Iterative Feedback Tuning for Positive Feedback Time Delay Controller

Kai-Ming Tsang, Ahmad B. Rad* and Wai-Lok Chan

Abstract: Closed-loop model-free optimization of positive feedback time delay controllers for dominant time delay systems is presented. Iterative feedback tuning (IFT) is applied to the tuning of positive feedback time delay controller. Three experiments are carried out to perform the model-free gradient descent optimization. The initial controller parameters and duration in specifying the cost function are suggested. The effects of step size, filter function and time weighting function on the performance of the optimized controller are given. Simulation and experimental studies are included to demonstrate the effectiveness of the tuning scheme.

Keywords: Iterative feedback tuning, time delay controller.

1. INTRODUCTION

Although the Ziegler and Nichols PID tuning formula is well known and robust, it is well documented that the performance of the controller deteriorates rapidly and may become unstable for systems with dominant time delay. The predictive PI controller [1], delay compensated PID controller [2], and positive feedback time delay controller [3] produce good responses for dominant time delay processes. The performances of these controllers highly depend on the accurate knowledge of the system time delay. Most of the times on-line identification [4-6] and experiments [3] are needed to derive the open-loop models. However controllers derived from the fitted models may not produce optimal responses.

An alternative way to tune a controller is to optimize parameters of a controller of fixed structure using information on the closed-loop system and the control performance will then be optimal. The optimization of such control performance criterion typically requires iterative gradient-based minimization procedures. The major obstacle for the solution of this type is the computation of the gradient of the criterion function with respect to the controller parameters. The iterative feedback tuning (IFT) algorithm presented in Hjalmarsson et al. [7] is a model-free gradient descent algorithm where the cost function gradient can be obtained by doing some experiments on the closed-loop process. The implementation of IFT for the tuning of time delay controllers is therefore investigated. The weighting functions filter function in the cost function, step size and duration of optimization will also be studied. Simulation and experimental studies are included to demonstrate the effectiveness of the tuning algorithm.

2. TIME DELAY CONTROLLER

The time delay controller uses a delay element in positive feedback to tackle the dead time of the process and a compensator to improve the speed of response of the processes shown in Fig. 1. The transfer function of the time delay controller can be written as

$$G_\tau(s) = \frac{G_c(s)}{1 - e^{-s\tau} G_a(s)},$$

where $G_c(s)$ is the compensator transfer function, $G_a(s)$ is a low pass filter and is the time delay of the controller. The proposed $G_c(s)$ and $G_a(s)$ are given by

$$G_c(s) = \frac{1}{K}\frac{1+Ts}{1+T_0 s},$$

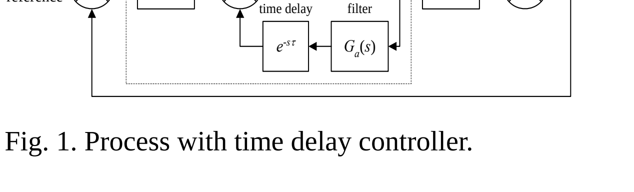

Fig. 1. Process with time delay controller.

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$$G_a(s) = \frac{1}{1+T_0 s},$$

where K is the static gain, T is the apparent time constant, T_0 is selected between 0.1 to 0.5 of T and τ is the apparent time delay of the process $G(s)$, respectively. With the proposed values, the time delay controller becomes

$$G_\tau(s) = \frac{1+Ts}{K(1+T_0 s - e^{-s\tau})}, \tag{1}$$

and the process output is given by

$$\begin{aligned} C(s) &= \frac{G_\tau(s)G(s)}{1+G_\tau(s)G(s)}R(s) + \frac{1}{1+G_\tau(s)G(s)}D(s) \\ &= H_0(s)R(s) + S_0(s)D(s), \end{aligned} \tag{2}$$

where $C(s)$ is the process output, $R(s)$ is the reference, $G(s)$ is the plant transfer function, and $D(s)$ is the output disturbance. Robustness of the positive feedback time delay controller for deadtime processes were presented in [3]. It has been shown that the closed loop system will still be stable provided the fitted T is less than twice the system time constant, or the fitted K is bigger than half the system static gain, or the fitted τ is bigger than one-third of the system time delay.

