\\dot{\boldsymbol{q}}=\dot{\boldsymbol{q}}(k-1)}}\varsigma_2T_s\dot{\boldsymbol{q}}(k-1)\\&\quad-\frac{\partial}{\partial\dot{\boldsymbol{q}}^T}\left(w_{ij}(\varsigma_1\boldsymbol{q},\varsigma_1\dot{\boldsymbol{q}})\right)\Big|_{\substack{\boldsymbol{q}=\boldsymbol{q}(k-1)\\\dot{\boldsymbol{q}}=\dot{\boldsymbol{q}}(k-1)}}\varsigma_3T_s\ddot{\boldsymbol{q}}(k-1)\end{aligned} \tag{3}$$

with $0\le\varsigma_1\le1$, and $\varsigma_2,\varsigma_3\approx1$ for a small sample time T_s. From (3), it can be seen that $\Delta\boldsymbol{F}_1(k)\to\boldsymbol{0}$ as T_s converges to zero in a compact set of $(\boldsymbol{q},\dot{\boldsymbol{q}})$. Similar properties can also be achieved for the coefficient matrixes $\boldsymbol{f}_2(k)$, $\boldsymbol{G}'(k)$ and $\boldsymbol{\beta}(k)$.

In a compact set of $(\boldsymbol{q},\dot{\boldsymbol{q}})$, the following properties can be deduced from (3) and the expressions of the coefficient matrixes of (2b):

Property 1: If the sample time T_s is small enough, then all coefficient matrixes of (2b) are slowly time-varying;

Property 2: $\boldsymbol{f}_1(k)\to2\boldsymbol{I}$, $\boldsymbol{f}_2(k)\to-\boldsymbol{I}$ and $\boldsymbol{f}_1(k)+\boldsymbol{f}_2(k)\to\boldsymbol{I}$, as the sample time T_s converges to zero. Then we can define the discrete equation (2b) with Properties 1 and *2* as the *robotic manipulator characteristic model.*

3. MULTI-VARIABLE GSA CONTROLLER WITH NN COMPENSATION

Discrete equation (2b) can be expressed as follows:

$$\boldsymbol{q}(k+1)=\boldsymbol{\Theta}^T(k)\boldsymbol{\Phi}(k)+\boldsymbol{e}(k)\,, \tag{4}$$

where

$$\boldsymbol{\Theta}(k)=\left[\boldsymbol{f}_1(k),\boldsymbol{f}_2(k),\boldsymbol{\beta}(k),\boldsymbol{G}'(k)\right]^T,$$
$$\boldsymbol{\Phi}(k)=\left[\boldsymbol{q}^T(k),\boldsymbol{q}^T(k-1),\boldsymbol{u}^T(k),-\boldsymbol{1}^T\right]^T,$$

$\boldsymbol{e}(k)$ denotes the vector of white noise with zero mean. In the case of $\boldsymbol{G}'(k)\equiv\boldsymbol{0}$, $\boldsymbol{\Theta}(k)$ and $\boldsymbol{\Phi}(k)$ can be reduced to

$$\boldsymbol{\Theta}(k)=\left[\boldsymbol{f}_1(k),\boldsymbol{f}_2(k),\boldsymbol{\beta}(k)\right]^T,$$
$$\boldsymbol{\Phi}(k)=\left[\boldsymbol{q}^T(k),\boldsymbol{q}^T(k-1),\boldsymbol{u}^T(k)\right]^T,$$

