Game Model Based Co-evolutionary Solution for Multiobjective Optimization Problems

Kwee-Bo Sim, Ji-Yoon Kim, and Dong-Wook Lee

Abstract: The majority of real-world problems encountered by engineers involve simultaneous optimization of competing objectives. In this case instead of single optima, there is a set of alternative trade-offs, generally known as Pareto-optimal solutions. The use of evolutionary algorithms Pareto GA, which was first introduced by Goldberg in 1989, has now become a sort of standard in solving Multiobjective Optimization Problems (MOPs). Though this approach was further developed leading to numerous applications, these applications are based on Pareto ranking and employ the use of the fitness sharing function to maintain diversity. Another scheme for solving MOPs has been presented by J. Nash to solve MOPs originated from Game Theory and Economics. Sefrioui introduced the Nash Genetic Algorithm in 1998. This approach combines genetic algorithms with Nash's idea. Another central achievement of Game Theory is the introduction of an Evolutionary Stable Strategy, introduced by Maynard Smith in 1982. In this paper, we will try to find ESS as a solution of MOPs using our game model based co-evolutionary algorithm. First, we will investigate the validity of our co-evolutionary approach to solve MOPs. That is, we will demonstrate how the evolutionary game can be embodied using co-evolutionary algorithms and also confirm whether it can reach the optimal equilibrium point of a MOP. Second, we will evaluate the effectiveness of our approach, comparing it with other methods through rigorous experiments on several MOPs.

Keywords: Co-evolutionary algorithm, evolutionary game theory, multiobjective optimization problem, Nash genetic algorithm, pareto optimal set.

1. INTRODUCTION

Most of the real-world problems encountered by engineers involve simultaneous optimization of several competitive objective functions [1]. The traditional optimization problems attempt to simultaneously minimize cost and maximize fiscal return. However, in these and most other cases, it is unlikely that each objective would be optimized by the same parameter choices. Hence, some trade-off between the criteria is needed to ensure a satisfactory design.

In searching for solutions to these problems, we find that there is no single optimal solution but rather a set of solutions. These solutions are optimal in the sense that no other solutions in the search space are superior to them when all objectives are considered. They are generally known as Pareto-optimal solutions [2]. Though numerous approaches for solving MOPs are in existence, we bring evolutionary algorithms into focus.

This chapter introduces previously established evolutionary approaches. The second chapter explains two optimization approaches based on Game Theory to solve MOPs. The first of these is a Nash genetic algorithm (Nash GA) proposed by Sefrioui and the second is game model based co-evolutionary algorithm proposed by us. In the final chapter, we compare optimized solutions using the game model based co-evolutionary algorithm with other algorithms through several test problems.

1.1. Evolutionary approaches: Non-Pareto approaches

The first exploration for treating objective functions separately using evolutionary algorithms was launched by Schaffer. In his 1984 dissertation [3], and later in [4], Schaffer proposed the Vector Evaluated Genetic Algorithm (VEGA) for finding a solution set to solve MOPs. He created VEGA to find and maintain multiple classification rules in a set-covering problem. VEGA attempted to achieve this goal by selecting a fraction of the next generation using one of each of the attributes (e.g., cost, reliability) [5]. Other ap-

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Kwee-Bo Sim, Ji-Yoon Kim and Dong-Wook Lee are with the School of Electrical and Electronic Engineering, Chung-Ang University, 221, Huksuk-Dong, Dongjak-Ku, Seoul 156-756, Korea (e-mail: kbsim@cau.ac.kr, jonathan@wm.cau.ac.kr, and dwlee@wm.cau.ac.kr).

proaches that search populations for multiple nondominated solutions include those of Fourman [6], Kursawe [7], and Hajela and Lin [8]. However, as none of them makes direct use of the actual definition of Pareto-optimality, different nondominated individuals are generally assigned different fitness values [9].

1.2. Evolutionary approaches: Pareto-based approaches

Goldberg first proposed Pareto-based fitness assignment approaches known as Pareto GA. The idea of this algorithm is to assign high probability to all non-dominated individuals in the population [10]. This method consists of assigning rank 1 to the non-dominated individuals and removing them from contention, then finding a new set of non-dominated individuals, ranked 2, and so forth. He named this ranking as Pareto ranking.

