Design of an Adaptive Fuzzy Controller and Its Applications to Controlling Uncertain Chaotic Systems

Chang-Woo Park, Chang-Hoon Lee, Jung-Hwan Kim, Seungho Kim, and Mignon Park

Abstract: In this paper, in order to control uncertain chaotic system, an adaptive fuzzy control(AFC) scheme is developed for the multi-input/multi-output plants represented by the Takagi-Sugeno(T-S) fuzzy models. The proposed AFC scheme provides robust tracking of a desired signal for the T-S fuzzy systems with uncertain parameters. The developed control law and adaptive law guarantee the boundedness of all signals in the closed-loop system. In addition, the chaotic state tracks the state of the stable reference model(SRM) asymptotically with time for any bounded reference input signal. The suggested AFC design technique is applied for the control of an uncertain Lorenz system based on T-S fuzzy model such as stabilization, synchronization and chaotic model following control(CMFC).

Keywords: Chaos, fuzzy control, Takagi-Sugeno fuzzy model, nonlinear system

I. Introduction

Many nonlinear systems have been shown to exhibit chaotic dynamics as well as period oscillations. A chaotic system is a nonlinear deterministic system which is very sensitive to small perturbation in its initial condition and its long time behavior is unpredictable. Although the model description of some chaotic systems is simple, the dynamic behavior is very complex. Since the research of Ott, Grebogi and Yorke(OGY)[1], many researchers have managed to use modern elegant theories to control chaotic systems, most of them based on exact chaotic model. Linear state feedback[2] is very simple and easily implemented for the nonlinear chaotic systems. The Lyapunov-type method[3] is a more general synthesis approach for nonlinear controller design. The feedback linearization[4] technique is an effective nonlinear geometric theory for nonlinear chaos control. However, if the chaos system is partly known, for example the differential equation of it is known but some or all of the parameters are unknown, these exact model based control methods may be infeasible. Recently it has been shown that it is possible to solve the problem of controlling chaos by using an appropriately chosen adaptive strategy[5][6]. On the other hand, an intelligent modelling and control methodology for chaotic system such as fuzzy logic or neural network have received an increasing interest[7][8]. K.Tanaka et al. derived the fuzzy models for various chaotic system and designed the fuzzy model based controllers for controlling chaotic fuzzy system such as stabilization, synchronization and chaotic model following control(CMFC) problems in [8]. However there are some drawback to these kinds of techniques, lack of consideration of parameter uncertainties in the fuzzy model. It is difficult to identify chaotic system exactly via fuzzy logic because the dynamics of chaos are very fast and some small perturbation in parameters leads to different chaotic behavior. Hence, in this paper, we will tackle the problem of controlling chaotic system with uncertainties and propose a complete solution to it.

Fuzzy logic controllers are generally considered applicable to plants that are mathematically poorly understood and where the experienced human operators are available. However, the fuzzy control has not been regarded as a rigorous science due to the lack of the guarantee of the global stability and acceptable performance. To overcome this drawback, since Takagi-Sugeno(T-S) fuzzy model which can express a highly nonlinear functional relation in spite of a small number of fuzzy implication rules was proposed in [9], there have been significant research on the stability analysis and systematic design of fuzzy controllers[10][11]. In their research, the nonlinear plant is represented by a T-S fuzzy model and the control design is carried out based on the fuzzy model via the so-called parallel distributed compensation(PDC) scheme[10]. The main idea of the PDC fuzzy controller design is to derive each rule to compensate each rule of a T-S fuzzy system. In literature [11], the stability analysis and design of the fuzzy control system using PDC fuzzy controller were cast to linear matrix inequality(LMI) problems and much systematic design of the fuzzy controller can be possible.

In order to deal with the uncertainties of nonlinear systems, in the fuzzy control system literature, a considerable amount of adaptive schemes have been suggested [12]-[18]. An adaptive fuzzy system is a fuzzy logic system equipped with an adaptive law. The major advantage of adaptive fuzzy controller over the conventional adaptive fuzzy controller is that the adaptive fuzzy controller is capable of incorporating linguistic fuzzy information from human operators. However most of them have considered only SISO plants and a complete analysis of the adaptive control problem for T-S fuzzy model has been given only in a few cases[15][16]. An adaptive scheme for the control of uncertain SISO plants whose structure was represented by T-S fuzzy model[16] and an indirect adaptive controller based on T-S model using feedback linearization scheme[15] have been presented. In this paper, to control

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Chang-Woo Park, Chang-Hoon Lee, Jung-Hwan Kim, Mignon Park: Electrical and Electronic Eng. Yonsei Univ.(cwpark@yeics.yonsei.ac.kr/chlee@yeics.yonsei.ac.kr/jhkim@yeics.yonsei.ac.kr/mignpark@yeics.yonsei.ac.kr)
Seungho Kim: Korea Atomic Energy Research Institute (robotkim@kaeri.re.kr)
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the uncertain chaotic system with not only single input but also multi input, we present alternative adaptive fuzzy controller(AFC) for MIMO plants subjected to parameter uncertainties. We utilize T-S fuzzy model for uncertain chaotic system and PDC as the basis of our control scheme which is different structure from those used in the above studies based on T-S fuzzy model. In contrast to the existing PDC of which structure has only state feedback gain, we present extended PDC with feedback and feedforward gain for tracking problems and the adaptation law for adjusting the parameters in feedback and feedforward gain of it is designed so that the plant output tracks the stable reference model(SRM) output. Since the extended PDC adopted in this paper has the structure in which the state feedback and reference input forwarding are performed, we can consider the reference input which makes the reference model follow the desired model. The developed control law and adaptive law guarantee the boundness of all the signals in the closed loop system. In addition, the plant state tracks the state of the reference model asymptotically with time for any bounded reference input signal.

The effectiveness of the suggested approach to controlling chaos via T-S fuzzy model and adaptive scheme is illustrated by its implementations to a Lorenz system with multi-input. The proposed controller is applied not only to robust stabilization and synchronization but also to the chaotic model following control(CMFC) of the chaotic system with parameter uncertainties.

II. Takagi-Sugeno model based fuzzy control

Consider a continuous-time nonlinear system described by the T-S fuzzy model[9]. The ith rule of a continuous-time T-S model is of the following form.

$$\begin{aligned} R^i &: \text{ If } x_1(t) \text{ is } M_1^i \text{ and } \cdots \text{ and } x_n(t) \text{ is } M_n^i \\ & \text{then } \quad \dot{\boldsymbol{x}}(t) = A_i \boldsymbol{x}(t) + B_i \boldsymbol{u}(t) . \end{aligned} \tag{1}$$

where $R^i\ (i = 1, 2, \cdots, l)$ denotes the ith implication, l is the number of fuzzy implications, M_j^i are fuzzy sets and

$$\boldsymbol{x}^T(t) = [x_1(t),\ x_2(t),\ \cdots,\ x_n(t)],$$

$$\boldsymbol{u}^T(t) = [u_1(t),\ u_2(t),\ \cdots,\ u_m(t)].$$

Given a pair of input $(\boldsymbol{x}(t),\ \boldsymbol{u}(t))$, the final output of the fuzzy system is inferred as follows:

$$\dot{\boldsymbol{x}}(t) = \frac{\sum_{i=1}^{l} w_i(t)\{A_i \boldsymbol{x}(t) + B_i \boldsymbol{u}(t)\}}{\sum_{i=1}^{l} w_i(t)}, \tag{2}$$

where $w_i(t) = \prod_{j=1}^{n} M_j^i(x_j(t))$, $M_j^i(x_j(t))$ is the grade of membership of $x_j(t)$ in M_j^i and it is assumed that

$$\sum_{i=1}^{l} w_i(t) > 0, \quad w_i(t) \geq 0, \quad \text{for } i = 1, 2, \cdots, l.$$