Then the elements of $\boldsymbol{q}(k+1)$ can be expressed as

$$q_i(k+1))=\boldsymbol{\Phi}^T(k)\boldsymbol{\theta}_i(k)+e_i(k)\,, \tag{5}$$

where $i=1,\cdots,n$, $q_i(k+1))$ is the element i of $\boldsymbol{q}(k+1)$, $e_i(k)$ is the element i of $\boldsymbol{e}(k)$ and $\boldsymbol{\theta}_i(k)$ is the column i of the matrix $\boldsymbol{\Theta}(k)$. When the coefficient matrixes are unknown, it can be estimated by

$$\hat{\boldsymbol{\Theta}}(k)=\pi\left(\boldsymbol{q}(k),\boldsymbol{q}(k-1),...,\boldsymbol{u}(k-1),...\right), \tag{6a}$$

where $\hat{\boldsymbol{\Theta}}(k)$ is the estimated coefficient matrix of $\boldsymbol{\Theta}(k)$ at the instant $t=kT_s$, and $\pi(\bullet)$ denotes an estimation operator. Considering the coefficient matrixes of the characteristic model being slowly time-varying, we can obtain the selected estimation operator by the weighted least squares method (WLS) [13], namely

$$\begin{aligned}&\hat{\boldsymbol{\theta}}_i(k+1)=\\&\hat{\boldsymbol{\theta}}_i(k)+\frac{\boldsymbol{P}(k-1)\boldsymbol{\Phi}(k)}{\lambda(k)+\boldsymbol{\Phi}^T(k)\boldsymbol{P}(k-1)\boldsymbol{\Phi}(k)}\bullet\\&\left(q_i(k+1)-\boldsymbol{\Phi}^T(k)\hat{\boldsymbol{\theta}}_i(k)\right),\end{aligned} \tag{6b}$$

$$\boldsymbol{P}(k)=\lambda^{-1}(k)\bullet\left(\boldsymbol{P}(k-1)-\frac{\boldsymbol{P}(k-1)\boldsymbol{\Phi}^T(k)\boldsymbol{\Phi}^T(k)\boldsymbol{P}(k-1)}{\lambda(k)+\boldsymbol{\Phi}^T(k)\boldsymbol{P}(k-1)\boldsymbol{\Phi}(k)}\right)$$

with $\lambda(k+1)=\mu_0\lambda(k)+(1-\mu_0)$, $0<\mu_0\le 1$, and $\hat{\boldsymbol{\theta}}_i(k)$ the column i of the matrix $\hat{\boldsymbol{\Theta}}(k)$.

Given a desired smooth trajectory $\boldsymbol{q}_d(t)$, the adaptive control controller is designed as follows

$$\boldsymbol{u}(k)=\boldsymbol{u}_T(k)+\boldsymbol{u}_G(k)+\boldsymbol{u}_c(k) \tag{7}$$

with the feed forward control law designed as

$$\begin{aligned}\boldsymbol{u}_T(k)&=\left(\hat{\boldsymbol{\beta}}(k)+\varepsilon(k)\boldsymbol{I}\right)^{-1}\bullet\\&\left(\boldsymbol{q}_d(k+1)-\hat{\boldsymbol{f}}_1(k)\boldsymbol{q}_d(k)-\hat{\boldsymbol{f}}_2(k)\boldsymbol{q}_d(k-1)\right)\end{aligned} \tag{7a}$$

and the multi-variable GSAC feedback law as

$$\begin{aligned}\boldsymbol{u}_G(k)&=\left(\hat{\boldsymbol{\beta}}(k)+\varepsilon(k)\boldsymbol{I}\right)^{-1}\bullet\\&\left(L_1\hat{\boldsymbol{f}}_1(k)\tilde{\boldsymbol{q}}(k)+L_2\hat{\boldsymbol{f}}_2(k)\tilde{\boldsymbol{q}}(k-1)+\hat{\boldsymbol{G}}'(k)\right),\end{aligned} \tag{7b}$$

where $\tilde{\boldsymbol{q}}(k)=\boldsymbol{q}_d(k)-\boldsymbol{q}(k)$ is the tracking error and $\varepsilon(k)>0$ is a small scalar that avoids the estimated matrix $\hat{\boldsymbol{\beta}}(k)$ being singular. The term of $\boldsymbol{u}_c(k)$ will be designed later; L_1 and L_2 are golden-section coefficients, that is,

$$L_1=\frac{3-\sqrt{5}}{2}\approx 0.382\,,\quad L_2=\frac{\sqrt{5}-1}{2}\approx 0.618\,,$$

which satisfy the relationship $L_1+L_2=1$ and $L_1=L_2^2$.

