Structuring Element Representation of an Image and Its Applications

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Abstract: In this paper we present the linear combination of a fuzzy opening and closing filter with locally adaptive structuring elements that can preserve the geometrical features of an image. Based on the adaptation algorithm of linear combination of the fuzzy opening and closing filter, the optimal structuring element for image representation is obtained. The optimal structuring element is an indicator of the shape and direction of an object's image, which is useful in filtering, multi resolution, segmentation, and recognition of an image.

Keywords: Fuzzy morphological filter, image representation, structuring element.

1. INTRODUCTION

Since a morphological filter can analyze the geometric features of an image, it is widely used in image filtering, segmentation, and edge detection. Compared with classical morphology, fuzzy morphology [1] provides a more intuitive and less restrictive fitting. Recently, a despeckling technique using fuzzy morphology has been proposed in [2]. In [2], it is shown that fuzzy morphology can cope with the ambiguous and obscure ultrasonic image, and further that linear combination of fuzzy opening-closing and fuzzy closing-opening (LFOCCO) is used for speckle reduction. Since the structuring element is large enough to include noise features and its shape is adapted to the geometry of the image features to be preserved, the selection of the structuring element with optimal shape is important. That is, the design of a morphological filter is needed in selecting the shape of structuring element that is adapted to the local geometry of the image being processed. The structuring element for a morphological filter can be directly obtained from input image [3] or weighted ones can be used [4]. Recently, implementation of the adaptation of the structuring element in fuzzy morphology has been proposed in [5]. Based on the result in [5], linear combination of an adaptive fuzzy opening and closing (LAFOC) filter is proposed. The optimal structuring elements obtained from LAFOC filter represent the shape and direction of an object's image, which is useful in the applications of filtering, segmentation, and recognition of an image.

In this paper, the fuzzy morphology [1] and its adaptation [5] are briefly reviewed. Then the linear combination of a fuzzy opening and closing filter with a locally adaptive structuring element is proposed. Using the optimal structuring elements obtained from the adaptation algorithm, an image is represented in form of the set of optimal structuring elements. Simulation results of structuring element representation of images are given.

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2. FUZZY MORPHOLOGICAL FILTER

2.1. Fuzzy morphology

In this section, some basic definitions of morphological operators are introduced. The maximum and minimum operators will be denoted as $\vee$ and $\wedge$, respectively. Fuzzy mathematical morphology [1] has been developed using the notion of fuzzy fitting. Fuzzy fitting of a fuzzy set A into a fuzzy set B is characterized by an inclusion indicator $I(A,B) \in [0,1]$.

$$I(A,B) = \wedge_{x \in X}\left[1 \wedge \left(1 - \mu_A(x) + \mu_B(x)\right)\right],$$

where $\mu(\cdot)$ denotes a membership function. The value of $I(A,B)$ between 0 (no fit) and 1 (perfect fit) indicates the degree of fitting of A into B.

Using $I(A,B)$, operators such as erosion (Θ), dilation ($\oplus$), opening ($\circ$) and closing($\bullet$) are defined as follows:

$$\mu_{f\Theta k}(n) = \wedge_{m \in K, n+m \in F}\left[1 \wedge \left(1 - \mu_k(m) + \mu_f(n+m)\right)\right],$$

$$\mu_{f\oplus k}(n) = \vee_{m \in K, n-m \in F}\left[0 \vee \left(\mu_k(m) + \mu_f(n-m) - 1\right)\right],$$

$$\mu_{f\circ k}(n) = \mu_{(f\Theta k)\oplus k}(n),$$

$$\mu_{f\bullet k}(n) = \mu_{(f\oplus k)\Theta k}(n),$$

where $\mu_f(n)$ and $\mu_k(n)$ with support regions F and K are membership functions of signal $f(n)$ and structuring element $k(n)$, respectively.