Fonseca and Fleming have proposed another different scheme, whereby an individual's rank corresponds to the number of individuals in the current population by which it is dominated [11]. Therefore, non-dominated individuals are assigned the same rank, while dominated ones are penalized according to the population density of the corresponding region of the trade-off surface [12]. Horn and Nafpliotis also proposed tournament selection based on Pareto dominance [13].

Moreover, distributive search is very important in Pareto GA. The goal of Pareto GA is to search all Pareto optimal solution sets distributed along the Pareto frontier. To achieve this subject Goldberg and Richardson introduced the concept of fitness sharing in their paper [14]. It is within the range of possibility to search distributive solutions through the fitness sharing that permits highly fitted candidates to share fitness with others in their surroundings [5].

With the introduction of non-dominance Pareto-ranking and fitness sharing, Pareto GAs have now become a sort of standard in the sense that the Pareto GAs provide a very efficient way to find a wide range of solutions to a given problem. Although this approach proposed by Goldberg was further developed in [15], and led to many applications [1, 16,17], all of these approaches are based on the concept of Pareto ranking and use either sharing or mating restrictions to ensure diversity. Regardless of how multiobjective optimization schemes based on Pareto optimality were developed, we introduce two different approaches to solve MOPs [18] based on Game Theory.

2. EVOLUTIONARY GAME THEORETICAL APPROACHES

Since mathematical basis founded by Von Neumann in the late 1920s', Game Theory has contributed to the study for solving MOPs that are indulged in the sphere of mathematics and economics. Game Theory introduces the notion of games and players associated to an optimization problem. In the case of a multiobjective design through Game Theory, each player involved has his own criterion. During the game, namely Nash Game, players try to make improvements until the system reaches the state of equilibrium.

In this chapter, we introduce two searching algorithms for finding an optimized equilibrium solution of MOPs through the evolutionary game. The first algorithm launched from the idea regarding the solution of a non-cooperative game was introduced early in the 1950's by J. F. Nash. This approach has brought in the concept of 'Game Player' for solving MOPs involved in Game Theory and Economics [18]. The second algorithm is a co-evolutionary algorithm using the game model, which is a newly proposed approach in this paper. This approach attempts to search the Evolutionary Stable Strategy (ESS) of MOPs combining the co-evolutionary algorithm with the evolutionary game theory.

2.1. Nash Genetic Algorithm (Nash GA)

The idea of Nash GA is to bring together genetic algorithms and Nash strategy in order to cause the genetic algorithm to build the Nash Equilibrium. In the following, we present how such merging can be achieved with 2 players trying to optimize 2 different objectives.

Let $s = XY$ be the string representing the potential solution for a dual objective optimization problem. Then X denotes the subset of variables handled by Player 1 and optimized along criterion 1. Similarly Y denotes the subset of variables handled by Player 2 and optimized along criterion 2. Thus, as advocated by Nash theory, Player 1 optimizes s with respect to the first criterion by modifying X while Y is fixed by Player 2. Symmetrically, Player 2 optimizes s with respect to the second criterion by modifying Y, while X is fixed by Player 1.

The next step consists of creating two different populations, one for each player. Player 1's optimization task is performed by Population 1 whereas Player 2's optimization task is performed by Population 2. Let X_{k-1} be the best value found by Player 1 at generation $k-1$ and Y_{k-1} be the best value found by Player 2 at generation $k-1$. At generation k, Player 1 optimizes X_k while using Y_{k-1} in order to evaluate s (in this case, $s = X_k Y_{k-1}$). Simultaneously, Player 2 optimizes Y_k while using X_{k-1} in order to evaluate s (in this case, $s = X_{k-1} Y_k$). After the optimization process,