Substituting (7) into (2), we can get

$$\begin{aligned}&\boldsymbol{M}(\boldsymbol{q}(k))\left(\tilde{\boldsymbol{q}}(k+1)-L_2\boldsymbol{f}_1\tilde{\boldsymbol{q}}(k)-L_1\boldsymbol{f}_2\tilde{\boldsymbol{q}}(k-1)\right)\\&=\boldsymbol{\Delta}_1(k)-T_s^2\boldsymbol{u}_c(k)\end{aligned} \tag{8}$$

with

$$\begin{aligned}\boldsymbol{\Delta}_1(k)=&\left(\boldsymbol{M}(\boldsymbol{q}(k))-\hat{\boldsymbol{M}}(k)\right)\boldsymbol{q}_d(k+1)\\&-\left(\boldsymbol{M}(\boldsymbol{q}(k))\boldsymbol{f}_1-\hat{\boldsymbol{M}}(k)\hat{\boldsymbol{f}}_1(k)\right)\left(L_2\boldsymbol{q}_d(k)+L_1\boldsymbol{q}(k)\right)\\&-\left(\boldsymbol{M}(\boldsymbol{q}(k))\boldsymbol{f}_2-\hat{\boldsymbol{M}}(k)\hat{\boldsymbol{f}}_2(k)\right)\bullet\\&\quad\left(L_1\boldsymbol{q}_d(k-1)+L_2\boldsymbol{q}(k-1)\right)\\&+\left(\boldsymbol{M}(\boldsymbol{q}(k))\boldsymbol{G}(k)-\hat{\boldsymbol{M}}(k)\hat{\boldsymbol{G}}(\boldsymbol{q}(k))\right)\end{aligned}$$

and $\hat{\boldsymbol{M}}(k)=T_s^2\left(\hat{\boldsymbol{\beta}}(k)+\varepsilon(k)\boldsymbol{I}\right)^{-1}$. Defining the tracking filtered error $\boldsymbol{s}(k+1)$ as $\boldsymbol{s}(k+1)\triangleq\tilde{\boldsymbol{q}}(k+1)-L_2\tilde{\boldsymbol{q}}(k)$ and using the relationships $L_1+L_2=1$ and $L_1=L_2^2$, (8) can be expressed as

$$\begin{aligned}\boldsymbol{M}(\boldsymbol{q}(k))\boldsymbol{s}(k+1)&=-\boldsymbol{M}(\boldsymbol{q}(k))L_2\boldsymbol{f}_2(k)\boldsymbol{s}(k)\\&-\boldsymbol{M}(\boldsymbol{q}(k))L_2\left(\boldsymbol{I}-\boldsymbol{f}_1(k)-\boldsymbol{f}_2(k)\right)\tilde{\boldsymbol{q}}(k)\\&+\boldsymbol{\Delta}_1(k)-T_s^2\boldsymbol{u}_c(k)\\&=-\boldsymbol{M}(\boldsymbol{q}(k))L_2\boldsymbol{f}_2(k)\boldsymbol{s}(k)\end{aligned} \tag{9}$$

$$+\hat{\boldsymbol{M}}(\boldsymbol{q}(k))L_2\left(\boldsymbol{I}-\hat{\boldsymbol{f}}_1(k)-\hat{\boldsymbol{f}}_2(k)\right)\tilde{\boldsymbol{q}}(k)$$
$$+\Delta(k)-T_s^2\mathbf{u}_c(k),$$

which

$$\boldsymbol{\Delta}(k)=\boldsymbol{\Delta}_1(k)-\boldsymbol{M}(\boldsymbol{q}(k))L_2\left(\boldsymbol{I}-\boldsymbol{f}_1(k)-\boldsymbol{f}_2(k)\right)\tilde{\boldsymbol{q}}(k)$$
$$+\hat{\boldsymbol{M}}(\boldsymbol{q}(k))L_2\left(\boldsymbol{I}-\hat{\boldsymbol{f}}_1(k)-\hat{\boldsymbol{f}}_2(k)\right)\tilde{\boldsymbol{q}}(k).$$