2.2. Adaptation of structuring element in fuzzy morphology

The design of fuzzy morphological filters consists in adapting the shape of structuring elements to the local geometry of an image. Recently, the optimization method in fuzzy morphology is proposed in [5]. More details can be found in [5]. By minimizing the inequality index, $J(n)=\left|\mu_t(n)-\mu_{f\Diamond k}(n)\right|$, the algorithm for updating the structuring element is provided by

$$\mu_k^{(i+1)}=\mu_k^{(i)}+\eta\,\mathrm{sgn}\left[\mu_t(n)-\mu_{f\Diamond k}(n)\right]\frac{\partial\mu_{f\Diamond k}(n)}{\partial\mu_k^{(i)}}, \quad (1)$$

where $\mu_t(n)$ denotes target membership function, $\Diamond$ denotes morphological operator, $\mathrm{sgn}(\cdot)$ denotes the sign function, i is iteration number, and η is training rate. The gradient for the erosion filter, $\dfrac{\partial\mu_{f\Theta k}(n)}{\partial\mu_k^{(i)}}\equiv\dot{e}_{f(n)}^{(i)}$, is

$$\dot{e}_{f(n)}^{(i)}=\left[e_{f(n)}^{(i)}(-M)\cdots e_{f(n)}^{(i)}(0)\cdots e_{f(n)}^{(i)}(M)\right]^T,$$

where $e_{f(n)}^{(i)}(m)=-U\left[\mu_k^{(i)}(m)-\mu_f(n+m)\right]$ if $\left[1\wedge\left(1-\mu_k(m)+\mu_f(n+m)\right)\right]$ is minimum, 0 otherwise and $\{-M,\cdots,0,\cdots,M\}\in K$. The gradient for the dilation filter, $\dfrac{\partial\mu_{f\oplus k}(n)}{\partial\mu_k^{(i)}}\equiv\dot{d}_{f(n)}^{(i)}$, is also given by

$$\dot{d}_{f(n)}^{(i)}=\left[d_{f(n)}^{(i)}(-M)\cdots d_{f(n)}^{(i)}(0)\cdots d_{f(n)}^{(i)}(M)\right]^T,$$

where $d_{f(n)}^{(i)}(m)=U\left[\mu_k^{(i)}(m)+\mu_f(n-m)-1\right]$ if $\left[0\vee\left(\mu_k(m)+\mu_f(n-m)-1\right)\right]$ is maximum, 0 otherwise. Note that $U[a]=1$ for $a\geq 0$, 0 for $a<0$.

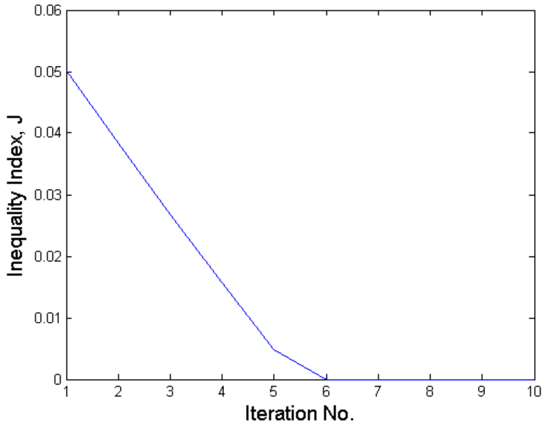
3. LINEAR COMBINATION OF ADAPTIVE FUZZY MORPHOLOGICAL OPERATORS

3.1. Linear combination of adaptive fuzzy opening and closing

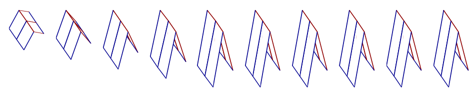
The linear combination of the fuzzy opening and closing filter is defined as

$$\mu_{f\otimes k}(n)=\frac{1}{2}\left(\mu_{f\circ k}(n)+\mu_{f\bullet k}(n)\right). \quad (2)$$

The linear combination of morphological filters can eliminate the bias of the individual morphological filter. For a flat structuring element, i.e., $\mu_k(m)=1,\forall m$, (2) becomes a pseudomedian filter [6]. From (1), the algorithm for updating the structuring element of LAFOC can be easily derived as



(a) Inequality index.



(b) Updated SEs per iteration.

