Fuzzy Estimator for Gain Scheduling and its Application to Magnetic Suspension

Seon-Ho Lee and Jong-Tae Lim

Abstract: The external force disturbance is the one of the main causes that deteriorate the performance of the magnetic suspension. Thus, this paper develops a fuzzy estimator for gain scheduling control of magnetic suspension systems suffering from the unknown disturbance. The proposed fuzzy estimator computes the disturbance injected to the plant and the gain scheduled controller generates the corresponding stabilizing control input associated with the estimated disturbance. In the simulation results we confirm the novelty of the proposed control scheme comparing with the other method using a feedback linearization.

Keywords: fuzzy estimator, gain scheduling, adaptive law, external force disturbance, magnetic suspension

I. Introduction

The gain scheduled controllers proved to be a successful design methodology have been developed by many researchers [1]-[3]. Furthermore, in order to improve its regulation performance for fast scheduling parameters, [4] and [5] have proposed extended control laws using the derivative information of the time-varying parameter. Nowadays, H_∞ control theory has become an effective design methodology in tracking the problem of stability and performance robustness under plant uncertainties. Thus, many researchers have been interested in the fusion technique of H_∞ synthesis and gain scheduling [6]-[8]. However, since the stabilizing control gains of all the gain scheduled controllers mentioned above are driven by the measured scheduling parameter, these developed controllers become unavailable for the unmeasurable scheduling parameter, which should be estimated and engaged appropriately.

In spite of the high nonlinearity and the unstable dynamics, the magnetic suspension systems have been researched with merits that the absence of contact reduces noise, component wear, vibration, and maintenance costs. It is well known that the external force disturbance and mass uncertainty dominantly deteriorate the quality of system performance. In order to solve such problems, the classical state feedback control with pole-placement has been applied into the linearized model corresponding to a specific operating condition [9]. However, since the operating condition changes according to the disturbance and the uncertainty, the local controller couldn't achieve the satisfactory performance in the global operating points. Nonlinear control schemes have also been reported in the literature such as the gain scheduling [10] and the feedback linearization methodology [11] [12]. However, the force disturbance estimator [10] utilized the exact inverse dynamic of the given plant model using the derivative information of the state vectors. Thus, it was confined to solve the regulation problem only when the exact derivative information of the scheduling parameter is available in the considered plant. Moreover, though the feedback linearization canceled the nonlinear nature in the magnetic suspension, it also had a limitation to tackle the unknown parameter variation and/or disturbance [11] [12]. Most recently, the self-tuning controller was developed for the unknown mass variation and showed improved performance [13]. However, the problem of determining an adequate adaptation rate and combining the cost functions still remained so as to apply to the general nonlinear plants.

The proposed fuzzy-estimated gain scheduled controller in this paper consists of the fuzzy estimator and the gain scheduled controller. The fuzzy estimator tracks the unknown scheduling parameter (external force disturbance) in the closed-loop control system and the stabilizing gains of the gain scheduled controller are appropriately scheduled according to the estimated disturbance. The proposed control scheme has several advantages over the related literature. The developed fuzzy estimator does not require the time derivative of the state vector that induces noise in practice. Moreover, the fuzzy estimator computes the adaptation rate and combine the adaptation laws in the manner of achieving the control objective. Thus, it is applicable to the various nonlinear plants in company with the gain scheduling methodologies.