Assuming $\hat{\boldsymbol{M}}(k)=\boldsymbol{M}(\boldsymbol{q}(k))$, $\hat{\boldsymbol{M}}(k)\hat{\boldsymbol{f}}_1(k)=\boldsymbol{M}(\boldsymbol{q}(k))$ $\boldsymbol{f}_1(k)$ and $\hat{\boldsymbol{M}}(k)\hat{\boldsymbol{f}}_2(k)=\boldsymbol{M}(\boldsymbol{q}(k))\boldsymbol{f}_2(k)$, if $\boldsymbol{u}_c(k)$ is selected as $\boldsymbol{u}_c(k)=T_s^{-2}\hat{\boldsymbol{M}}(\boldsymbol{q}(k))L_2\left(\boldsymbol{I}-\hat{\boldsymbol{f}}_1(k)-\hat{\boldsymbol{f}}_2(k)\right)$ $\tilde{\boldsymbol{q}}(k)$, then $\boldsymbol{\Delta}(k)=\boldsymbol{0}$, and then (9) can be written as

$$\boldsymbol{s}(k+1)=-L_2\boldsymbol{f}_2(k)\boldsymbol{s}(k). \tag{10}$$

Since in Property 2 $\boldsymbol{f}_2(k)\to-\boldsymbol{I}$ as $T_s\to 0$ in a compact set of $(\boldsymbol{q},\dot{\boldsymbol{q}})$, a small sample time T_s can be selected such that the inequality $L_2\|\boldsymbol{f}_2(k)\|<1$ can be satisfied. Therefore, the tracking filtered error $\boldsymbol{s}(k)$ asymptotically converges to zero in this case. The convergence of $\boldsymbol{s}(k)$ to zero in turn guarantees the convergence of $\tilde{\boldsymbol{q}}(k)$ to zero. Because of the dynamics of the estimator and the time-varying coefficients of the characteristic model, it is almost impossible to satisfy the above assumptions. Therefore, we can design a suitable compensation control law $\boldsymbol{u}_c(k)$ to avoid possibly the case that the control performance is deteriorated or that the close-loop system is even unstable due to the estimation errors. Hence $\boldsymbol{u}_c(k)$ is designed as

$$\boldsymbol{u}_c(k)=T_s^{-2}\left(\hat{\boldsymbol{M}}(k)L_2\left(\boldsymbol{I}-\hat{\boldsymbol{f}}_1(k)-\hat{\boldsymbol{f}}_2(k)\right)\tilde{\boldsymbol{q}}(k)+\hat{\boldsymbol{\Delta}}(k)\right), \tag{11}$$

where $\hat{\boldsymbol{\Delta}}(k)$ is the estimate of $\boldsymbol{\Delta}(k)$.

Assuming $\boldsymbol{\Delta}(k)$ is smooth enough and bounded, it then can be approximated by the linearly parameterized NN to any required degree of accuracy [6,14]. Then the element $\Delta_i(k)$ of $\boldsymbol{\Delta}(k)$ can be expressed as