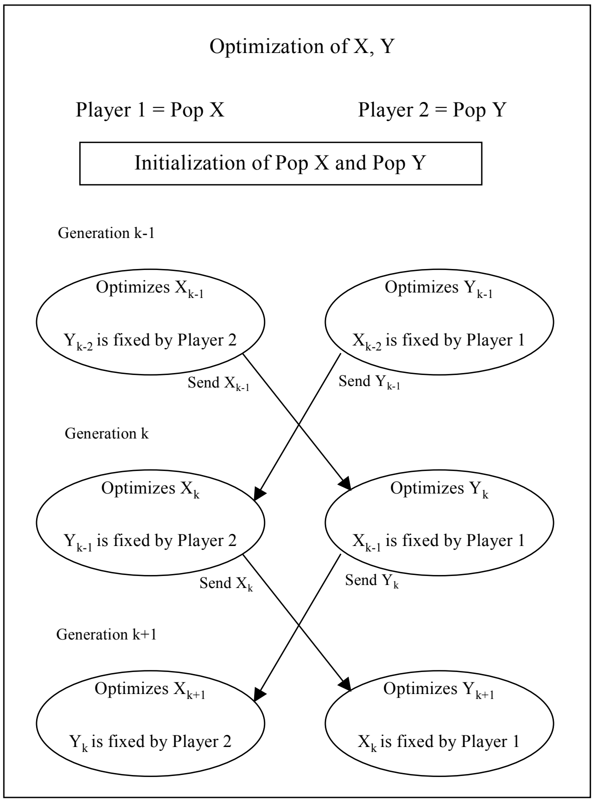

Fig. 1. Block diagram of Nash Genetic Algorithm.

Player 1 sends the best value X_k to Player 2 who will use it at generation $k+1$. Similarly, Player 2 sends the best value Y_k to Player 1 who will use it at generation $k+1$. Nash equilibrium is reached when neither Player 1 nor Player 2 can further improve their criteria [18].

For this algorithm Sefrioui also uses distance-dependent mutation, which is a technique evolved to maintain diversity in small populations. Instead of a fixed mutation rate, each offspring has its mutation rate computed after each mating. This mutation rate depends on the distance between the two parents [19].

2.2. Evolutionary Stable Strategy (ESS)

The primary contribution of evolutionary game theory (EGT) is the concept of the Evolutionary Stable Strategy (ESS). ESS was originally proposed by a world renowned biologist named Maynard Smith based on EGT and defined as an unchangeable strategy by other strategies [20]. Unchangeable strategy means that no matter how outstanding a particular strategy may be, it cannot maintain predominance over other inferior strategies permanently. In the context of an actual ecosystem, more evolutionary stable species can be reserved than superior species, in other words an evolution chooses the strategy that not only executes progressive direction but also moves the equilibrium state.

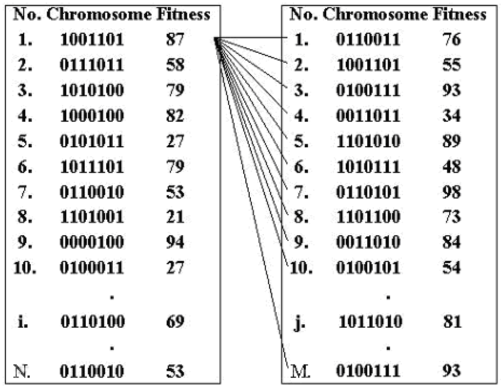

Fig. 2. Population of co-evolutionary algorithm for game.

The ESS is a refinement of Nash equilibrium that dispenses with the traditional assumption of agent rationality. Instead, Maynard Smith shows that game-theoretic equilibriums can be achieved through the process of Darwinian selection [21]. Nevertheless, the ESS is defined as a static concept, and since its introduction many other stability concepts have been proposed [22], including those that are more properly rooted in dynamical systems theory [23]. The ESS corresponds to a dynamical attractor [24].

For the game modeled in this paper, the evolutionary game is embodied by the co-evolutionary algorithm. Each population corresponds to player and fitness of individuals in the population is evaluated as a reward of the game. This reward results from fitness of an opponent player in another population.

2.3. Game model based co-evolutionary algorithm

In this section, the co-evolutionary algorithm designed for searching ESS of MOP is explained. Throughout the game, players for each objective function try to optimize their own objectives and all individuals in a population set are rewarded. The reward value is determined by the percentage of victories during the game.

To design the co-evolutionary algorithm based on Game Theory, we first established a game player with randomly generated populations. All individuals in each population are rewarded 'fitness' that will be used during the selection procedure. During the game each individual in the first population plays the game with others in the remaining populations and is paid the fitness calculated from (1), (2) and (3). Other individuals in the remaining populations execute the game in the same manner by turns. Using the fitness, the next generation individuals are produced in each population independently through crossover and mutation.

In Fig. 2, 'No.' signifies the number of individuals within each population. We use binary chromosome and normalized fitness. The example of MOPs having

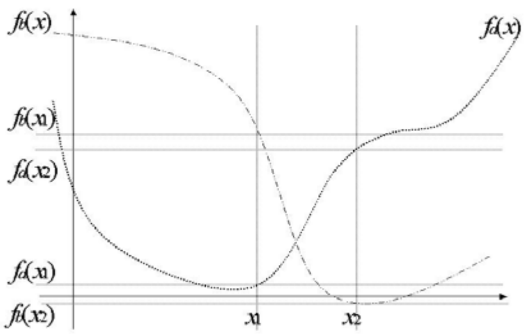

Fig. 3. Objective function graph for calculating gain value.

two variables is given to help and this is the minimization problem.