II. Gain scheduled controller

The single-axis magnetic suspension in Fig. 1 is modeled by the following nonlinear dynamic equation [9]

$$\begin{aligned} \dot{x} &= f(x,u,f_d) = \begin{bmatrix} x_2 \\ -\frac{C}{4m}(\frac{x_3}{x_1})^2 + g + \frac{f_d}{m} \\ \frac{x_2 x_3}{x_1} - \frac{2R}{C}x_1 x_3 + \frac{2x_1}{C}u \end{bmatrix} \\ y &= h(x) = x_1 \end{aligned} \tag{1}$$

with $C = \mu_o A N^2$ where x_1 is the vertical air-gap, x_2 is the vertical velocity, $x_3 = i(t)$ is the magnet current, $u = v(t)$ is the applied voltage, f_d is the external force disturbance, $m = 15kg$ is the total mass, $N = 2000$ is the number of turns of the coil wrapped around the magnet, $A = 0.0012m^2$ is the pole area, $\mu_o = 4\pi \times 10^{-7}H/m$ is the permeability of free space,

Fig. 1. Magnetic suspension system.

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$R = 8\Omega$ is the coil resistance, $g = 9.8m/sec^2$ is the gravity constant, and r_d is the reference air-gap. Defining the disturbance f_d as the scheduling parameter, the smooth functions $\mathbf{x}(f_d)$ and $\mathbf{u}(f_d)$ satisfying

$$0 = f(\mathbf{x}(f_d), \mathbf{u}(f_d), f_d) \text{ and } r_d = h(\mathbf{x}(f_d)) \qquad (2)$$

for $f_d \in \Gamma \subset R$ are computed by

$$\mathbf{x}(f_d) = \begin{bmatrix} r_d \\ 0 \\ \frac{2r_d w(f_d)}{\sqrt{C}} \end{bmatrix} \text{ and } \mathbf{u}(f_d) = \frac{2r_d R w(f_d)}{\sqrt{C}} \qquad (3)$$

where $w(f_d) = \sqrt{f_d + mg}$. The control objective is to obtain $\lim_{t\to\infty} \|r_d - y\| = 0$ while rejecting the disturbance f_d. Then, for each fixed $f_d \in \Gamma$, the corresponding linearized closed loop system with a nonlinear state feedback control law $u = k(x, f_d)$ is written by

$$\begin{aligned} \dot{x}_\delta &= A(f_d)x_\delta + B(f_d)u_\delta \\ y_\delta &= C(f_d)x_\delta \\ u_\delta &= K(f_d)x_\delta \end{aligned} \qquad (4)$$

where the deviation variables are defined by $x_\delta = x - \mathbf{x}(f_d)$, $u_\delta = u - \mathbf{u}(f_d)$, and $y_\delta = y - r_d$. Moreover, the linearized system coefficients are given by

$$\begin{aligned} A(f_d) &= \frac{\partial f(\mathbf{x}(f_d), \mathbf{u}(f_d), f_d)}{\partial x} \\ &= \begin{bmatrix} 0 & 1 & 0 \\ \frac{2w(f_d)^2}{mr_d} & 0 & -\frac{\sqrt{C}w(f_d)}{mr_d} \\ 0 & \frac{2w(f_d)}{\sqrt{C}} & -\frac{2rR}{C} \end{bmatrix} \qquad (5) \\ B(f_d) &= \frac{\partial f(\mathbf{x}(f_d), \mathbf{u}(f_d), f_d)}{\partial u} = \begin{bmatrix} 0 & 0 & \frac{2r_d}{C} \end{bmatrix}' \qquad (6) \\ C(f_d) &= \frac{\partial h(\mathbf{x}(f_d))}{\partial x} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \qquad (7) \\ K(f_d) &= \frac{\partial k(\mathbf{x}(f_d), f_d)}{\partial x} \qquad (8) \end{aligned}$$

In order to achieve the desired pole-placement in the linearized closed-loop system, $K(f_d)$ is determined so that the eigenvalues of $A(f_d)+B(f_d)K(f_d)$ have specified values with negative real parts for each $f_d \in \Gamma$ with the controllable pair of $A(f_d)$ and $B(f_d)$ [1]. Then,

$$k(x, f_d) = \mathbf{u}(f_d) + K(f_d)(x - \mathbf{x}(f_d)) \qquad (9)$$

is obtained with $K(f_d) = \left[\frac{-\sqrt{C}\lambda(6w(f_d)^2+\lambda^2 mr_d)}{2r_d w(f_d)}, \frac{3\sqrt{C}\lambda^2 m}{2w(f_d)}, \frac{3C\lambda}{2r_d} + R\right]$ where $\lambda = -40$ is the triple eigenvalues of the linearized closed loop system. The unknown disturbance f_d in (9) is substituted with the estimated value $\hat{f}_d$ in the closed-loop control system as shown in Fig. 2.