Fig. 1. Convergence of LAFOC for 2-D concave signal.

$$\mu_k^{(i+1)}=\mu_k^{(i)}+\frac{\eta}{2}\mathrm{sgn}\left[\mu_t(n)-\mu_{f\otimes k}(n)\right]\left\{\frac{\partial\mu_{f\circ k}(n)}{\partial\mu_k^{(i)}}+\frac{\partial\mu_{f\bullet k}(n)}{\partial\mu_k^{(i)}}\right\}. \quad (3)$$

The gradients of opening and closing are given by

$$\frac{\partial\mu_{f\circ k}(n)}{\partial\mu_k^{(i)}}=\left(\dot{E}_f^{(i)}R+\mathrm{I}\right)\dot{d}_{f\Theta k(n)}^{(i)},$$

$$\frac{\partial\mu_{f\bullet k}(n)}{\partial\mu_k^{(i)}}=\left(\mathrm{I}-\dot{D}_f^{(i)}R\right)\dot{e}_{f\oplus k(n)}^{(i)},$$

where $\dot{E}_f^{(i)}=\left[\dot{e}_{f(n-M)}^{(i)}\cdots\dot{e}_{f(n)}^{(i)}\cdots\dot{e}_{f(n+M)}^{(i)}\right]$, $\dot{D}_f^{(i)}=\left[\dot{d}_{f(n-M)}^{(i)}\cdots\dot{d}_{f(n)}^{(i)}\cdots\dot{d}_{f(n+M)}^{(i)}\right]$, R and I are $(2M+1)\times(2M+1)$ reflection and identity matrices, respectively.

3.2. Convergence

Fig. 1 illustrates the shape of updated structuring elements (SE) for the LAFOC filter using an initial flat structuring element for a 2-Dimensional concave signal. As one can see, the shape of the updated structuring element for the concave signal is concave, which is optimal for both opening and closing. Fig. 1

also illustrates fast convergence with less iteration.

4. SIMULATION RESULTS

To obtain locally optimized structuring elements, the image is partitioned into regions, and an optimal structuring element corresponding to each region is obtained by (3). In this simulation, a 5x5 flat structuring element is used as the initial SE in the adaptation process.

4.1. Experiment 1: fingerprint image

The proposed filter is tested on a fingerprint image as illustrated in Fig. 2(a). Fig. 2(b) and (c) indicate the optimal structuring element representation of a fingerprint image. The set of optimal structuring elements obtained from the proposed method is an indicator of the direction of the fingerprint, and very similar to the fingerprint directional map [7,8]. We thus develop a structural representation of fingerprint images that might be useful in the recognition of fingerprints.

4.2. Experiment 2: ultrasonic image

Fig. 3 shows structuring element representation of an ultrasonic image. This example also indicates that the set of optimal structuring elements as prior information provides a structural characterization of the images that is useful in the removal of speckle noise [2].

To evaluate the performance of the LAFOC filter, the smoothing measurement proposed in [11] is used. Figure 4 shows a scatter plot of the gradient magnitude of the original ultrasonic image (x-axis) versus the gradient magnitude of the filtered image (y-axis). Sharpened and smoothed pixels are grouped into two sets C (above the line $y = x$) and D (below the line $y = x$), respectively. Note that in general $|C| << |D|$. The line $y = a_D x + b_D$ can be obtained from the curve fitting method. The slope a_D offers an indication of the smoothing induced by the filter. Also the offset b_D gives an indication of the bias induced by the filter. The smoothing measurement is provided by

$$Smoothing = \left(\frac{1}{a_D} - 1 \right) \frac{|D|}{|C| + |D|}.$$

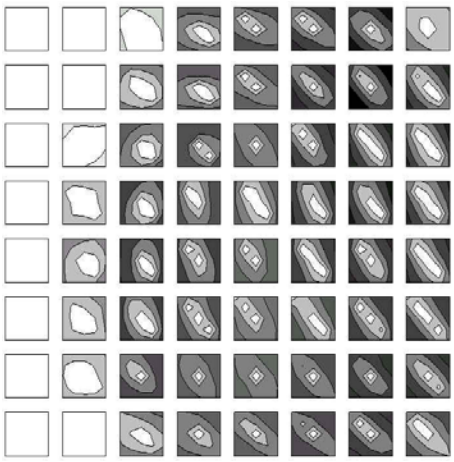
As shown in Fig. 4 and Table 1, compared to median and pseudomedian filters, the LAFOC filtered image is less smoothed, i.e., the detail edges are preserved. In addition, the filtered image by the LAFOC filter is less biased.