Step 1: Two populations are randomly generated like in Fig. 2.

Step 2: The first individual in the primary population plays with each individual in the other population and is evaluated for level of fitness.

Throughout the game by turns, the fitness of the opponent individual in the second population is calculated in the same manner.

Step 3: The process of Step 2 is executed for all individuals of the first population one by one.

Step 4: The processes of Step 2 and Step 3 are executed for all individuals of the second population analogously.

$$Fitness(x_i) = \lambda + \sum_{n=1}^{M} \frac{f_a(y_n) - f_a(x_i) + f_b(y_n) - f_b(x_i)}{MAX(x_i, y_n)} \tag{1}$$

$$Fitness(y_j) = \lambda + \sum_{n=1}^{N} \frac{f_a(x_n) - f_a(y_j) + f_b(x_n) - f_b(y_j)}{MAX(y_j, x_n)} \tag{2}$$

$$\begin{aligned} MAX(x_i, y_j) = {} & f_{a\_\max}(y_j) - f_{a\_\max}(x_i) \\ & + f_{b\_\max}(y_j) - f_{b\_\max}(x_i) \end{aligned} \tag{3}$$

In (1) and (2), x_i and y_j signify the individual in the first population and the second population. The $f_a(y_j)$, $f_a(x_i)$, $f_b(y_j)$, and $f_b(x_i)$ are the values calculated when $x = x_i$ or y_j for $f_a(x)$, $f_b(x)$ from Fig. 3. And in (3), $f_{a\_\max}(x)$ and $f_{b\_\max}(x)$ indicate the maximum values of $f_a(x)$, $f_b(x)$ for the given variable.

Step 5: Using $Fitness(x_i)$ and $Fitness(y_j)$ deter - mined from the previous procedures, each population produces next generation individuals independently.

Step 6: Until ending condition is satisfied the procedures from Step 2 to Step 5 are reiterated.

Keeping these ideas, by the comparison experimental result of co-evolutionary approach with Nash GA, we show that these approaches can be regarded as appropriate and the stable equilibrium points of MOPs can be found. To achieve these goals we choose several MOPs. One out of these is the multiobjective function proposed by Sefrioui in his paper to evaluate Nash GA and another is the function proposed by Schaffer, which is generally used in solving MOPs by genetic algorithms [10]. Various other test problems are also adopted.

In this paper, we introduced several approaches to solving MOPs. In the introduction, established optimization algorithms based on the concept of Pareto optimal set are introduced. Contrary to these algorithms, in this chapter, we introduce theoretical backgrounds of Nash Genetic Algorithm (Nash GA) and Evolutionary Stable Strategy (ESS), which are based on EGT. Moreover, ESS is the basis of the co-evolutionary algorithm using the game model as newly proposed in this paper. But generally ESS, the equilibrium solution of the co-evolutionary game model, exists more than once so that we apply elitism to search the most optimized equilibrium solution of MOPs. In the next chapter, from the experimental results we confirm that co-evolutionary algorithm based on EGT can search the optimal equilibrium solutions of MOPs.

3. TEST PROBLEMS AND EVALUATION

While various evolutionary approaches (and variations of them) to solve MOPs were successfully applied to these problems, in recent years some researchers have investigated particular topics of evolutionary multiobjective search. In spite of this variety of approaches, there is a lack of studies that compare the performance and the singular aspects of these approaches. In this chapter, we used several problems. The problems considered here are Sefrioui's problem used in his paper, Schaffer's problem, and Zitzler's test MOPs, which are proposed for a systematic comparison of several multiobjective EAs in his paper [25].

3.1. Optimization of problem used in Sefrioui's paper

As the first experiment, the multiobjective problem to minimize two objective functions is used and this problem is introduced in Sefrioui's paper to evaluate Nash GA.

$$f_1(x, y) = (x-1)^2 + (x-y)^2, \tag{4}$$

$$f_2(x, y) = (y-3)^2 + (x-y)^2, \tag{5}$$

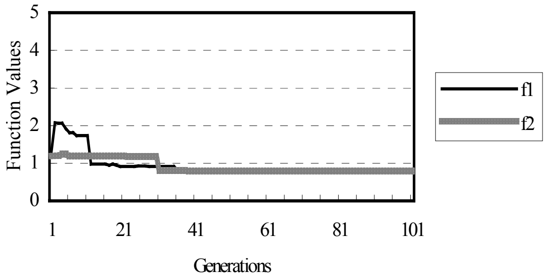

Fig. 4. The change of f_1, f_2 values for every generation by Nash GA.