In order to design fuzzy controllers to stabilize the fuzzy system (2), we utilize the concept of PDC[10]. The PDC fuzzy controller shares the same fuzzy sets with fuzzy model (2) to construct its premise part. That is, the PDC fuzzy controller is of the following form.

$$\begin{aligned} R^i &: \text{ If } x_1(t) \text{ is } M_1^i \text{ and } \cdots \text{ and } x_n(t) \text{ is } M_n^i \\ & \text{then } \quad \boldsymbol{u}(t) = -K_i \boldsymbol{x}(t), \end{aligned} \tag{3}$$

where $\boldsymbol{x}^T(t) = [x_1(t), x_2(t), \cdots, x_n(t)]$,

$\boldsymbol{u}^T(t) = [u_1(t),\ u_2(t),\ \cdots,\ u_m(t)]$ and

$i = 1, \cdots, l$.

Given a state feedback $\boldsymbol{x}(t)$, the final output of the PDC fuzzy controller (3) is inferred as

$$\boldsymbol{u}(t) = -\frac{\sum_{i=1}^{l} w_i(t) K_i \boldsymbol{x}(t)}{\sum_{i=1}^{l} w_i(t)},$$

where

$$w_i(t) = \prod_{j=1}^{n} M_j^i(x_j(t)). \tag{4}$$

By substituting the controller (4) into the fuzzy model (2), we can construct the closed-loop fuzzy control system as the following equation.

$$\dot{\boldsymbol{x}}(t) = \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(t) w_j(t) \{A_i - B_i K_j\}\boldsymbol{x}(t)}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(t) w_j(t)}. \tag{5}$$

A sufficient condition for ensuring the stability of the closed-loop fuzzy system (5) is given in Theorem 1, which was derived in [10].

Theorem 1-Stability of T-S fuzzy system: The equilibrium of a fuzzy control system (5) is asymptotically stable in the large if there exists a common positive definite matrix P such that

$$G_{ij}^T P + P G_{ij} = -Q_{ij}, \tag{6}$$

for all $i, j = 1, 2, \cdots, l$

where $G_{ij} = A_i - B_i K_j$ and Q_{ij} is a symmetric positive definite matrix.

The design problem of model based fuzzy control is to select $K_j\ (j = 1, 2, \cdots, l)$ which satisfy the stability conditions(6). In [11], the common P problem was solved efficiently via convex optimization techniques for LMI's.

III. Adaptive control based on T-S fuzzy models

In this section, we present a model reference AFC scheme for T-S system with multi-input/multi-output. Consider the following nonlinear plant represented by the T-S fuzzy model

$$\dot{\boldsymbol{x}} = \frac{\sum_{i=1}^{l} w_i(\boldsymbol{x})(A_i\boldsymbol{x} + B_i\boldsymbol{u})}{\sum_{i=1}^{N} w_i(\boldsymbol{x})}, \tag{7}$$

where state $\boldsymbol{x} \in R^n$ is available for measurement, $A_i \in R^{n\times n}$, $B_i \in R^{n\times q}$ $(i = 1, \cdots, l)$ are unknown constant matrices and (A_i, B_i) are controllable. The control objective is to choose the input vector $\boldsymbol{u} \in R^q$ such that all the signals in the closed-loop plant are bounded and the plant state $\boldsymbol{x}$ follows the state $\boldsymbol{x}_m \in R^n$ of a stable reference model(SRM) specified by the system

$$\dot{\boldsymbol{x}}_m = \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})\{(A_m)_{ij}\boldsymbol{x}_m + (B_m)_{ij}\boldsymbol{r}\}}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})}, \tag{8}$$

where $(A_m)_{ij} \in R^{n\times n}$ $(i, j = 1, \cdots, l)$ satisfy the stability condition of fuzzy system given in Theorem 1, i.e., there exists a common symmetric positive definite matrix $P = P^T > 0$ such that $(A_m)_{ij}^T P + P(A_m)_{ij} = -Q_{ij}$ for $Q_{ij} = Q_{ij}^T > 0$, $(B_m)_{ij} \in R^{n\times q}$ and $\boldsymbol{r} \in R^q$ is a bounded reference input vector. The SRM and input $\boldsymbol{r}$ are chosen so that $\boldsymbol{x}_m(t)$ represents a desired trajectory that $\boldsymbol{x}$ has to follow.

1. Control law

We design the control law as the following using PDC conception.

$$R^i : \text{ If } x_1(t) \text{ is } M_1^i \text{ and } \cdots \text{ and } x_n(t) \text{ is } M_n^i \text{ then } \boldsymbol{u}(t) = -K_i^*\boldsymbol{x}(t) + L_j^*\boldsymbol{r}(t) \tag{9}$$

where $\boldsymbol{x}^T(t) = [x_1(t), x_2(t), \cdots, x_n(t)]$,

$\boldsymbol{r}^T(t) = [r_1(t), r_2(t), \cdots, x_q(t)]$ and $i = 1, \cdots, l$.

It can be inferred as

$$\boldsymbol{u} = \frac{\sum_{j=1}^{l} \hbar_j(\boldsymbol{x})(-K_j^*\boldsymbol{x} + L_j^*\boldsymbol{r})}{\sum_{j=1}^{l} \hbar_j(\boldsymbol{x})}, \tag{10}$$

where $\hbar_j(\boldsymbol{x}) = w_j(\boldsymbol{x})$. If the matrices A_i, B_i were known, we could apply the control law and obtain the closed-loop plant

$$\dot{\boldsymbol{x}} = \frac{\sum_{i=1}^{l} w_i(\boldsymbol{x})\left\{A_i\boldsymbol{x} + B_i\frac{\sum_{j=1}^{l}\hbar_j(\boldsymbol{x})(-K_j^*\boldsymbol{x} + L_j^*\boldsymbol{r})}{\sum_{j=1}^{l}\hbar_j(\boldsymbol{x})}\right\}}{\sum_{i=1}^{l} w_i(\boldsymbol{x})} = \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})\{(A_i - B_iK_j^*)\boldsymbol{x} + B_iL_j^*\boldsymbol{r}\}}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})} \tag{11}$$

Hence, if $K_j^* \in R^{q\times n}$, and $L_j^* \in R^{q\times q}$ are chosen to satisfy the algebraic equations

$$A_i - B_iK_j^* = (A_m)_{ij},\ B_iL_j^* = (B_m)_{ij}, \tag{12}$$

then the transfer matrix of the closed-loop plant is the same as that of the SRM and $\boldsymbol{x}(t) \to \boldsymbol{x}_m(t)$ exponentially fast for any bounded reference input signal $\boldsymbol{r}(t)$.