4.3. Experiment 3: Nonflat area detection

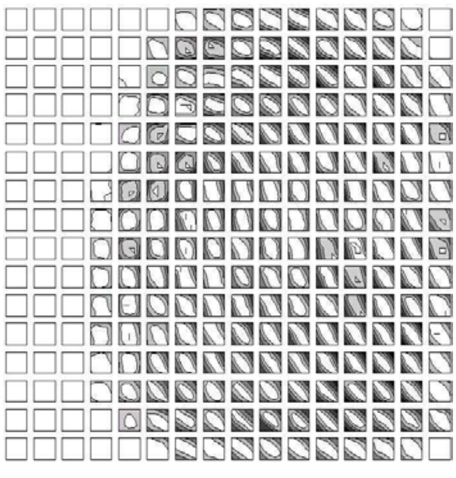
Since $I(A, B)$ indicates the degree of fitting between two sets, it can be extended to an averaged degree of fitting between A set and other sets $B_l, l = 1, \cdots, L$ as follows:

$$I(A) = \frac{\sum_l I(A, B_l)}{L}.$$

(a) Fingerprint image.



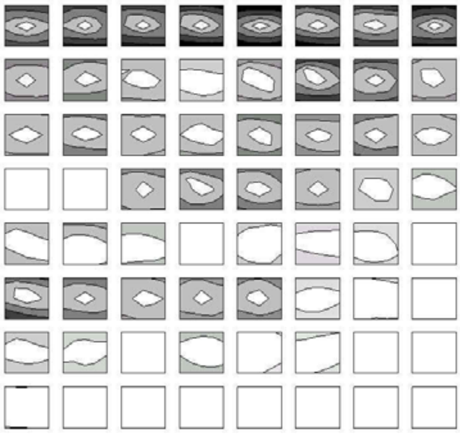
(b) 8x8 region.



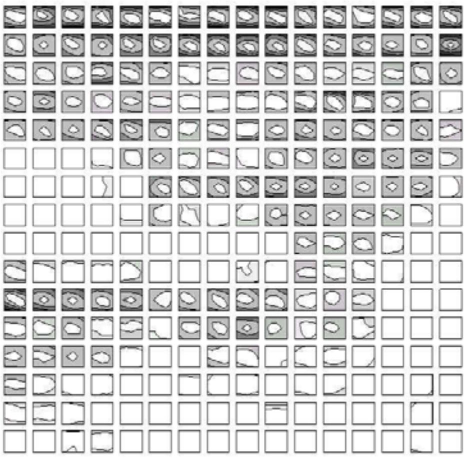
(c) 16x16 region.

Fig. 2. Optimal structuring element representation.

(a) Ultrasonic image.



(b) 8x8 region.

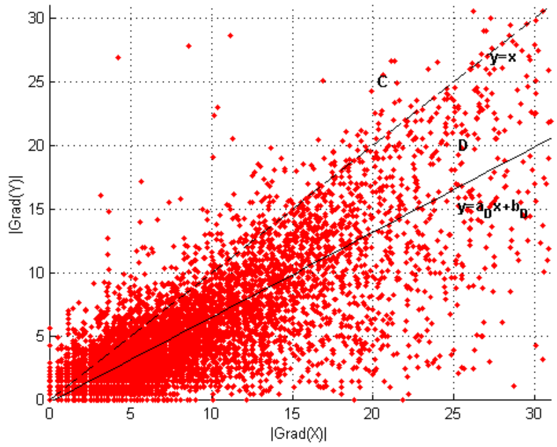


(c) 16x16 region.