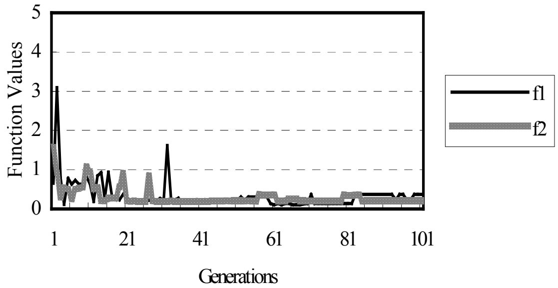

Fig. 5. The change of f_1, f_2 values for every generation by Co-evolutionary algorithm.

Table 1. The simulated solutions of Sefrioui's multi-objective function found by Nash GA and Game model based co-evolutionary algorithm.

Nash GA	X_1	Y_1	F_1
	1.6673	2.3297	0.8840
	X_2	Y_2	F_2
	1.6697	2.3393	0.8849
Game model based co-evolutionary algorithm	X_1	Y_1	F_1
	1.3033	0.7723	0.3740
	X_2	Y_2	F_2
	2.4340	2.5570	0.2114

where constraint is $-5 \le x, y \le 5$. To solve the MOPs shown above using genetic algorithms, we allotted two populations to each objective function for searching solutions.

Table 1 presents the most optimized solutions for this problem by using Nash GA and the co-evolutionary algorithm. Hence, X_1, Y_1, F_1 are the optimized values by Population 1 and X_2, Y_2, F_2 are the optimized values by Population 2. In this experiment the optimized values by Sefrioui using Nash GA are the identical values found in Sefrioui's paper. This also presents Nash equilibrium point.

Fig. 4 illustrates the change of f_1, f_2 values for every generation by Nash GA and Fig. 5 displays the change of f_1, f_2 values using our game model based co-evolutionary algorithm. From these results, we can determine that the game model based co-evolutionary algorithm search more optimized solutions than the solutions found by Nash GA and these solutions also exist in the boundary of the Pareto frontier.

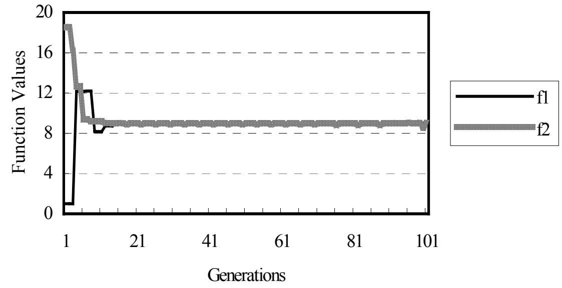

Fig. 6. The change of f_1, f_2 values for every generation by Nash GA.

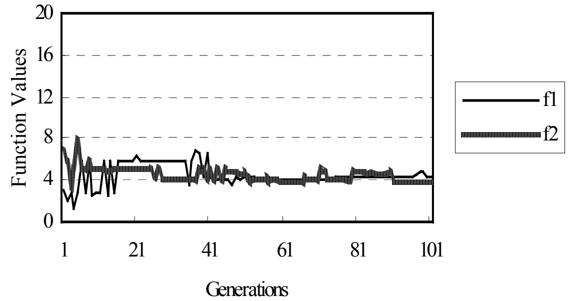

Fig. 7. The change of f_1, f_2 values for every generation by Co-evolutionary algorithm.

3.2. Optimization of Schaffer's MOP

The second experiment is to solve a double objective minimization problem, proposed by Schaffer. This problem is composed of two functions having the second order.

$$f_1(x, y) = (x-1)^2 + (y-1)^2, \quad (6)$$

$$f_2(x, y) = (x-4)^2 + (y-4)^2, \quad (7)$$

where $-5 \le x, y \le 5$ is the constraint. Like as in the previous experiment, we allotted two populations for each objective function.

The experimental results searched by applying each algorithm to this problem are presented in Table 2, Fig. 6 and 7. As seen in these results, our game model based co-evolutionary algorithms have found more optimized solutions for this problem. Moreover this point exists in the Pareto optimal front, just as in the previous experiment.