Fig. 2. Overall control system of fuzzy-estimated gain scheduling.

In order to compare the simulation result of the proposed control scheme in this paper, we introduce the feedback linearizing controller from [11]. It is easily shown that $\{g, ad_f g, ad_f^2 g\}$ is linear independent and the distribution $span\{g, ad_f g\}$ is involutive where

$$\begin{aligned} g &= \begin{bmatrix} 0 \\ 0 \\ \frac{2x_1}{C} \end{bmatrix} \qquad (10) \\ ad_f g &= \begin{bmatrix} 0 \\ \frac{x_3}{mx_1} \\ \frac{4Rx_1^2}{C^2} \end{bmatrix} \qquad (11) \\ ad_f^2 g &= \begin{bmatrix} \frac{-x_3}{mx_1} \\ 0 \\ \frac{4R}{C^2}x_1x_2 - \frac{x_3^2}{mx_1^2} + \frac{8R^2x_1^3}{C^3} \end{bmatrix} \qquad (12) \end{aligned}$$

Moreover, for an input-state feedback linearization, a diffeomorphism is computed by

$$z = T(x) = \begin{bmatrix} x_1 - r_d \\ x_2 \\ -\frac{Cx_3^2}{4mx_1^2} + g \end{bmatrix} \qquad (13)$$

Then, the resulting feedback linearizing controller is obtained by

$$u = m\frac{x_1}{x_3}(k_1z_1 + k_2z_2 + k_3z_3 + \frac{Rx_3^2}{mx_1}) \qquad (14)$$

where $k_1 = 343000$, $k_2 = 14700$, and $k_3 = 210$.

Fig. 3. Structure of fuzzy estimator.

III. Fuzzy estimator

Since $\hat{f}_d$ drives the stabilizing gain of the controller $k(x, \hat{f}_d)$, it should be appropriately estimated in the way of stabilizing the closed loop system. Since both x_{δ_3} and u_δ are the functions of f_d, we define $\psi \in R^2$ such that

$$\psi(x, u, f_d) = \begin{bmatrix} x_{\delta_3} \\ u_\delta \end{bmatrix} = \begin{bmatrix} x_3 - \frac{2r_d w(f_d)}{\sqrt{C}} \\ u - \frac{2r_d R w(f_d)}{\sqrt{C}} \end{bmatrix} \qquad (15)$$

Here, we note that $\psi(\cdot)$ is a vector composed of the elements of x_δ and u_δ whose derivatives with respect to f_d are nonzero. In order to estimate f_d, we define cost functions

$$e_i = \frac{1}{\Omega_i}\psi_i(x, u, \hat{f}_d) \qquad (16)$$

for $i \in \{1, 2\}$ with $\Omega_i = \frac{\partial\psi_i(x,u,0)}{\partial f_d}$ where e_i is engaged to the input of the fuzzy rule-table and $\hat{f}_d$ is the estimate of f_d. Furthermore, based on (16) we propose an adaptation law of ϕ_i such that

$$\phi_i = -\eta_i \int_o^t e_i d\tau \qquad (17)$$

for $i \in \{1, 2\}$ where ϕ_i is the estimated portion of f_d contributed from ψ_i, and η_i is an adaptation rate computed by the fuzzy logic in Fig. 3.