$$\Delta_i(k)=\boldsymbol{\psi}_i^T\boldsymbol{y}(k)+\delta_i(k), \tag{12}$$

where $i=1,\cdots,n$, $\boldsymbol{\psi}_i\in R^{n_\psi}$ is the column i of the optimal NN weight matrix, $\boldsymbol{\psi}=[\boldsymbol{\psi}_1,\cdots,\boldsymbol{\psi}_{n_\psi}]^T$. Activation functions $\boldsymbol{y}(k)=[y_1(k),\cdots,y_{n_\psi}(k)]^T$ represent the basis function vector, which can be selected as any one of Gaussian radial basis, B-spline basis, Wavelet basis, and etc. [14], and $\delta_i(k)$ denotes the element i of the NN reconstruction error vector $\boldsymbol{\delta}(k)$, namely $\boldsymbol{\delta}(k)=[\delta_1(k),\cdots,\delta_n(k)]^T$.

Using compensation control law $\boldsymbol{u}_c(k)$, (9) can be written as

$$\boldsymbol{M}(\boldsymbol{q}(k))\boldsymbol{s}(k+1)=-\boldsymbol{M}(\boldsymbol{q}(k))L_2\boldsymbol{f}_2(k)\boldsymbol{s}(k)$$
$$+[\boldsymbol{\psi}_1-\hat{\boldsymbol{\psi}}_1,\cdots,\boldsymbol{\psi}_{n_\psi}-\hat{\boldsymbol{\psi}}_{n_\psi}]^T\boldsymbol{y}(k)+\boldsymbol{\delta}(k), \tag{13}$$

where $\hat{\boldsymbol{\psi}}_i(k)$ is the estimate of $\boldsymbol{\psi}_i$, and $\delta_{\max}=\sup_k\|\boldsymbol{\delta}(k)\|<\infty$.

An estimate $\hat{\boldsymbol{\psi}}(k)$ is now obtained by minimizing the cost function

$$J=\frac{1}{2}\boldsymbol{s}^T(k+1)\boldsymbol{M}(\boldsymbol{q}(k))\boldsymbol{s}(k+1). \tag{14}$$

After substituting (13) into (14), the gradient of the cost function in (14) is derived as

$$\frac{\partial J}{\partial\hat{\boldsymbol{\psi}}}=-\boldsymbol{y}(k)\boldsymbol{s}^T(k+1). \tag{15}$$

According to the gradient descent method the NN weight adaptation law can be designed as

$$\hat{\boldsymbol{\psi}}(k+1)=\hat{\boldsymbol{\psi}}(k)+\alpha\boldsymbol{y}(k)\boldsymbol{s}^T(k+1) \tag{16}$$

with $\alpha>0$. Then the compensation control law $\boldsymbol{u}_c(k)$ in (11) can be written as

$$\boldsymbol{u}_c(k)=\frac{1}{T_s^2}\hat{\boldsymbol{M}}(k)L_2\left(\boldsymbol{I}-\hat{\boldsymbol{f}}_1(k)-\boldsymbol{f}_2(k)\right)\tilde{\boldsymbol{q}}(k)$$
$$+\frac{1}{T_s^2}\hat{\boldsymbol{\psi}}^T(k)\boldsymbol{y}(k). \tag{17}$$

In view of the case $\boldsymbol{I}-\hat{\boldsymbol{f}}_1(k)-\hat{\boldsymbol{f}}_2(k)\approx\boldsymbol{0}$, the term $\boldsymbol{u}_c(k)$ can be simplified as

$$\boldsymbol{u}_c(k)=\frac{1}{T_s^2}\hat{\boldsymbol{\psi}}^T(k)\boldsymbol{y}(k).$$