From the previously executed experimental results we confirmed that it is regarded as appropriate to search

Table 2. The simulated solutions of Schaffer's multiobjective function found by Nash GA and Game model based co-evolutionary algorithm.

Nash GA	X_1	Y_1	F_1
	0.9933	4.0004	9.0023
	X_2	Y_2	F_2
	0.9933	4.0004	9.0403
Game model based co-evolutionary algorithm	X_1	Y_1	F_1
	2.3499	2.6825	4.6531
	X_2	Y_2	F_2
	2.4994	2.4641	4.6107

optimal equilibrium strategy by applying the co-evolutionary algorithm combined EGT model and this can be regarded as the solutions of MOPs. In particular, we found that the co-evolutionary algorithm based on the EGT model can search more optimized solutions in spite of its simplicity.

3.3. Test functions proposed in Zitzler's paper [1]

In the previous chapter, we introduced variousestablished evolutionary algorithms for solving multiobjective problems. In spite of this variety, there is a lack of studies that compare the performance and different aspects of these approaches. Among these studies we introduce several researches. On the theoretical side, Fonseca and Fleming discussed the influence of different fitness assignment strategies on the selection process [12]. On the practical side, Zitzler and Thiele used a NP-hard 0/1 knapsack problem to compare several multiobjective EAs [26,27]. In these papers, Zitzler provides a systematic comparison of multiobjective EAs, including a random search strategy as well as a single-objective EA using objective aggregation. The basis of this empirical study is formed by a set of well-defined, domain-independent test functions that allow the investigation of independent problem features. We thereby draw upon results presented in Deb, where problem features that may make convergence of EAs to the Pareto-optimal front difficult are identified and, furthermore, methods of constructing appropriate test functions are suggested [28]. The functions considered here cover the range of convexity, nonconvexity and discrete Pareto fronts. Deb has identified several features that may cause difficulties for multiobjective EAs in converging to the Pareto-optimal front and maintaining diversity within the population [29]. Each of the test functions defined below is structured in the same manner and consists itself of three functions f_1, g, h.

Minimize $t(x)=(f_1(x_1), f_2(x_2))$

Subject to

$$f_2(x)=g(x_2,\cdots,x_n)\cdot h(f_1(x_1), g(x_2,\cdots,x_n)), \quad (8)$$

where $x=(x_1,\cdots,x_n)$. The function f_1 is a function of the first decision variable only, g is a function of the remaining $m-1$ variables, and the parameters of h are the function values of f_1 and g. The test functions differ in these three functions as well as in the number of variables m and in the values the variables may take [28].

The test function T_1 has a convex Pareto-optimal front

$$f_1(x_1)=x_1,$$
$$g(x_2,\cdots,x_n)=1+9\cdot\frac{\sum_{i=2}^{n} x_i}{n-1},$$
$$h(f_1,g)=1-\sqrt{\frac{f_1}{g}},$$

where $n=30$, and $x_i\in[0,1]$.

The test function T_2 has a nonconvex Pareto-optimal front

$$f_1(x_1)=x_1,$$
$$g(x_2,\cdots,x_n)=1+9\cdot\frac{\sum_{i=2}^{n} x_i}{n-1},$$
$$h(f_1,g)=1-\left(\frac{f_1}{g}\right)^2,$$

where $n=30$, and $x_i\in[0, 1]$.

The test function T_3 represents the discreteness feature; its Pareto-optimal front consists of several noncontiguous convex parts:

$$f_1(x_1)=x_1,$$
$$g(x_2,\cdots,x_n)=1+9\cdot\frac{\sum_{i=2}^{n} x_i}{n-1},$$
$$h(f_1,g)=1-\sqrt{\frac{f_1}{g}}-\left(\frac{f_1}{g}\right)\cdot\sin(10\pi f_1),$$

where $n=30$, and $x_i\in[0, 1]$.

Figs. 9, 11, and 13 show the simulated solution of each test function using our game model based co-evolutionary algorithm. Figs. 8, 10, and 12 are cited pictures from Zitzler's paper to compare with the results of our algorithm. These pictures are a searched solution of each test function and shown in criterion space (f_1-f_2) [25].

In these cited pictures,

- RAND: A random search algorithm.
- FFGA: Fonseca and Fleming's multiobjective EA.
- NPGA: The Niched Pareto Genetic Algorithm.