However, the design of the control parameters is not possible for the systems whose parameters are unknown. To overcome this drawback, in this research, following AFC is developed for the plant models of which parameters are unknown.

$$\boldsymbol{u} = \frac{\sum_{j=1}^{l} \hbar_j(\boldsymbol{x})(-K_j(t)\boldsymbol{x} + L_j(t)\boldsymbol{r})}{\sum_{j=1}^{N} \hbar_j(\boldsymbol{x})}, \tag{13}$$

where, $K_j(t)$, $L_j(t)$ are the estimates of K_j^*, L_j^*, respectively, to be generated by an appropriate adaptive law.

2. Adaptive law

By adding and subtracting the desired input term multiplied by B_i, namely,

$$\sum_{j=1}^{l}\hbar_j(\boldsymbol{x})\{-B_i(K_j^*\boldsymbol{x} - L_j^*\boldsymbol{r})\} / \sum_{j=1}^{l}\hbar_j(\boldsymbol{x}),$$

in the plant equation(17) and using (12), we obtain

$$\dot{\boldsymbol{x}} = \frac{\sum_{i=1}^{l} w_i(\boldsymbol{x})\left\{A_i\boldsymbol{x} + B_i\boldsymbol{u} + B_i\frac{\sum_{j=1}^{l}\hbar_j(\boldsymbol{x})(-K_j^*\boldsymbol{x} + L_j^*\boldsymbol{r} + K_j^*\boldsymbol{x} - L_j^*\boldsymbol{r})}{\sum_{j=1}^{l}\hbar_j(\boldsymbol{x})}\right\}}{\sum_{i=1}^{l} w_i(\boldsymbol{x})}$$

$$= \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})\{(A_i - B_iK_j^*)\boldsymbol{x} + B_iL_j^*\boldsymbol{r} + B_i(K_j^*\boldsymbol{x} - L_j^*\boldsymbol{r} + \boldsymbol{u})\}}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})}$$

$$\dot{\boldsymbol{x}} = \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})(A_m)_{ij}}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})}\boldsymbol{x} + \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})(B_m)_{ij}}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})}\boldsymbol{r} + \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})B_i(K_j^*\boldsymbol{x} - L_j^*\boldsymbol{r} + \boldsymbol{u})}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})}, \tag{14}$$

Furthermore, by similar adding and subtracting the estimated input term multiplied by $\sum_{i=1}^{l} w_iB_i / \sum_{i=1}^{l} w_i$, namely, adding

$$\frac{\sum_{i=1}^{l} w_iB_i}{\sum_{i=1}^{l} w_i}\left\{\frac{\sum_{j=1}^{l}\hbar_j(\boldsymbol{x})\{(K_j(t)\boldsymbol{x} - L_j(t)\boldsymbol{r})\}}{\sum_{j=1}^{l}\hbar_j(\boldsymbol{x})} + \boldsymbol{u}\right\}$$

in the SRM (8), we obtain

$$\dot{\boldsymbol{x}}_m = \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})(A_m)_{ij}}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})}\boldsymbol{x}_m + \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})(B_m)_{ij}}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})}\boldsymbol{r} + \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})B_i(K_j(t)\boldsymbol{x} - L_j(t)\boldsymbol{r} + \boldsymbol{u})}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})}. \tag{15}$$

By using (14) and (15), we can express the equation of the tracking error defined as $\boldsymbol{e} \triangleq \boldsymbol{x} - \boldsymbol{x}_m$, i.e.,

$$\dot{\boldsymbol{e}} = \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})(A_m)_{ij}}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})}\boldsymbol{e} + \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})B_i(-\tilde{K}_j\boldsymbol{x} + \tilde{L}_j\boldsymbol{r})}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\hbar_j(\boldsymbol{x})}, \tag{16}$$

where $\tilde{K}_j = K_j(t) - K_j^*$ and $\tilde{L}_j = L_j(t) - L_j^*$.

In the dynamic equation (16) of tracking error, B_i are unknown. Hence, we assume that L_j^* are either positive definite or negative definite and define $\Gamma_j^{-1} = L_j^* sgn(l_j)$, where $l_j = 1$ if L_j^* is positive definite and $l_j = -1$ if L_j^* is negative definite. Then $B_i = (B_m)_{ij} L_j^{*-1}$ and (16) becomes

$$\dot{\boldsymbol{e}} = \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})(A_m)_{ij}}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})}\boldsymbol{e} + \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})(B_m)_{ij} L_j^{*-1}(-\tilde{K}_j \boldsymbol{x} + \tilde{L}_j \boldsymbol{r})}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})}. \tag{17}$$

Now, by using the tracking error dynamics (17), we derive the adaptive law for updating the desired control parameters K_j^*, L_j^* so that the closed-loop plant model (14) follows the SRM (8). We assume that the adaptive law has the general structure

$$\dot{K}_j(t) = F_j(\boldsymbol{x}, \boldsymbol{x}_m, \boldsymbol{e}, \boldsymbol{r}),\ \dot{L}_j = G_j(\boldsymbol{x}, \boldsymbol{x}_m, \boldsymbol{e}, \boldsymbol{r}), \tag{18}$$

where F_j and G_j $(i = 1, \cdots, l)$ are functions of known signals that are to be chosen so that the equilibrium

$$K_{je} = K_j^*,\ L_{je} = L_j^*,\ \boldsymbol{e}_e = \boldsymbol{0}. \tag{19}$$

We propose the following Lyapunov function candidate

$$V(\boldsymbol{e}, \tilde{K}_j, \tilde{L}_j) = \boldsymbol{e}^T P \boldsymbol{e} + \sum_{j=1}^{l} tr(\tilde{K}_j^T \Gamma_j \tilde{K}_j + \tilde{L}_j^T \Gamma_j \tilde{L}_j), \tag{20}$$

where $P = P^T > 0$ is a common positive definite matrix of the Lyapunov equations

$$(A_m)_{ij}^T P + P(A_m)_{ij} = -Q_{ij}$$
$$\text{for } Q_{ij} = Q_{ij}^T > 0 \ (i, j = 1, \cdots, l),$$

whose existence is guaranteed by the stability assumption for A_m and $tr(\cdot)$ denotes the trace of a matrix $(\cdot)$. Then, the time derivative $\dot{V}$ of V along the trajectory of (17), (18) is given by