4. SIMULATION RESULTS

Consider a planar, two-link, articulated manipulator as in [3] (as presented in Fig. 1), whose dynamics can be written explicitly as

$$\begin{bmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{bmatrix}\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix}$$

$$+\begin{bmatrix} -h\dot{q}_2 & -h(\dot{q}_1+\dot{q}_2) \\ h\dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}=\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad (18)$$

where

$$M_{11}=a_1+2a_3\cos q_2+2a_3\sin q_2,$$
$$M_{12}=M_{21}=a_2+a_3\cos q_2+a_4\sin q_2,$$
$$M_{22}=a_2,$$
$$h=a_3\sin q_2-a_4\cos q_2$$

with $a_1=I_1+m_1l_{c1}^2+I_e+m_el_{ce}^2+m_el_1^2$, $a_2=I_e+m_el_{ce}^2$, $a_3=m_el_1l_{ce}\cos\delta_e$, $a_4=m_el_1l_{ce}\sin\delta_e$, $m_1=1$, $l_1=1$, $m_e=2$, $\delta_e=30^o$, $I_1=0.12$, $l_{c1}=0.5$, $I_e=0.25$, and $l_{ce}=0.6$.

In the simulation, the sample time $T_s=2ms$, the initial values and the parameters of the estimator and the controller are selected as $\boldsymbol{P}(0)=1\times10^3\boldsymbol{I}$, $\lambda(0)=0.96$, $\mu_0=0.98$, the anti-singularity factor $\varepsilon(k)$ can be designed as $\varepsilon(k)=5\times10^{-6}\exp(-kT_s)$.

According to the Property 2, the initial estimate values of the characteristic model coefficient matrixes are chosen as $\hat{\boldsymbol{f}}_1(0)=2\boldsymbol{I}$, $\hat{\boldsymbol{f}}_2(0)=-\boldsymbol{I}$.

A basis set of activation function $\boldsymbol{y}(k)$ can be selected as in the Random Vector Function Link net [16], namely,

$$\boldsymbol{y}(k)=\sigma\left(\boldsymbol{V}^T\boldsymbol{X}(k)\right) \quad (19)$$

with $\boldsymbol{V}$ a randomly selected matrix and $\boldsymbol{X}(k)$ the NN input vector. $\sigma(\bullet)$ can be chosen as the hyperbolic tangent function, and $\boldsymbol{X}(k)$ can be taken as

$$\boldsymbol{X}(k)=\left[\boldsymbol{q}_d^T(k),\ \boldsymbol{q}_d^T(k-1),\ \boldsymbol{q}^T(k),\ \boldsymbol{q}^T(k-1),\ 1\right]^T.$$

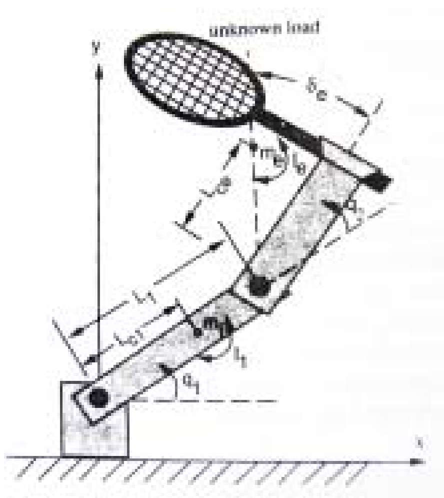

Fig. 1. An articulated two-link manipulator [3].

The adaptation gain for the NN weight tuning is taken as $\alpha=0.005$, and the initial values of the weights are set to zeros.

The desired trajectory is chosen as

$$\boldsymbol{q}_d(t)=[30^o\left(1-\cos2\pi t\right),\ 45^o\left(1-\cos2\pi t\right)]^T. \quad (20)$$

The corresponding simulation results are plotted in Figs. 2 to 7. The tracking errors of joint 1 and joint 2 are plotted in Figs. 2 and 3, corresponding joint torques are plotted in Figs. 4 and 5, and the estimates

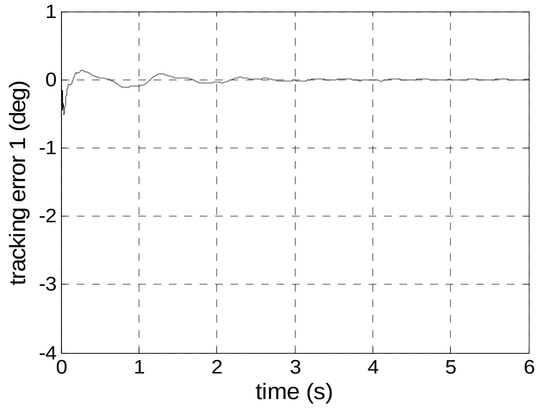

Fig. 2. Tracking error of joint 1 under GSAC.