Before we proceed further, we consider the case when η_2 is fixed to zero for simplicity, i.e., $\phi_2 = 0$ in order to give an intuition of the adaptation law. For the notational convenience, we let

$$\xi = -\frac{\Omega_1}{\eta_1}\phi_1 = \Omega_1 \int_0^t e_1 d\tau \qquad (18)$$

Table 1. Fuzzy rule-table for $i \in \{1, 2\}$.

$e_i \setminus \dot{e}_i$	NB	NS	ZE	PS	PB
NB	NB	NS	ZE	PS	PB
NS	NS	NS	ZE	PS	PS
ZE	ZE	ZE	ZE	ZE	ZE
PS	PS	PS	ZE	NS	NS
PB	PB	PS	ZE	NS	NB

Then, from (18) we note that if $\phi_1 = f_d$ is achieved, ξ converges to $\xi^{eq} = -\frac{\Omega_1 f_d}{\eta_1}$. Furthermore, using the relation of $\hat{f}_d = \phi_1 = -\frac{\eta_1}{\Omega_1}\xi$ Taylor series expansion of ξ about $\hat{f}_d = 0$ in (18) gives

$$\begin{aligned}\dot{\xi} &= \psi_1(x, u, 0) + \frac{\partial \psi_1(x, u, 0)}{\partial \hat{f}_d}\hat{f}_d + o(\hat{f}_d) \\ &= \psi_1(x, u, 0) - \eta_1\xi + o(\hat{f}_d) \qquad (19)\end{aligned}$$

where $o(\hat{f}_d) = \psi_1(x, u, \hat{f}_d) - \psi_1(x, u, 0) - \frac{\partial \psi_1(x,u,0)}{\partial \hat{f}_d}\hat{f}_d$. In order to investigate the convergence of the adaptation law of the proposed scheme intuitively, we assume that for a sufficiently large η_1, ξ has faster dynamics than x_δ. Then, we evaluate the time derivative of $V = \frac{1}{2}(\xi - \xi^{eq})^2$ and obtain

$$\begin{aligned}\dot{V} &= (\xi - \xi^{eq})\dot{\xi} \\ &= (\xi - \xi^{eq})(\psi_1(x, u, 0) - \eta_1\xi + o(\hat{f}_d)) \qquad (20)\end{aligned}$$

Since $\psi_1(x, u, 0) - \eta_1\xi^{eq} + o(\hat{f}_d) = 0$ at equilibrium point $\xi = \xi^{eq}$, by choosing a sufficiently large value of η_1 it can be derived that

$$\psi_1(x, u, 0) - \eta_1\xi + o(\hat{f}_d) \begin{cases} > 0 & \text{if } \xi < \xi^{eq} \\ = 0 & \text{if } \xi = \xi^{eq} \\ < 0 & \text{if } \xi > \xi^{eq} \end{cases} \qquad (21)$$

Then, $\dot{V} < 0$ is guaranteed regardless of ξ and we conclude that $\lim_{t\to\infty} \xi(t) = \xi^{eq}$ and $\lim_{t\to\infty} \phi_1 = f_d$ in (18). The above explanation can also be made for the case when η_1 is fixed to zero similarly. However, it is noted that there are two remaining problems in the developed adaptation scheme above. First, although ϕ_i, $i \in \{1, 2\}$ are computed for the two adaptation laws, there is no rule to combine the adaptation laws. Second, although the adaptation rate η_i are required to be sufficiently large, it is a plant-dependent problem requiring much trial and error procedure in real practice. Thus, in order to overcome these problems we introduce a fuzzy logic to combine the adaptation laws and to determine the adaptation rates simultaneously.