$$\begin{aligned}
\dot{V} &= \dot{\boldsymbol{e}}^T P \boldsymbol{e} + \boldsymbol{e}^T P \dot{\boldsymbol{e}} + \sum_{j=1}^{l} tr(2\tilde{K}_j^T \Gamma_j \dot{\tilde{K}}_j + 2\tilde{L}_j^T \Gamma_j \dot{\tilde{L}}_j)\\
&= \left\{\frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})(A_m)_{ij}}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})}\boldsymbol{e} + \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})(B_m)_{ij} L_j^{*-1}(-\tilde{K}_j \boldsymbol{x} + \tilde{L}_j \boldsymbol{r})}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})}\right\}^T P\boldsymbol{e}\\
&\quad + \boldsymbol{e}^T P\left\{\frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})(A_m)_{ij}}{\sum_{i=1}^{N}\sum_{j=1}^{N} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})}\boldsymbol{e} + \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})(B_m)_{ij} L_j^{*-1}(-\tilde{K}_j \boldsymbol{x} + \tilde{L}_j \boldsymbol{r})}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})}\right\}\\
&\quad + \sum_{j=1}^{l} tr(2\tilde{K}_j^T \Gamma_j \dot{\tilde{K}}_j + 2\tilde{L}_j^T \Gamma_j \dot{\tilde{L}}_j)\\
&= \boldsymbol{e}^T \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})\{(A_m)_{ij}^T P + P(A_m)_{ij}\}}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})}\boldsymbol{e}\\
&\quad + \boldsymbol{e}^T P \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})(B_m)_{ij} L_j^{*-1}(-\tilde{K}_j \boldsymbol{x} + \tilde{L}_j \boldsymbol{r})}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})}\\
&\quad + \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})(-\boldsymbol{x}^T \tilde{K}_j^T + \boldsymbol{r}^T \tilde{L}_j^T)\{(B_m)_{ij} L_j^{*-1}\}^T}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})} P\boldsymbol{e}\\
&\quad + \sum_{j=1}^{l} tr(2\tilde{K}_j^T \Gamma_j \dot{\tilde{K}}_j + 2\tilde{L}_j^T \Gamma_j \dot{\tilde{L}}_j)\\
&= -\boldsymbol{e}^T \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x}) Q_{ij}}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})}\boldsymbol{e}\\
&\quad + 2\boldsymbol{e}^T P\left\{-\frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})(B_m)_{ij} L_j^{*-1}\tilde{K}_j}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})}\boldsymbol{x} + \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})(B_m)_{ij} L_j^{*-1}\tilde{L}_j}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})}\boldsymbol{r}\right\}\\
&\quad + 2\sum_{j=1}^{l} tr(\tilde{K}_j^T \Gamma_j \dot{\tilde{K}}_j + \tilde{L}_j^T \Gamma_j \dot{\tilde{L}}_j)
\end{aligned} \tag{21}$$

By using the following properties of trace to manipulate (21),

i) $tr(AB) = tr(BA)$
ii) $tr(A + B) = tr(A) + tr(B)$ for any $A, B \in R^{n \times n}$
iii) $tr(y x^T) = x^T y$ for any $x, y \in R^{n \times 1}$,

we have

$$\begin{aligned}
&\boldsymbol{e}^T P \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})(B_m)_{ij} L_j^{*-1}\tilde{K}_j}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})}\boldsymbol{x}\\
&= \boldsymbol{x}^T \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})\tilde{K}_j^T \Gamma_j (B_m)_{ij}^T sgn(l_j)}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})} P\boldsymbol{e}\\
&= tr\left\{\frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})\tilde{K}_j^T \Gamma_j (B_m)_{ij}^T sgn(l_j)}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})} P\boldsymbol{e}\boldsymbol{x}^T\right\}.
\end{aligned} \tag{22}$$

and

$$\begin{aligned}
&\boldsymbol{e}^T P \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})(B_m)_{ij} L_j^{*-1}\tilde{L}_j}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})}\boldsymbol{r}\\
&= \boldsymbol{r}^T \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})\tilde{L}_j^T \Gamma_j (B_m)_{ij}^T sgn(l_j)}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})} P\boldsymbol{e}\\
&= tr\left\{\frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})\tilde{L}_j^T \Gamma_j (B_m)_{ij}^T sgn(l_j)}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x}) h_j(\boldsymbol{x})} P\boldsymbol{e}\boldsymbol{r}^T\right\}.
\end{aligned} \tag{23}$$

By substituting (22) and (23) into (21), we obtain

$$\begin{aligned}\dot V = & -e^T \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(x)\mu_j(x)\,Q_{ij}}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(x)\mu_j(x)}\, e \\ & + 2tr\Bigg\{ -\frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(x)\mu_j(x)\,\tilde K_j^{T}\Gamma_j (B_m)_{ij}^T\, sgn(l_j)}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(x)\mu_j(x)}\, P e x^T \\ & + \sum_{j=1}^{l} \tilde K_j^{T}\Gamma_j \dot{\tilde K}_j \Bigg\} \\ & + 2tr\Bigg\{ \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(x)\mu_j(x)\,\tilde L_j^{T}\Gamma_j (B_m)_{ij}^T\, sgn(l_j)}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(x)\mu_j(x)}\, P e r^T \\ & + \sum_{j=1}^{l} \tilde L_j^{T}\Gamma_j \dot{\tilde L}_j \Bigg\}. \end{aligned} \tag{24}$$

In the last two terms of (24), if we let

$$\sum_{j=1}^{l} \tilde K_j^{T}\Gamma_j \dot{\tilde K}_j = \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(x)\mu_j(x)\,\tilde K_j^{T}\Gamma_j (B_m)_{ij}^T\, sgn(l_j)}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(x)\mu_j(x)}\, P e x^T . \tag{25a}$$

$$\sum_{j=1}^{l} \tilde L_j^{T}\Gamma_j \dot{\tilde L}_j = -\frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(x)\mu_j(x)\,\tilde L_j^{T}\Gamma_j (B_m)_{ij}^T\, sgn(l_j)}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(x)\mu_j(x)}\, P e r^T . \tag{25b}$$

we can make $\dot V$ to be negative, i.e.,

$$\dot V = -e^T \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(x)\mu_j(x)\,Q_{ij}}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(x)\mu_j(x)}\, e \leq 0 . \tag{26}$$

Hence, the obvious choice for adaptive law to make $\dot V$ negative is

$$\begin{aligned}\dot{\tilde K}_j = \dot K_j(t) &= \frac{\sum_{i=1}^{l} w_i(x)\mu_j(x)\,(B_m)_{ij}^T\, sgn(l_j)}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(x)\mu_j(x)}\, P e x^T \\ &= \left\{\frac{\sum_{i=1}^{l} w_i (B_m)_{ij}^T}{\sum_{i=1}^{l} w_i}\right\}\left\{\frac{\mu_j}{\sum_{j=1}^{l}\mu_j}\right\} sgn(l_j)\, P e x^T ,\end{aligned} \tag{27a}$$

$$\begin{aligned}\dot{\tilde L}_j = \dot L_j(t) &= -\frac{\sum_{i=1}^{l} w_i(x)\mu_j(x)\,(B_m)_{ij}^T\, sgn(l_j)}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(x)\mu_j(x)}\, P e r^T \\ &= -\left\{\frac{\sum_{i=1}^{l} w_i (B_m)_{ij}^T}{\sum_{i=1}^{l} w_i}\right\}\left\{\frac{\mu_j}{\sum_{j=1}^{l}\mu_j}\right\} sgn(l_j)\, P e r^T ,\end{aligned} \tag{27b}$$

Note that all the quantities in the right-hand sides of (27a) and (27b) are known or available for measurement. Therefore, the adaptive law (27) for model reference AFC of T-S fuzzy system can be implementable.

Using arguments previously discussed, we establish the following theorem which shows the properties of the AFC derived in this section. The control law (13) together with the adaptive law (27) guarantee boundedness of all the signals in the closed-loop system. In addition, the plant state x tracks the state of the SRM, x_m asymptotically with time for any bounded reference input signal r.

Theorem 2: Stability of the AFC scheme for T-S fuzzy model: Consider the plant model (7) and the reference fuzzy model (8) with the control law (13) and adaptive law (27). Assume that the reference input r and the state x_m of the SRM are uniformly bounded. Then the control law (13) and the adaptive law (27) guarantee that

i) $K(t)$, $L(t)$, $e(t)$ are bounded

ii) $e(t) \to 0$ as $t \to \infty$

Proof: The proof of this theorem will be given in Appendix A.