3. ITERATIVE FEEDBACK TUNING

If the output error signal is defined as

$$\tilde{c}(t) = c(t) - c_d(t), \tag{3}$$

where $c(t)$ is the process output response and $c_d(t)$ is the desired process output response, respectively. To attenuate the high frequency additive noise, the output error is low-pass filtered to give

$$\tilde{C}_f(s) = F(s)\tilde{C}(s), \tag{4}$$

where $F(s)$ is a low-pass filter. The control design objective is to minimize a cost function

$$J(\theta) = \frac{1}{2}E\left(w(t)\tilde{c}_f^2(t)\right), \tag{5}$$

where $E(.)$ denotes expectation and $w(t)$ is a time weighting function, with respect to the controller parameter $\theta = [K \quad T \quad \tau]^T$. The standard solution to the problem is to solve

$$0 = \frac{\partial J(\theta)}{\partial \theta} = E\left(w(t)\tilde{c}_f(t)\frac{\partial \tilde{c}_f(t)}{\partial \theta}\right). \tag{6}$$

If the gradient $\frac{\partial J(\theta)}{\partial \theta}$ is available, the solution of (6) can be obtained by the iterative algorithm

$$\theta_{i+1} = \theta_i - \gamma_i R_i^{-1}\frac{\partial J(\theta_i)}{\partial \theta}. \tag{7}$$

Here θ_i denotes the controller parameter vector θ at iteration i, R_i is some appropriate positive definite matrix (typically an estimate of the Hessian of $J(\theta)$) and γ_i is a sequence of positive numbers that determines the step size. (7) is essentially a Newton-Raphson search algorithm. It typically converges to a solution of (5) provided that the step size γ_i is chosen such that $J(\theta_{i+1}) < J(\theta_i)$ and R_i is nonsingular [8]. Numerical problems may occur if R_i is singular or close to singular (when the data are not sufficiently informative or the controller is over parameterized).

3.1. Generation of the gradient signal

In order to obtain an estimate of $\frac{\partial J(\theta)}{\partial \theta}$, estimates of the signal $\tilde{c}_f(t)$ and its gradient $\frac{\partial \tilde{c}_f(t)}{\partial \theta}$ are needed. One of the obstacle in solving the optimal controller parameter is the computation of the gradient $\frac{\partial \tilde{c}_f(t)}{\partial \theta}$. Hjalmarsson et al.[7] showed that the gradient could be obtained by performing some experiments on the closed loop system. From (3) and (4),

$$\frac{\partial \tilde{C}_f(s)}{\partial \theta} = F(s)\frac{\partial C(s)}{\partial \theta}, \tag{8}$$

and from (2)

$$\begin{aligned} \frac{\partial C(s)}{\partial \theta} &= \frac{G(s)}{1+G_\tau(s)G(s)}\frac{\partial G_\tau(s)}{\partial \theta}R(s) - \frac{G_\tau(s)G^2(s)}{\left(1+G_\tau(s)G(s)\right)^2}\frac{\partial G_\tau(s)}{\partial \theta}R(s) \\ &\quad - \frac{G}{\left(1+G_\tau(s)G(s)\right)^2}\frac{\partial G_\tau(s)}{\partial \theta}D(s) \\ &= \frac{1}{G_\tau(s)}\frac{\partial G_\tau(s)}{\partial \theta}\left(H_0(s)R(s) - H_0^2(s)R(s) - H_0(s)S_0(s)D(s)\right) \\ &= \frac{1}{G_\tau(s)}\frac{\partial G_\tau(s)}{\partial \theta}\left(H_0(s)R(s) - H_0(s)C(s)\right). \end{aligned} \tag{9}$$

Since $G_\tau(s)$ and $\frac{\partial G_\tau(s)}{\partial \theta}$ are known functions, from (1)

$$\begin{aligned} \frac{\partial G_\tau(s)}{\partial K} &= -\frac{1+Ts}{K^2\left(1+T_0 s - e^{-s\tau}\right)}, \\ \frac{\partial G_\tau(s)}{\partial T} &= \frac{s}{K\left(1+T_0 s - e^{-s\tau}\right)}, \\ \frac{\partial G_\tau(s)}{\partial \tau} &= -\frac{(1+Ts)se^{-s\tau}}{K\left(1+T_0 s - e^{-s\tau}\right)^2}, \end{aligned} \tag{10}$$

and (9) becomes

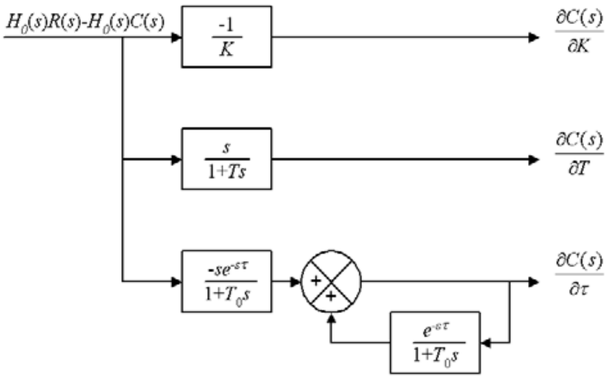

Fig. 2. Generation of the change of process output with.