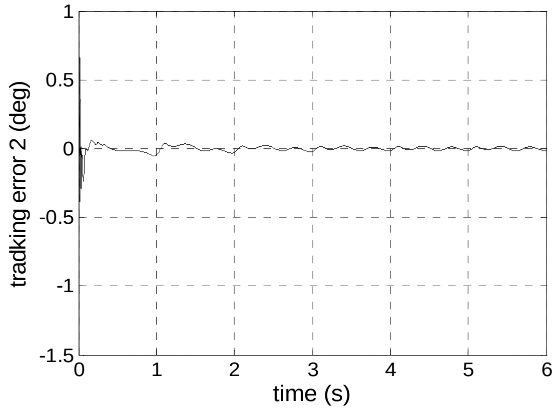

Fig. 3. Tracking error of joint 2 under GSAC.

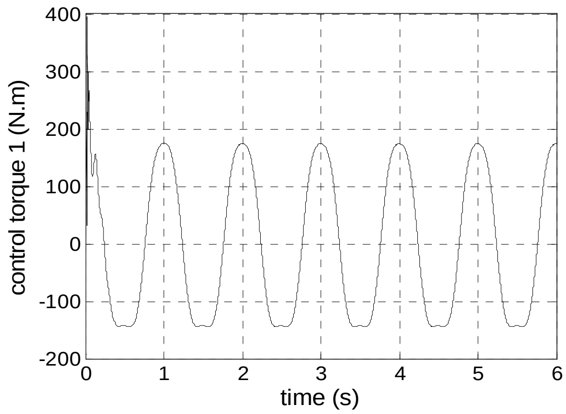

Fig. 4. Control torque of joint 1 under GSAC.

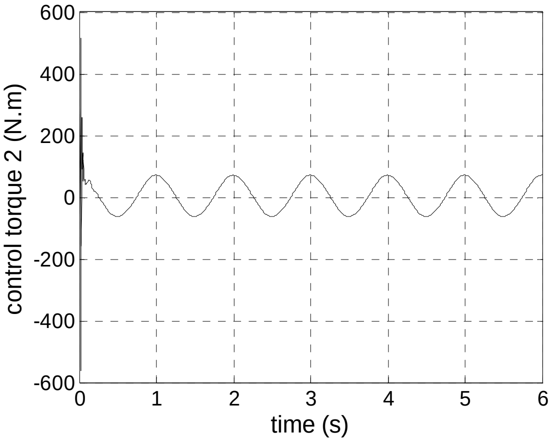

Fig. 5. Control torque of joint 2 under GSAC.

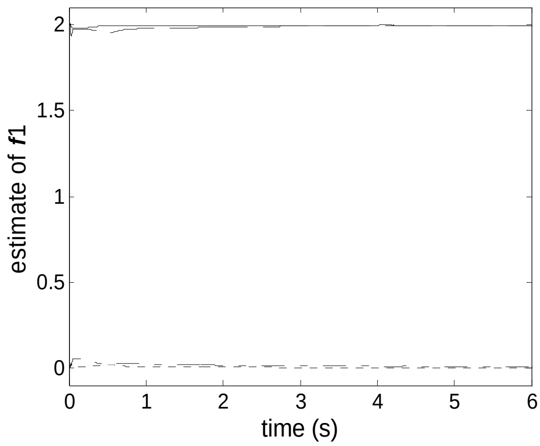

Fig. 6. Estimates of $f_1(k)$ elements.