IV. Control simulations of uncertain chaotic system

In this section, the validity and effectiveness of the proposed AFC are examined through the simulation of uncertain chaotic system control. A Lorenz system with three input terms is chosen to demonstrate the ability of the proposed scheme.

The uncontrolled model for a Lorenz system is given by

$$\begin{aligned}\dot x_1(t) &= -ax_1(t) + ax_2(t) + u(t)\\ \dot x_2(t) &= cx_1(t) - x_2(t) - x_1(t)x_3(t)\\ \dot x_3(t) &= x_1(t)x_2(t) - bx_3(t).\end{aligned} \tag{28}$$

The Lorenz model is used for the fluid conviction description, especially for some feature of the atmospheric dynamics[19]. x_1, x_2, and x_3 represent measures of fluid velocity and horizontal and vertical temperature variations, respectively. $a, c,$ and b are positive parameters representing the Prandtl number, Rayleigh number and geometric factor, respectively. In these simulations, we will consider the system with $a = 10$, $b = 8/3$, $c = 28$ which is assumed to be not known exactly.

1. Fuzzy modelling of chaotic systems

In order to apply the suggested AFC, we need a T-S fuzzy model representation of the chaotic systems. K.Tanaka has derived the fuzzy models for various chaotic systems in [8]. Among those models, we consider a Lorenz system with three input terms. An exact fuzzy modelling[21] has been employed to construct the fuzzy models for the Lorenz system. It utilize the concept of sector nonlinearity. The following fuzzy model exactly represents the nonlinear equation of the Lorenz system under the assumption that $x_1(t) \in [-d \;\; d]$

Rule 1: *If* $x_1(t)$ *is* M_1 *then* $\dot x(t) = A_1 x(t) + B u(t)$

Rule 2: *If* $x_1(t)$ *is* M_2 *then* $\dot x(t) = A_2 x(t) + B u(t)$ (29)

where $u(t) = [u_1(t)\; u_2(t)\; u_3(t)]^T$

and $x(t) = [x_1(t)\; x_2(t)\; x_3(t)]^T$,

$$A_1 = \begin{bmatrix} -a & a & 0\\ c & -1 & -d\\ 0 & d & -b\end{bmatrix},\quad A_2 = \begin{bmatrix} -a & a & 0\\ c & -1 & d\\ 0 & -d & -b\end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$M_1(x_1(t)) = \frac{1}{2}\left(1 + \frac{x_1(t)}{d}\right), \quad M_2(x_1(t)) = \frac{1}{2}\left(1 - \frac{x_1(t)}{d}\right),$$

where it is assumed that $d = 30$. Figure 1 shows the state trajectory of the fuzzy modelled chaotic system(29) with the initial condition $[x_1\ x_2\ x_3] = [5\ 5\ 25]$.

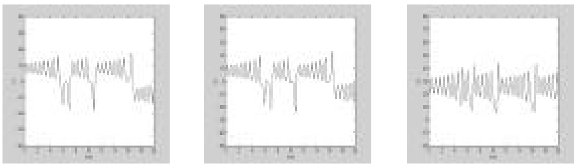


Fig. 1. State trajectory of fuzzy modelled Lorenz system.

2. Stabilization

In this subsection, the control objective is to suppress chaos, that is, to drive a system from a chaotic regime to regular attractor such as fixed points. Such an objective is justified because, in some cases, the onset of chaos has been associated with abnormal behavior[20].

To apply the proposed AFC scheme, the SRM for the state x of the chaotic system to follow should be specified.

In this simulation, the closed loop eigenvalues for each subsystem are chosen to be the same as

$$\lambda = [-1,\ -1-i,\ -1+i],$$

which, in turn, make the SRM for each fuzzy subspace to be the same and linear one as the following,

$$\dot{\mathbf{x}}_m = A_m \mathbf{x}_m + B_m \mathbf{r} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -3 \end{bmatrix} \mathbf{x}_m + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{r}, \qquad (30)$$

that is,

$$(A_m)_{ij} = A_m \ \text{ and } \ (B_m)_{ij} = B_m \quad (i, j = 1, 2).$$

We design the PDC type fuzzy controller to choose the input vector $\mathbf{u}$ such that all the signals in the closed-loop system are bounded and the state $\mathbf{x}$ of the chaotic system follows the state $\mathbf{x}_m$ of the SRM(30). As mentioned earlier, the idea is to design a compensator for each rule of the fuzzy model and the resulting overall controller, which is nonlinear, is a fuzzy blending of each individual linear controller. The PDC fuzzy controller shares the same fuzzy sets with fuzzy model to construct its premise part.

$$\begin{aligned} &Rule\ 1:\ If\ x_1(t)\ is\ M_1\ \ then\ \ \mathbf{u}(t) = -K_1(t)\,\mathbf{x}(t) + L_1\,\mathbf{r}(t) \\ &Rule\ 2:\ If\ x_1(t)\ is\ M_2\ \ then\ \ \mathbf{u}(t) = -K_2(t)\,\mathbf{x}(t) + L_2\,\mathbf{r}(t) \end{aligned} \qquad (31)$$

The feedback and feedforward control gains $K_i, L_i\ (i = 1, 2)$ of each sub-controller are updated by an adaptive law so that the closed-loop system follows the SRM. The initial values of $K_{i0}, L_{i0}\ (i = 1, 2)$ are designed from the nominal parameters of the plant model to be controlled. We design the initial $K_{i0}, L_{i0}\ (i = 1, 2)$ so that the closed loop system including designed fuzzy feedback controller has the same eigenvalues as SRM. The nominal parameters for the chaos fuzzy model of the Lorenz system are assumed to be $a = 8$, $b = 5/3$, $c = 30$, $d = 30$ in this simulation. Hence, the initial values of $K_i, L_i\ (i = 1, 2)$ can be given as the following equations.

$$\begin{aligned} &A_{10} - BK_{10} = A_{20} - BK_{20} = A_m,\ \ BL_{10} = BL_{20} = B_m \\ &K_{i0} = B(A_{i0} - A_m),\quad L_{i0} = B^{-1}B_m,\ \ i = 1, 2 \end{aligned} \qquad (32)$$

$$K_1 = \begin{bmatrix} -10 & 9 & 0 \\ 28 & -1 & -31 \\ 2 & 34 & 0.3333 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -10 & 9 & 0 \\ 28 & -1 & 29 \\ 2 & -26 & 0.3333 \end{bmatrix},$$

$$L_1 = L_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Now by using (27), we derive the following adaptive law for updating the elements of K_i and L_i so that the closed-loop plant follows the SRM.

$$\dot{K}_j(t) = \left\{ \frac{\gamma_j}{\sum_{j=1}^{2} \gamma_j} \right\} sgn(l_j) B_m^T P\, \mathbf{e}\, \mathbf{x}^T \quad i = 1, 2, \qquad (33a)$$

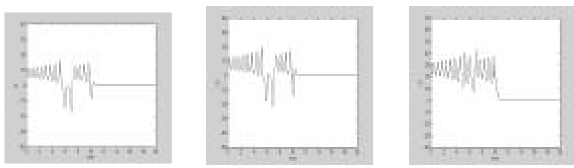
$$\dot{L}_j(t) = -\left\{ \frac{\gamma_j}{\sum_{j=1}^{2} \gamma_j} \right\} sgn(l_j) B_m^T P\, \mathbf{e}\, \mathbf{r}^T \quad i = 1, 2, \qquad (33b)$$

where $B_m^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $P = \begin{bmatrix} 1.95 & 1.4 & 0.25 \\ 1.4 & 2.475 & 0.475 \\ 0.25 & 0.475 & 0.325 \end{bmatrix}$.