$$\begin{aligned}\frac{\partial C(s)}{\partial K}&=-\frac{1}{K}\left(H_0(s)R(s)-H_0(s)C(s)\right),\\ \frac{\partial C(s)}{\partial T}&=\frac{s}{1+Ts}\left(H_0(s)R(s)-H_0(s)C(s)\right),\\ \frac{\partial C(s)}{\partial \tau}&=-\frac{se^{-s\tau}}{1+T_0 s-e^{-s\tau}}\left(H_0(s)R(s)-H_0(s)C(s)\right).\end{aligned} \tag{11}$$

Hence the gradient functions of (11) can be obtained provided $H_0(s)R(s)$ and $H_0(s)C(s)$ are available. Performing three experiments on the closed loop system with reference inputs $R_1(s)$, $R_2(s)$ and $R_3(s)$, the corresponding output responses are

$$\begin{aligned}&C_1(s)=H_0(s)R(s)+S_0(s)D_1(s),\ \ R_1(s)=R(s),\\ &C_2(s)=H_0(s)C_1(s)+S_0(s)D_2(s),\ \ R_2(s)=C_1(s),\\ &C_3(s)=H_0(s)R(s)+S_0(s)D_3(s),\ \ R_3(s)=R(s).\end{aligned} \tag{12}$$

Estimates of $H_0(s)R(s)$ and $H_0(s)C(s)$ can be replaced by $C_3(s)$ and $C_2(s)$ and approximations of the change of output with respect to the change of parameters vector is shown in Fig. 2.

3.2. Estimation of the gradient and Hessian

With the experiments performed in (12), an estimate of the gradient can be obtained as

$$\frac{\partial \hat{J}(\theta)}{\partial \theta}=\frac{1}{N}\sum_{k=1}^{N} w(k)\tilde{c}_f(k)\frac{\partial c_f(k)}{\partial \theta}, \tag{13}$$

where N is the number of data records collected for the optimization problem, $\tilde{c}_f(k)$ and $\frac{\partial c_f(k)}{\partial \theta}$ are sampled values of the filtered output and change of output with respect to the change of parameters vector. A good approximation of the Hessian can be obtained as

$$\hat{R}=\frac{1}{N}\sum_{k=1}^{N}\frac{\partial c_f(k)}{\partial \theta}\left(\frac{\partial c_f(k)}{\partial \theta}\right)^T. \tag{14}$$

3.3. Iterative feedback tuning algorithm

The optimization procedures can be summarized as follows:

i. Apply relay auto-tuning [3] to obtain initial values of K, T and τ. Set the sampling time to be approximately equal to 1/40 of T, $T_0=0.4T$, duration for optimization to be $\tau+10T$, the weighting function $w(k)$, the filter function $F(s)$, the step size γ_i and the reference model $G_m(s)=\frac{e^{-0.5s\tau}}{1+0.2Ts}$.

ii. With the controller $G_\tau(s)$ in the loop, perform the three experiments described in (12).

iii. Compute the error signal $\tilde{c}(t)$ using (4) where $C_d(s)=G_m(s)R(s)$ and an estimate of $\frac{\partial C(s)}{\partial \theta}$ using (11).

iv. Filter the error and the change of outputs with the filter $F(s)$ and obtain samples of $\tilde{c}_f(k)$ and $\frac{\partial c_f(k)}{\partial \theta}$.

v. Compute an estimate of the gradient using (13) and the Hessian using (14).

vi. Update the parameter vector using (7).

vii. If $\frac{J(\theta_i)-J(\theta_{i+1})}{J(\theta_i)}<$ a small tolerance, stop the algorithm. Otherwise go to ii.