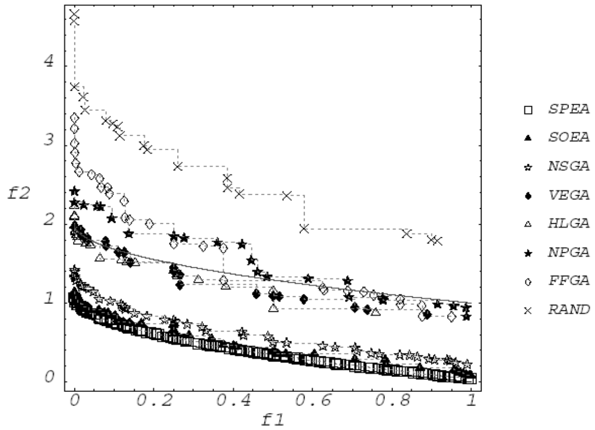

Fig. 8. The Pareto front of T_1 searched by other evolutionary algorithms (Cited from [1]).

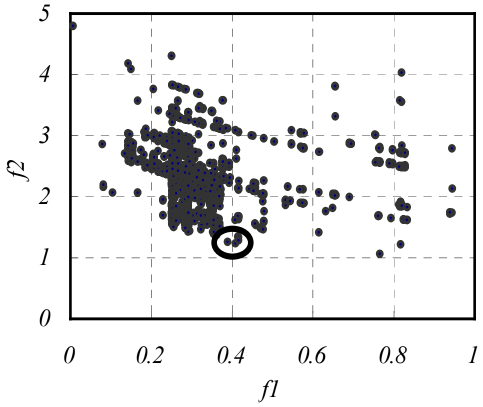

Fig. 9. The optimized solution of T_1 searched by the proposed co-evolutionary algorithm.

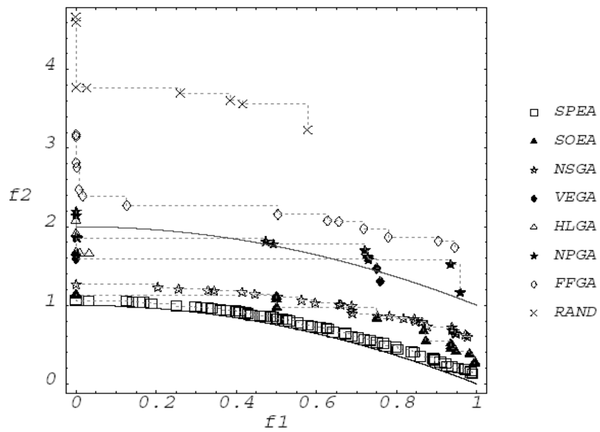

Fig. 10. The Pareto front of T_2 searched by other evolutionary algorithms (Cited from [1]).

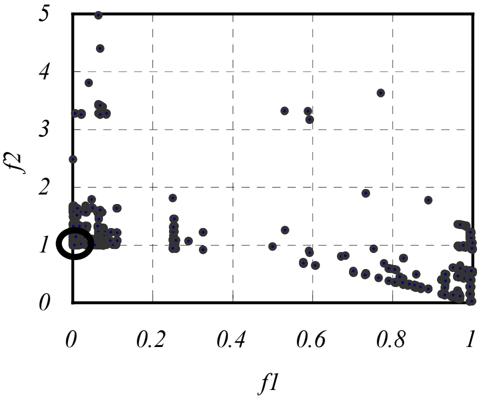

Fig. 11. The optimized solution of T_2 searched by the proposed co-evolutionary algorithm.

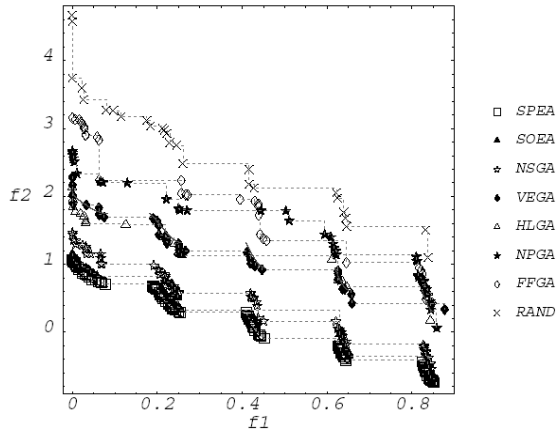

Fig. 12. The Pareto front of T_3 searched by other evolutionary algorithms (Cited from [1]).