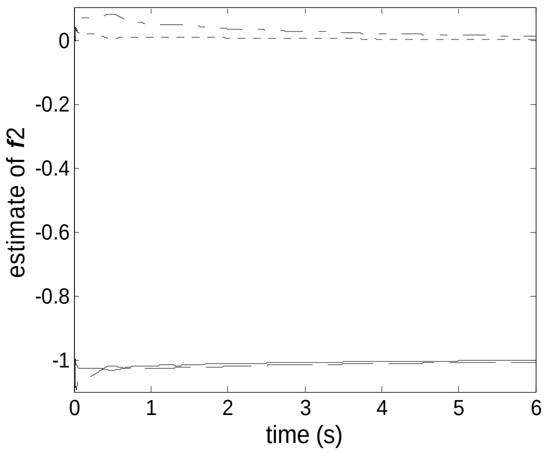

Fig. 7. Estimates of $f_2(k)$ elements.

of the elements of coefficient matrixes $f_1(k)$ and $f_2(k)$ are plotted in Figs. 6 and 7, respectively.

For comparison, Figs. 8 and 9 show the tracking errors using the Slotine's adaptive control method [3] on the same desired trajectory. Fig. 8 and Fig. 9 present the tracking errors of joint 1 and 2, respectively.

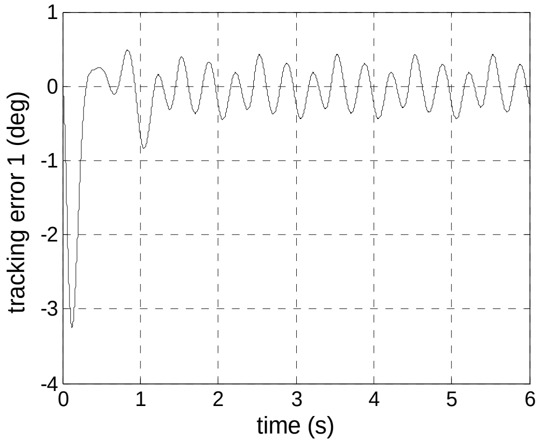

Fig. 8. Tracking error of joint 1 under Slotine's method.

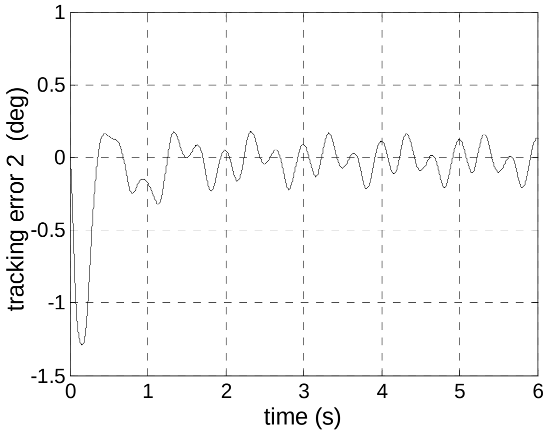

Fig. 9. Tracking error of joint 2 under Slotine's method.

5. CONCLUSIONS

In this paper, a robotic manipulator characteristic model is established and a corresponding multi-variable Golden-Section adaptive controller with NN compensation is developed for the control of manipulators. The proposed adaptive control scheme is easy to design without precisely knowing the prior knowledge of the systems to be controlled. GSAC is so robust and the number of its parameters to be tuned is so small that it can cope with both parametric and non-parametric uncertainties effectively, and furthermore, the introduced NN compensation controller can improve the dynamic characteristics of the close-loop system. Especially, the GSAC algorithm can be implemented using a microcomputer without excessive computation burden. Simulation results demonstrate that the proposed adaptive control techniques may be successfully applied to the control of the robotic manipulator.

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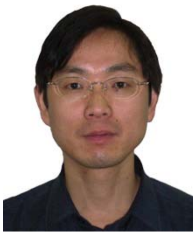

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