4. SIMULATION STUDIES

Consider the model

$$G(s)=\frac{1}{(s+1)^{10}}.$$

The static gain of the process is $K=1$ and the ultimate gain and frequency of oscillation of the process are $K_u=1.6517$ and $\omega_u=0.3249$ rad/s, respectively. The apparent time constant and time delay can be obtained as [3]

$$T=\frac{\sqrt{K_u^2K^2-1}}{\omega_u}=4.046 \text{ sec},$$

$$\tau=\frac{\pi\text{-}\tan^{-1}(T\omega_u)}{\omega_u}=6.836 \text{ sec}.$$

The sampling time for the process was set to 0.1 sec, the duration for optimization was set to 47.3 sec and the reference model was

$$G_m(s)=\frac{e^{-3.418s}}{1+0.8092s}.$$

The effects of the time weighting function $w(t)$, the step size γ_i and filter function $F(s)$ on the performance of the algorithm are investigated.

4.1. Time weighting function

The step size γ_i was fixed at 0.5 and the filter function $F(s)$ was set to 1. Weighting functions of the form

$$w(t) = 1,$$
$$w(t) = t,$$
$$w(t) = t^2,$$

were tested. The tolerance to stop the optimization procedure was set to 0.005. Fig. 3 shows the controller parameter vectors and the cost functions for the three weighting functions at different iterations. After four iterations, the ratios $\frac{J(\theta_i)-J(\theta_{i+1})}{J(\theta_i)}$ were all less than 0.005. Fig. 4 shows the performances of the three final optimal controllers. The performance of the controller with uniform weighting function gave a faster response but the overshoot was higher and the settling time was longer. The performance of the controller with $w(t)=t^2$ had a lower overshoot but the settling was faster. The performance of controller with $w(t)=t$ was between the two.

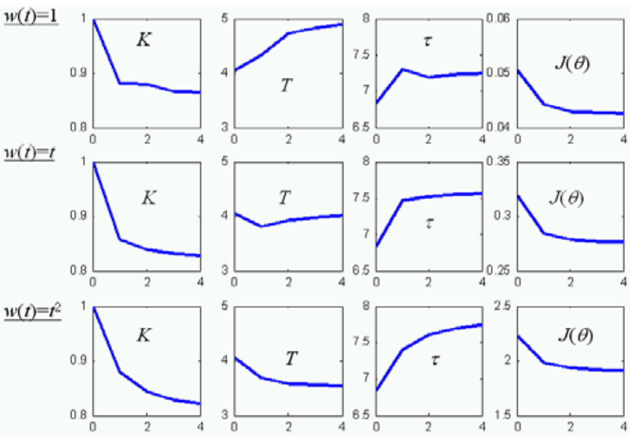

Fig. 3. Profiles of the controller parameters and cost functions.

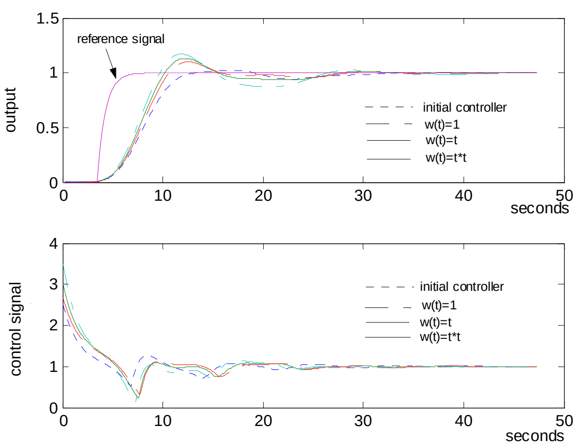

Fig. 4. Performances of the three optimal controllers.

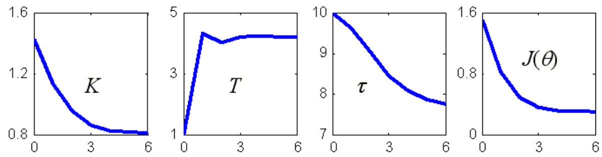

Fig. 5. Profiles of the controller parameters and cost function with variations over 20% for $w(t)=t$.

Fig. 5 shows the profile of the controller parameter vector and the cost function with a weighting function $w(t)=t$ and initial parameter variation over 20%. After six iterations, the ratios $\frac{J(\theta_i)-J(\theta_{i+1})}{J(\theta_i)}$ was less than 0.005 and the parameter vector converged to similar values obtained in Fig. 3.