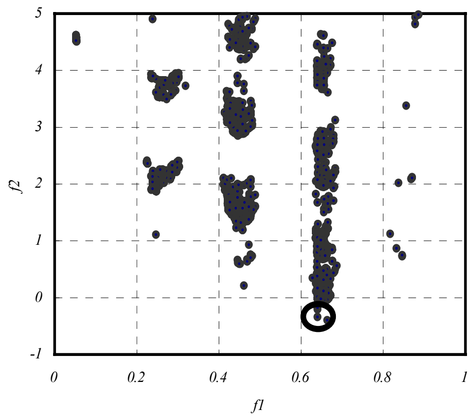

Fig. 13. The optimized solution of T_3 searched by the proposed co-evolutionary algorithm.

- HLGA: Hajela and Lin's weighted-sum based approach.
- VEGA: The Vector Evaluated Genetic Algorithm.
- NSGA: The Nondominated Sorting Genetic Algorithm.
- SOEA: A single-objective evolutionary algorithm using weighted-sum aggregation.
- SPEA: The Strength Pareto Evolutionary Algorithm.

In the pictures of simulated result by our proposed algorithm, black circles show finally found optimized solutions using our game model based co-evolutionary algorithm. Used parameters are as follows. The number of generation is 500. Population size is 100. One-point crossover rate is 0.8. Mutation rate is 0.01. These are the same parameter values used in Zitzler's experiment. From these results, we confirm that optimized solutions using game model based co-evolutionary algorithm are all existent in Pareto optimal front and this algorithm can search relatively more optimized solutions than those of the other eight evolutionary algorithms.

4. CONCLUSIONS

In this paper, we proposed a novel game model based co-evolutionary scheme to solve MOPs, which is different from conventional ones. In order to point out the difference, we first surveyed the conventional evolutionary algorithms that were proposed by others to search for the solution set of a MOP. Second, we showed the construction methodology combining the game theory with the co-evolutionary algorithm systematically. Finally, we investigated the performance efficiency of the designed algorithm rigorously. It was shown that the designed method not only successfully reaches the equilibrium points of MOPs but it also locates the superior optimal solution set in comparison with other conventional evolutionary algorithms. Especially in the case of Pareto front solution set, the better solutions were obtained by comparison with other approaches. Our method has an advantage over the Nash GA method because it uses a very simple genetic operator to evolve the solution unlike the genetic operators used in Nash GA. For instance, Nash GA uses a complex mutation operator, which requires a heavy computation burden to calculate the hamming distances of two individual binary chromosomes involving encoding and decoding of chromosomes.

Conclusively, our game model based co-evolutionary scheme is simple, robust in finding the optimal solution set, requires less computation, and it can be an alternative method in solving MOPs.

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Kwee-Bo Sim received the B.S. and M.S. degrees in the Department of Electronic Engineering from Chung-Ang University, Seoul, Korea, in 1984 and 1986 respectively, and the Ph.D. degree in the Department of Electronic Engineering from the University of Tokyo, Japan, in 1990. Since 1991, he has been a Faculty Member of the School of Electrical and Electronic Engineering at Chung-Ang University, where he is currently a Professor. His research interests are Artificial Life, Intelligent Robot, Multi-Agent System, Distributed Autonomous Robotic System, Learning & Adaptational Algorithms, Soft Computing (Neural Network, Fuzzy System, Evolutionary Computation), Artificial Immune System, Network Intrusion Detection System, Evolvable Hardware, Artificial Brain, etc. He is a member of IEEE, SICE, RSJ, KITE, KIEE, ICASE, and KFIS.

Ji-Yoon Kim received the B.S. and M.S. degrees in the School of Electrical and Electronic Engineering from Chung-Ang University, Seoul, Korea, in 2002 and 2004 respectively, and is currently working towards his doctorate. His research interests are Artificial Life, Evolutionary Computation, Distributed Auto-nomous Robotic Systems, Machine Learning, and Artificial Intelligence.

Dong-Wook Lee received the B.S., M.S., and Ph.D. degree in Department of Control and Instrumentation Engineering from Chung-Ang University in 1996, 1998, and 2000, respectively. Hi is currently Postdoctoral Researcher in School of Electrical and Electronic Engineering at Chung-Ang University. His interest includes Artificial Life, Evolutionary Computation, Evolutionary Robotics, Artificial Brain, and Artificial Immune System.