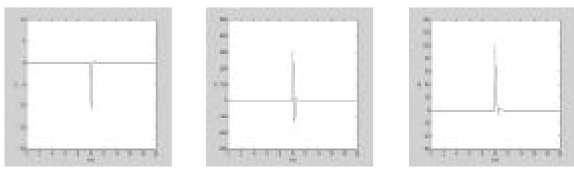
Figure 2 shows the simulation results of the response of state and control input with reference input $\mathbf{r}(t) = \mathbf{0} = [0\ 0\ 0]^T$ where the control input is added at $t > 10$. It can be seen that the designed AFC stabilizes the chaotic system, that is, $x_1(t) \to 0$, $x_2(t) \to 0$, and $x_3(t) \to 0$ although we can not know the parameters of the chaotic system exactly.

3. Synchronization

This subsection deals with synchronization problem of two chaotic systems with different initial conditions each other. First of all, the reference input $\mathbf{r}(t)$ should be determined to solve the synchronization problem. The reference input $\mathbf{r}(t)$ should be chosen so that $\mathbf{x}_m(t)$ represents a desired trajectory that $\mathbf{x}(t)$ has to follow. The exact linearization scheme which was proposed in [8] can be used to get the reference input.



(a) State trajectory $\mathbf{x}(t)$



(b) Control input $\mathbf{u}(t)$

Fig. 2. Control results of stabilization.

Consider a stable reference model(SRM) and a chaotic reference model(CRM) for $x_m(t)$ of the SRM to follow.

Stable reference model(SRM):

$$\begin{aligned}&\textit{Rule } i: \text{ If } x_{m1}(t) \text{ is } N^m_{i1} \text{ and } \cdots \text{ and } x_{mn}(t) \text{ is } N^m_{in}\\ &\quad \text{then } \dot{\boldsymbol{x}}_{\boldsymbol{m}}(t) = A_{mi}\boldsymbol{x}_{\boldsymbol{m}}(t) + B\boldsymbol{r}(t)\\ &\quad i = 1, 2, \cdots, l.\end{aligned} \tag{34a}$$

Chaotic reference model(CRM):

$$\begin{aligned}&\textit{Rule } i: \text{ If } x_{R1}(t) \text{ is } N^R_{i1} \text{ and } \cdots \text{ and } x_{Rn}(t) \text{ is } N^R_{in}\\ &\quad \text{then } \dot{\boldsymbol{x}}_{\boldsymbol{R}}(t) = D_i\boldsymbol{x}_{\boldsymbol{R}}(t)\\ &\quad i = 1, 2, \cdots, l.\end{aligned} \tag{34b}$$

The inferred output for SRM is represented as

$$\begin{aligned}\dot{\boldsymbol{x}}_{\boldsymbol{m}}(t) &= \frac{\sum_{i=1}^{l} w_i(t)\{A_{mi}\boldsymbol{x}_{\boldsymbol{m}}(t) + B\boldsymbol{r}(t)\}}{\sum_{i=1}^{l} w_i(t)}\\ &= \sum_{i=1}^{l} h_i(\boldsymbol{x}_{\boldsymbol{m}}(t))\{A_{mi}\boldsymbol{x}_{\boldsymbol{m}}(t) + B\boldsymbol{r}(t)\},\end{aligned} \tag{35a}$$

where $w_i(t) = \prod_{j=1}^{n} N^m_{ij}(x_{mj}(t))$ and

$$h_i(\boldsymbol{x}_{\boldsymbol{m}}(t)) = \frac{w_i(t)}{\sum_{i=1}^{l} w_i(t)}.$$

Similarly, the inferred output for CRM yield

$$\begin{aligned}\dot{\boldsymbol{x}}_{\boldsymbol{R}}(t) &= \frac{\sum_{i=1}^{l} \varpi_i(t) D_i \boldsymbol{x}_{\boldsymbol{R}}(t)}{\sum_{i=1}^{l} \varpi_i(t)}\\ &= \sum_{i=1}^{l} v_i(\boldsymbol{x}_{\boldsymbol{R}}(t)) D_i \boldsymbol{x}_{\boldsymbol{R}}(t),\end{aligned} \tag{35b}$$

where $\varpi_i(t) = \prod_{j=1}^{n} N^R_{ij}(x_{Rj}(t))$ and

$$v_i(\boldsymbol{x}_{\boldsymbol{R}}(t)) = \frac{\varpi_i(t)}{\sum_{i=1}^{l} \varpi_i(t)}.$$

Assume that $\boldsymbol{e}_{\boldsymbol{m}}(t) = \boldsymbol{x}_{\boldsymbol{m}}(t) - \boldsymbol{x}_{\boldsymbol{R}}(t)$. Then, from (35a) and (35b), we have

$$\begin{aligned}\dot{\boldsymbol{e}}_{\boldsymbol{m}}(t) &= \sum_{i=1}^{l} h_i(\boldsymbol{x}_{\boldsymbol{m}}(t)) A_{mi}\boldsymbol{x}_{\boldsymbol{m}}(t)\\ &\quad - \sum_{i=1}^{l} v_i(\boldsymbol{x}_{\boldsymbol{R}}(t)) D_i \boldsymbol{x}_{\boldsymbol{R}}(t) + B\boldsymbol{r}(t).\end{aligned} \tag{36}$$

Consider two subfuzzy controllers to design the reference input which makes $\boldsymbol{x}_{\boldsymbol{m}}(t)$ follows $\boldsymbol{x}_{\boldsymbol{R}}(t)$.

Subcontroller A

$$\begin{aligned}&\textit{Control rule } i: \text{ If } x_{m1}(t) \text{ is } N^m_{i1} \text{ and } \cdots \text{ and } x_{mn}(t) \text{ is } N^m_{in}\\ &\quad \text{then } \boldsymbol{r}_{\boldsymbol{A}}(t) = -F_i\boldsymbol{x}_{\boldsymbol{m}}(t)\\ &\quad i = 1, 2, \cdots, l.\end{aligned} \tag{37a}$$

Subcontroller B

$$\begin{aligned}&\textit{Control rule } i: \text{ If } x_{R1}(t) \text{ is } N^R_{i1} \text{ and } \cdots \text{ and } x_{Rn}(t) \text{ is } N^R_{in}\\ &\quad \text{then } \boldsymbol{r}_{\boldsymbol{B}}(t) = G_i\boldsymbol{x}_{\boldsymbol{R}}(t)\\ &\quad i = 1, 2, \cdots, l.\end{aligned} \tag{37b}$$

The parallel connection of the subcontroller A and the subcontroller B is represented as

$$\begin{aligned}\boldsymbol{r}(t) &= \boldsymbol{r}_{\boldsymbol{A}}(t) + \boldsymbol{r}_{\boldsymbol{B}}(t) =\\ &\quad -\sum_{i=1}^{l}\{h_i(\boldsymbol{x}_{\boldsymbol{m}}(t)) F_i \boldsymbol{x}_{\boldsymbol{m}}(t) - v_i(\boldsymbol{x}_{\boldsymbol{R}}(t)) G_i \boldsymbol{x}_{\boldsymbol{R}}(t)\}.\end{aligned} \tag{38}$$