As shown in Fig. 3 the fuzzy logic computes the increment of adaptation rate η_i^* by using both e_i and $\dot{e}_i (= \frac{de_i}{dt})$ as inputs and $\tilde{\eta}_i$ as a output of the rule-table in Table 1 for $i \in \{1, 2\}$. The associated membership functions $M_e^i(e_i)$, $M_{de}^i(\dot{e}_i)$, and $M_\eta^i(\tilde{\eta}_i)$ are constructed as shown in Fig. 4 where e_i^{max}, de_i^{max}, and $\tilde{\eta}_i^{max}$ are positive constants. The rule-table is constructed and is expandable by the following concept. In the case of $e_i\dot{e}_i < 0 (> 0)$, $\tilde{\eta}_i$ is selected by a positive (negative) fuzzy value, which means that η_i satisfying $e_i\dot{e}_i < 0 (> 0)$ is a winner (loser) and gains a corresponding positive (negative) increment η_i^*. Moreover, $|\tilde{\eta}_i|$ becomes decreased as $|e_i\dot{e}_i|$ decreases regardless of the sign of $e_i\dot{e}_i$, which means that as η_i becomes a winner (loser) more frequently, the amount of the increment η_i^* becomes smaller (larger). We note that the number of winners is not fixed to one but varying as time goes on. Thus, in this manner, the combined output membership function μ_i is obtained by reasoning process. In defuzzification, the center of gravity method is adopted such that $\eta_i^* = \frac{\int \mu_i(\tilde{\eta}_i)\tilde{\eta}_i d\tilde{\eta}_i}{\int \mu_i(\tilde{\eta}_i) d\tilde{\eta}_i}$ where η_i^* is the defuzzified value of the increment and $\mu_i(\tilde{\eta}_i)$ is the value of the combined output membership function corresponding to $\tilde{\eta}_i$. Then, the adaptation rate η_i is updated by the increment of η_i^*, i.e., $\dot{\eta}_i = \eta_i^*$ and the estimated disturbance is determined by $\hat{f}_d = \sum_{i=1}^{2} \phi_i$.

Table 2. Coefficients of membership functions.

i	e_{max}	de_{max}	$\tilde{\eta}_{max}$
1	0.02	0.002	50
2	0.001	0.0002	50

Fig. 4. Membership functions for $i \in \{1, 2\}$.

IV. Simulation results

The simulation of the magnetic suspension system is executed using the proposed control scheme (9) with the fuzzy estimator, while comparing with the feedback linearization method (14). Both control schemes are investigated for the cases of the constant and the sinusoidally varying external force disturbance. For the construction of the fuzzy estimator, its associated coefficients are chosen as Table 2. Fig. 5-6 shows that the output responses for the constant disturbance case. We note that the adaptation rates with initial values of zeros converge to a certain constant value in the steady state. Moreover, the fuzzy estimator shows good estimation of the unknown disturbance while the controlled output achieves the control objective (i.e., zero steady-state error), whereas the feedback linearizing controller does not. Here, we note that the steady-state error by the linearizing controller is computed as $\frac{k_3 f_d}{m k_1}$ from the result of [11]. In Fig. 7-8, the similar enhanced performance is observed for the sinusoidally varying disturbance, while achieving more reduced ultimate output error than the feedback linearization.

Fig. 5. Output response of the proposed control scheme for the constant disturbance: $f_d(t) = 5$.

Fig. 6. Output response of the proposed control scheme for the constant disturbance: $f_d(t) = 15$.

Fig. 7. Output response of the proposed control scheme for the sinusoidal disturbance: $f_d(t) = 15\sin(2\pi/30t)$.

Fig. 8. Output response of the proposed control scheme for the sinusoidal disturbance: $f_d(t) = 15\sin(2\pi/15t)$.

VI. Conclusions

In this paper, we develop a fuzzy estimator for the gain scheduling control of the magnetic suspension experiencing the external force disturbance. The fuzzy estimator computes the unknown disturbance injected to the plant and the gain scheduled controller is driven according to the estimated disturbance. The fuzzy logic combines the adaptation laws and determines the their rates adequately at the same time. Furthermore, since the fuzzy estimator is constructed based on the general gain scheduling idea, it is applicable to the other existing gain scheduled controllers.

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