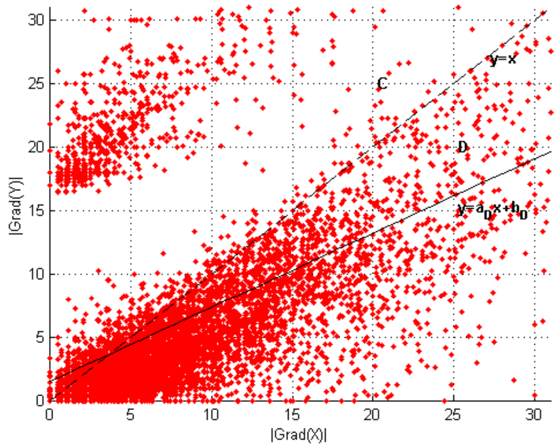
Fig. 3. Optimal structuring element representation.

Table 1. Smoothing results.

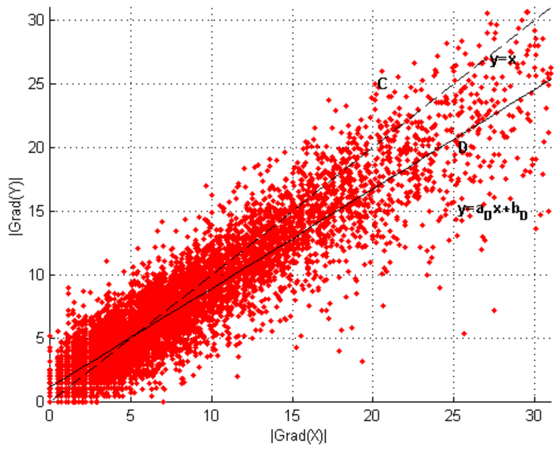
Median filter	Pseudomedian filter	LAFOC filter
0.4087	0.5830	0.1442



(a) Median filter (a_D =0.6687, b_D =-0.1874).



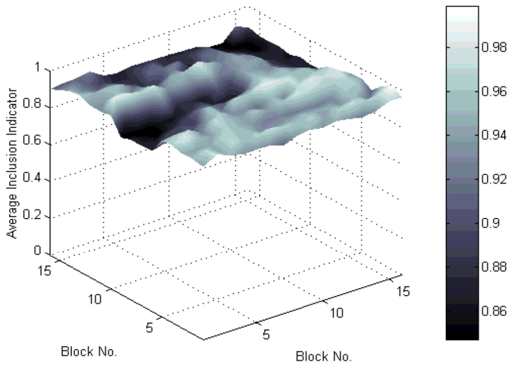
(b) Pseudomedian filter (a_D =0.5855, b_D =1.5176).



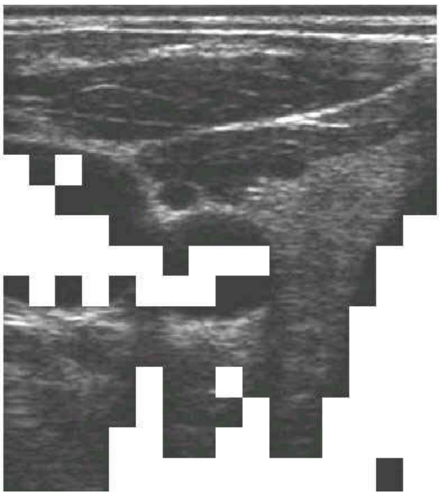
(c) LAFOC filter (a_D =0.7804, b_D =1.1466).

Fig. 4. Scatter plot of gradient magnitude.

That is, the value of $I(A)$ is indicative of how well the graph $\mu_A(x)$ fits beneath the graph $\mu_{B_l}(x), \forall l$,



(a) Average inclusion indicator.



(b) Nonflat area.

Fig. 5. $I(k_l)$ map of ultrasonic image and area detection.

in average sense. Using the set of optimal structuring elements obtained from each region, denoted as μ_{k_l} for the l-th region, each $I(k_l)$ is calculated and displayed in Fig. 5(a). As one can see, the values of $I(k_l)$ corresponding to flat SEs are low, while the values of $I(k_l)$ corresponding to detailed SEs are high. That is, the average inclusion indicator indicates the degree of relative flatness or extent of detail of an object's image. The nonflat area of an image is easily detected by the thresholding the value of $I(k_l)$ (see Fig. 5(b)). The flat area detected by $I(k_l)$ or the set of optimal structuring elements is also useful in flat zone filtering [9].

4.4. Experiment 4: morphological multi resolution decomposition

In mathematical morphology, a multi resolution analysis decomposes an image into different sub images where each sub image contains objects of a specific size.