By substituting (38) into (36) yields the overall error system as

$$\begin{aligned}\dot{\boldsymbol{e}}_{\boldsymbol{m}}(t) &= \sum_{i=1}^{l}\{h_i(\boldsymbol{x}_{\boldsymbol{m}}(t))[A_{mi} - BF_i]\boldsymbol{x}_{\boldsymbol{m}}(t)\\ &\quad - v_i(\boldsymbol{x}_{\boldsymbol{R}}(t))[D_i - BG_i]\boldsymbol{x}_{\boldsymbol{R}}(t)\}\end{aligned} \tag{39}$$

Theorem 3: Linearization of TS fuzzy system: The error system represented by (39) is exactly linearized via the fuzzy controller (38) as $\dot{\boldsymbol{e}}_{\boldsymbol{m}}(t) = H\boldsymbol{e}_{\boldsymbol{m}}(t)$, where $H = A_{m1} - BF_1 = A_{mi} - BF_i = D_j - BG_j$ ($i = 2, 3, \cdots, l$, $j = 1, 2, \cdots, l$) if there exists the feedback gains F_i and G_i such that

$$\begin{aligned}&\{(A_{m1} - BF_1) - (A_{mi} - BF_i)\}^T\\ &\cdot \{(A_{m1} - BF_1) - (A_{mi} - BF_i)\} = \mathbf{0} \quad i = 2, 3, \cdots, l,\end{aligned} \tag{40a}$$

$$\begin{aligned}&\{(A_{m1} - BF_1) - (D_j - BG_j)\}^T\\ &\cdot \{(A_{m1} - BF_1) - (D_j - BG_j)\} = \mathbf{0} \quad j = 1, 2, \cdots, l.\end{aligned} \tag{40b}$$

Proof: The proof of this theorem will be given in Appendix B.

In order to solve the synchronization problem, let the CRM which the state $\boldsymbol{x}_{\boldsymbol{m}}(t)$ of the SRM (30) has to follow be the following chaotic fuzzy model with initial condition $[x_{10}\ x_{20}\ x_{30}] = [5\ 5\ 25]$.

$$\begin{aligned}&\textit{Rule } 1: \text{ If } x_{R1}(t) \text{ is } N^R_1 \text{ then } \dot{\boldsymbol{x}}_{\boldsymbol{R}}(t) = D_1\boldsymbol{x}_{\boldsymbol{R}}(t)\\ &\textit{Rule } 2: \text{ If } x_{R1}(t) \text{ is } N^R_2 \text{ then } \dot{\boldsymbol{x}}_{\boldsymbol{R}}(t) = D_2\boldsymbol{x}_{\boldsymbol{R}}(t)\end{aligned} \tag{41}$$

where $\boldsymbol{x}(t) = [x_{R1}(t)\ x_{R2}(t)\ x_{R3}(t)]^T$,

$$D_1 = \begin{bmatrix} -a & a & 0\\ c & -1 & -d\\ 0 & d & -b \end{bmatrix}, \quad D_2 = \begin{bmatrix} -a & a & 0\\ c & -1 & d\\ 0 & -d & -b \end{bmatrix}$$

$$\begin{aligned}N^R_1(x_{R1}(t)) &= \frac{1}{2}\left(1 + \frac{x_{R1}(t)}{d}\right),\\ N^R_2(x_{R1}(t)) &= \frac{1}{2}\left(1 - \frac{x_{R1}(t)}{d}\right), \quad x_{R1}(t) \in [-d\ \ d].\end{aligned}$$

Then, we choose a stable matrix

$$H = -10 \cdot I = -10 \cdot \begin{bmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1 \end{bmatrix},$$

and design the reference input as follows.

$$\boldsymbol{r}(t) = \boldsymbol{r}_{\boldsymbol{A}}(t) + \boldsymbol{r}_{\boldsymbol{B}}(t).$$

Subcontroller A:

$$\begin{aligned}&\textit{Rule } 1: \text{ If } x_{m1} \text{ is } M_1 \text{ then } \boldsymbol{r}_{\boldsymbol{A}}(t) = -F_1 x_m(t)\\ &\textit{Rule } 2: \text{ If } x_{m1} \text{ is } M_2 \text{ then } \boldsymbol{r}_{\boldsymbol{A}}(t) = -F_2 x_m(t)\end{aligned} \tag{42a}$$

where $F_1 = F_2 = B^{-1}(A_m - H) = \begin{bmatrix} 10 & 1 & 0 \\ 0 & 10 & 1 \\ -2 & -4 & 7 \end{bmatrix}$.

Subcontroller B:

$$Rule\ 1:\ If\ x_{R1}\ is\ M_1\ then\ \boldsymbol{r_B}(t) = G_1 x_R(t)$$

$$Rule\ 2:\ If\ x_{R1}\ is\ M_2\ then\ \boldsymbol{r_B}(t) = G_2 x_R(t) \tag{42b}$$

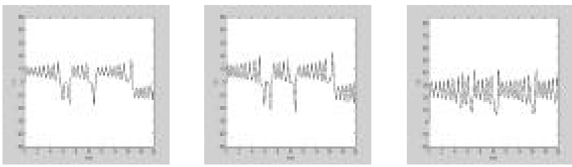
where $G_1 = B^{-1}(D_1 - H) = \begin{bmatrix} 0 & 10 & 0 \\ 28 & 9 & -30 \\ 0 & 30 & 7.3333 \end{bmatrix}$ and $G_2 = B^{-1}(D_2 - H) = \begin{bmatrix} 0 & 10 & 0 \\ 28 & 9 & 30 \\ 0 & -30 & 7.3333 \end{bmatrix}$.

Hence, the overall fuzzy controller can be given as

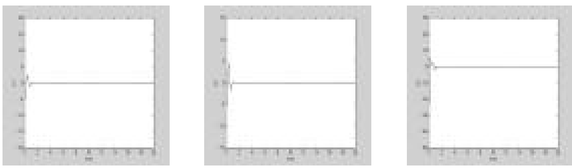
$$\boldsymbol{r}(t) = \boldsymbol{r_A}(t) + \boldsymbol{r_B}(t) = -\begin{bmatrix} 10 & 1 & 0 \\ 0 & 10 & 1 \\ -2 & -4 & 7 \end{bmatrix} \boldsymbol{x_m}(t) + \frac{\sum_{i=1}^{2} \mu_i(t) G_i \boldsymbol{x_R}(t)}{\sum_{i=1}^{2} \mu_i(t)} \tag{43}$$

where $\mu_i(t) = M_i(x_{R1}(t)),\quad i = 1, 2$.