4.2. Step size

With $w(t)$ set to t^2 and $F(s)$ set to 1, step sizes of 0.2, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9 were tested to test their convergent rates. Fig. 6 shows the cost functions at different iterations. Clearly a small step size resulted with a slow convergent rate. As the step size increased to 0.7, an optimal solution was reached after three iterations. The convergent rates using step sizes of 0.7, 0.8 and 0.9 were very similar but the parameter vectors during the optimization procedures were more fluctuated when step sizes of 0.8 and 0.9 were used.

4.3. Filter function

If output additive noise appeared in the process, a filter $F(s)$ is required to attenuate its effect in the optimization of the cost function $J(\theta)$. A low-pass filter of the form

$$F(s) = \frac{1}{1+\alpha Ts}$$

is suggested where α is between 0 to 0.5 such that the dynamics of the system could still be maintained and T could either be the initial estimate obtained from the relay test or estimate obtained from the optimization procedures. In this simulation study, T was taken as the initial estimate obtained from the relay test and different values of α were tried to see their effects on the optimization results. The variance of the output additive noise was set to 0.005, $w(t)$ was set to t^2 and γ_i was set to 0.7. Fig. 7 shows the profiles of the cost functions for different values of α. Clearly the optimization diverged if α was set to zero. The performances of the optimized controllers are shown in Fig. 8. The performances of the controllers with α set to 0.1 and 0.2 were very similar. As α was further increased, the first overshoot started to increase because more and more of the system dynamics were attenuated by the introduced filter.

Hence a good suggestion for the time weighting function, filter function and the step size are $w(t)=t^2$,

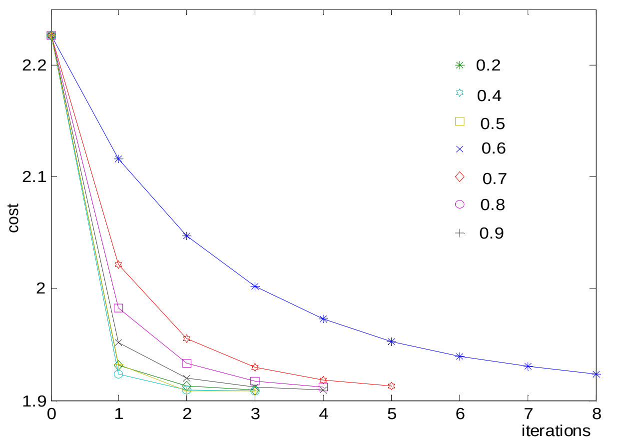

Fig. 6. Profiles of the cost function with different step sizes.

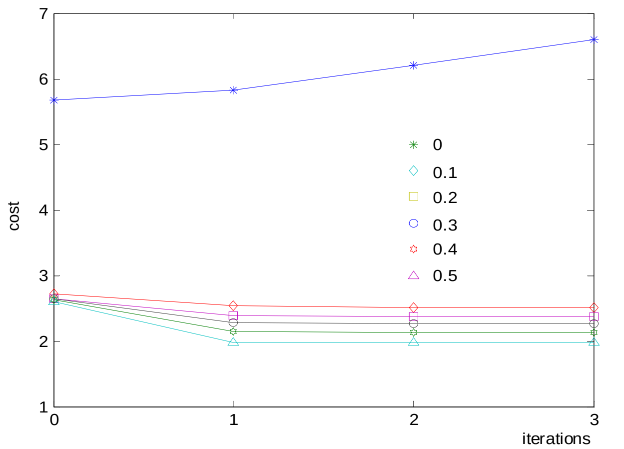

Fig. 7. Profiles of the cost function with different α.

α=0.2 and γ_i=0.7.

Remark 1: The use of relay auto-tuner for initial model is optional and is not required by the algorithm. It is just a means of speeding up the response.

Remark 2: T his method is fundamentally different from the other methods that approximate a higher-order system with a first-order with time delay. We are only interested in the closed-loop response.

5. EXPERIMENTAL STUDIES

The process simulator PCS327 [9] from Feedback Instruments Ltd. was used to demonstrate the performance of the tuning technique. The system was set to the 1 second operating condition and an extra time delay of 2 seconds was added to the process to make it delay dominant. From the relay test [3], the ultimate gain and ultimate period of oscillation were 1.5338 and 11.5649 sec, respectively. The estimated static gain, apparent time constant and apparent time delay of the process were given by K=1.0017, T=2.1469 sec, and τ=4.1957 sec, respectively [3]. To