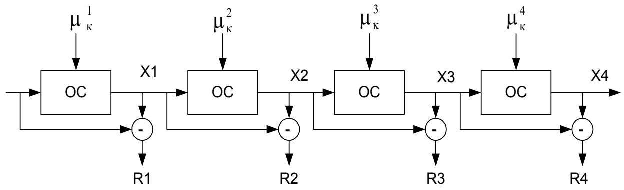
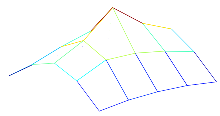
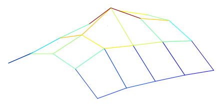


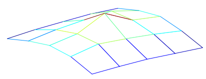
Fig. 6. Morphological multi resolution decomposition.



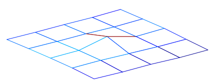
(a) $\mu_k^{\ 1}$.



(b) $\mu_k^{\ 2}$.



(c) $\mu_k^{\ 3}$.



(d) $\mu_k^{\ 4}$.

Fig. 7. Decomposition SEs for ultrasonic image.

Fig. 6 shows the standard morphological decomposition [10] where OC stands for opening followed by closing operation. In general, the size of the structuring element is increased in the subsequent decomposition stage $s+1$, i.e., $\mu_k^{\ S+1} \geq \mu_k^{\ S}$. Instead of using the predefined structuring element [10], the set of optimal structuring elements that contain the geometric features of an image can be used for decomposition. To acquire the structuring element for 4-stage decomposition, i.e., $\mu_k^{\ 1}$, $\mu_k^{\ 2}$, $\mu_k^{\ 3}$, and $\mu_k^{\ 4}$, the values of $I(k_l)$ are ranked as follows: $\Re\{I(k_l), l \in (1,\ldots,L)\} = [r_1, r_2, \ldots, r_L]$
where $\Re$ denotes ascending rank operator. And then,

$$\mu_k^{\ 1}(n) = \frac{\sum_l \mu_{k_l}(n)}{L/4} \text{ for } l \in \{I(k_l) \leq r_{L/4}\},$$

$$\mu_k^{\ 2}(n) = \frac{\sum_l \mu_{k_l}(n)}{L/4} \text{ for } l \in \{r_{L/4} < I(k_l) \leq r_{L/2}\},$$

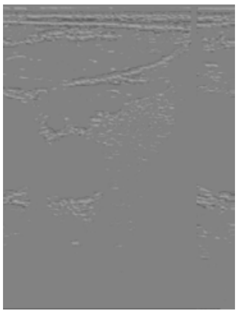
$$\mu_k^{\ 3}(n) = \frac{\sum_l \mu_{k_l}(n)}{L/4} \text{ for } l \in \{r_{L/2} < I(k_l) \leq r_{3L/4}\},$$

$$\mu_k^{\ 4}(n) = \frac{\sum_l \mu_{k_l}(n)}{L/4} \text{ for } l \in \{r_{3L/4} < I(k_l)\}.$$

Fig. 7 depicts the structuring element for decomposition obtained from Fig. 3(c) ($L = 16 \times 16$) by the above equations. Clearly, the structuring elements between stages are shown to have the increasing relationship, $\mu_k^{\ S+1} \geq \mu_k^{\ S}$. As one can see,

(a) R1.

(b) R2.



(c) R3.

(d) R4.

(e) X4.

Fig. 8. Morphological multi-resolution decomposition of ultrasonic image.

the structuring elements for decomposition have horizontal direction with different sizes. The morphological decomposition result is shown in Fig. 8. This decomposition result is revealed to filter out small objects of a certain size while preserving horizontal details at each stage.

5. CONCLUSIONS

In this paper we presented the linear combination of a fuzzy opening and closing filter with locally adaptive structuring elements. It is shown that the optimal structuring element from the adaptation algorithm of a linear combination fuzzy opening and closing filter represents the geometric features of an image. The average inclusion indicator also illustrates the degree of relative flatness or detail of an object in an image. Therefore, the structuring element representation of an image promises to be very suitable for future work in filtering, multi resolution, segmentation, and recognition of image.

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