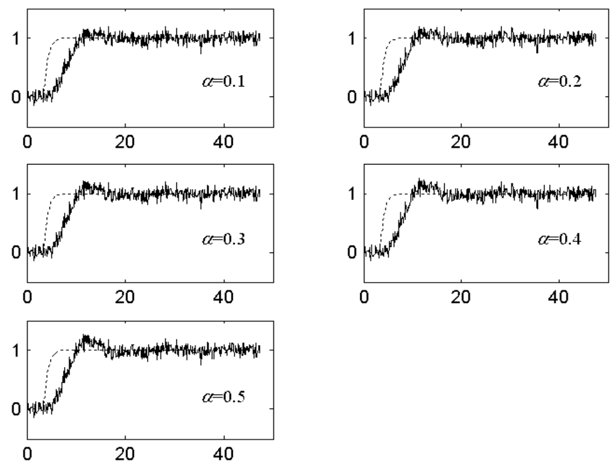

Fig. 8. Performances of different optimized controllers with different α.

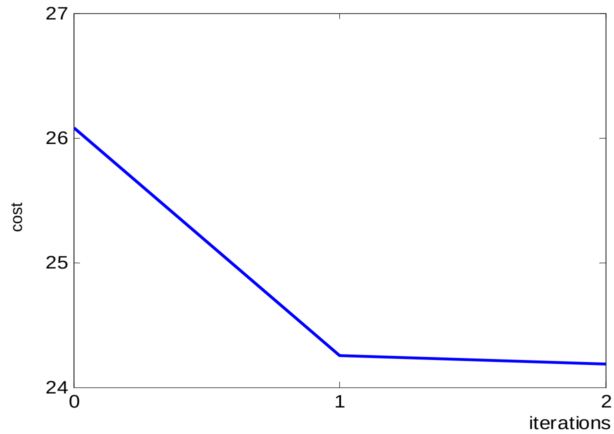

Fig. 9. Profile of the cost function.

carry out the iterative feedback tuning algorithm, the sampling frequency was set to 0.05 second, duration for the optimization was set to 25.7 sec, $w(t)$ was set to t^2, γ_i was set to 0.7 and α was set to 0.2. The procedures described in section 3.3 were executed and a tolerance of 0.005 was set for step vii. After two iterations, the algorithm stopped and the fitted K, T and τ were 0.8874, 2.0458 sec. and 4.4489 sec, respectively. The profile of the cost function is shown in Fig. 9. For the estimated ultimate gain and ultimate period of oscillation, the Ziegler and Nichols PID controller was given by

$$G_{PID}(s) = 0.9203\left(1 + \frac{1}{5.7824s} + 1.4456s\right).$$

Fig. 10 shows the initial time delay controller, optimized time delay controller and the Ziegler and Nichols PID controller in action. Clearly, the optimized time delay controller had a faster response that the initial time delay controller and outperformed the Ziegler and Nichols PID controller for time delay dominant processes.

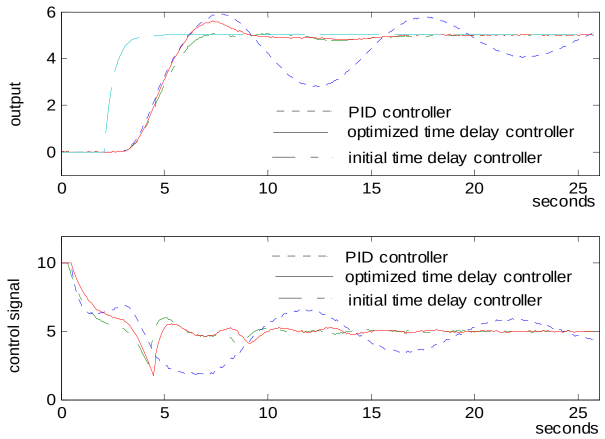

Fig. 10. Performances of the PID and time delay controllers on the process PCS327.

6. CONCLUSIONS

Iterative feedback tuning algorithm has been successfully implemented for the optimization of time delay controller. The gradient of the cost function can be obtained by performing three experiments on the closed-loop process. In setting the step size to 0.7, the bandwidth of the filter function to 5 times the estimated system bandwidth, the duration of the optimization to apparent time delay plus ten times the apparent time constant and the tolerance to stop the optimization to 0.005, optimal solution is reached in two to three iterations. The optimized time delay controller outperforms the Ziegler and Nichols PID controller for dominant time delay processes. The novelty of the tuning technique is that it is model-free, it could be run in closed-loop operation and optimal solution could be reached in two to three iterations.

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