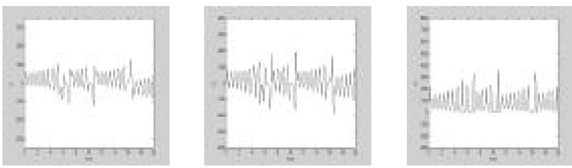
Figure 3 presents the trajectory of state $\boldsymbol{x_m}(t)$ of the SRM(30) with the reference input(43) and the CRM(41), where $\boldsymbol{x_m}(t)$ is shown by the solid line and CRMl state $\boldsymbol{x_R}(t)$ by the dotted line. It can be seen that the designed reference input makes the state of the stable reference model follow the CRM after short transient time. Therefore, it can be used as a reference input for the implementation of AFC. Figure 4 shows the control results for synchronization of the Lorenz system(29) with initial condition $[x_1\ x_2\ x_3] = [5.5\ 5.5\ 24]$ via proposed AFC, where the control input is added at $t > 10$ and actual $\boldsymbol{x}(t)$ is shown by the solid line,



(a) State trajectory $\boldsymbol{x_m}(t)$, $\boldsymbol{x_R}(t)$



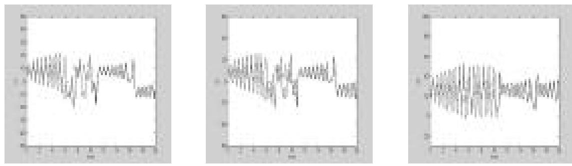
(b) Tracking error $\boldsymbol{e_m}(t)$



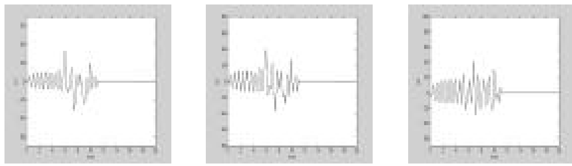
(c) Reference input $\boldsymbol{r}(t)$

Fig. 3. Control results of chaotic reference model following control.

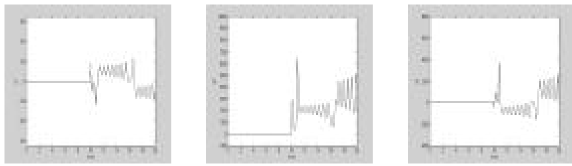
model state $\boldsymbol{x_m}(t)$ by the dotted line. We can see that the AFC synchronizes the uncertain Lorenz system with the CRM(41) which has the same model form except that the initial condition is slightly different from that of the Lorenz system, i.e., $[x_1\ x_2\ x_3] = [5\ 5\ 25]$.



(a) State trajectory $\boldsymbol{x}(t)$, $\boldsymbol{x_m}(t)$



(b) Tracking error $\boldsymbol{e}(t)$



(c) Control input $\boldsymbol{u}(t)$

Fig. 4. Control results of synchronization.

4. Chaotic model following control

In this subsection, CMFC problem, i.e., the control problem to drive a system to a chaotic regime is presented. This type of control could be important in a variety of situations where chaos is welcome such as, for example, some applications to human physiology, secure communication or heat transfer enhancement, etc.

Let us consider the following fuzzy model for another Lorenz system as follows.

$$Rule\ 1:\ If\ x_1(t)\ is\ M_1\ then\ \dot{\boldsymbol{x}}(t) = A_1 \boldsymbol{x}(t) + B\,\boldsymbol{u}(t)$$

$$Rule\ 2:\ If\ x_1(t)\ is\ M_2\ then\ \dot{\boldsymbol{x}}(t) = A_2 \boldsymbol{x}(t) + B\,\boldsymbol{u}(t) \tag{44}$$

where $\boldsymbol{u}(t) = [u_1(t)\ u_2(t)\ u_3(t)]^T$

and $\boldsymbol{x}(t) = [x_1(t)\ x_2(t)\ x_3(t)]^T$,

$$A_1 = \begin{bmatrix} -0.5a & 0.5a & 0 \\ 2c & -1 & -d \\ 0 & d & -0.5b \end{bmatrix},\quad A_2 = \begin{bmatrix} -0.5a & 0.5a & 0 \\ 2c & -1 & d \\ 0 & -d & -0.5b \end{bmatrix},$$

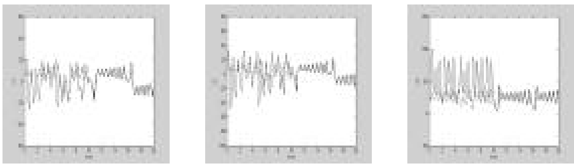
$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_1(x_1(t)) = \frac{1}{2}\left(1 + \frac{x_1(t)}{d}\right),\quad M_2(x_1(t)) = \frac{1}{2}\left(1 - \frac{x_1(t)}{d}\right)$$

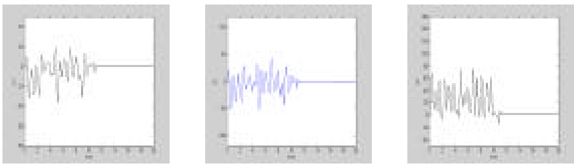
where the parameters a, b and c are assumed to be not known exactly.

The SRM is the same as (30) and the CRM which the states $\boldsymbol{x_m}(t)$ of the SRM has to follow is chosen as (41). Hence, we can design the reference input for the state

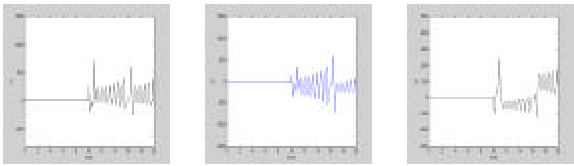
$x_m(t)$ to follow the CRM through the same procedure presented in the previous section.



(a) State trajectory $x(t)$, $x_m(t)$



(b) Tracking error $e(t)$



(c) Control input $u(t)$

Fig. 5. Control results of chaotic model following.

Figure 5 shows the simulation results of CMFC using the proposed AFC, where the control input is added at $t>10$ and actual $x(t)$ is shown by the solid line, model state $x_m(t)$ by the dotted line. As can be seen in these figures, the uncertain Lorenz system tracks the desired trajectory generated by the CRM, that is, almost $e_1(t)\rightarrow 0$, $e_2(t)\rightarrow 0$, and $e_3(t)\rightarrow 0$.

V. Conclusions

In this paper, we have developed an alternative T-S fuzzy model based adaptive control scheme for a chaotic systems with parameter uncertainty in their fuzzy model. The adaptation law adjusts the controller parameters on-line so that the plant output tracks the SRM output. The developed adaptive law guarantees the boundedness of all the signals in the closed-loop system and ensures that the chaotic state tracks the one of the reference model asymptotically with time for any bounded reference input signal. The proposed AFC scheme has been applied to stabilization, synchronization and CMFC of a Lorenz system to verify the validity and effectiveness of the control scheme. From the simulation results, we concluded that the suggested scheme can effectively solve the control problems of uncertain chaotic systems based on T-S fuzzy models.

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Appendix A

The proof of theorem 2: From (20) and (26), it directly follows that V is a Lyapunov function for the system (17), which implies that the equilibrium given by (19) is uniformly stable, which, in turn, implies that the trajectory $\tilde{K}(t)$, $\tilde{L}(t)$, $\boldsymbol{e}(t)$ is bounded for all $t>0$. Because $\boldsymbol{e}=\boldsymbol{x}-\boldsymbol{x}_m$ and $\boldsymbol{x}_m\in\mathcal{L}_\infty$, we have that $\boldsymbol{x}\in\mathcal{L}_\infty$. From (13) and $\boldsymbol{r}\in\mathcal{L}_\infty$, we also have that $\boldsymbol{u}\in\mathcal{L}_\infty$; therefore, all signals in the closed-loop are bounded.

Now, let us show that $\boldsymbol{e}\in\mathcal{L}_2$. From (20) and (26), we conclude that V has a limit, i.e.,

$$\lim_{t\to\infty} V(\boldsymbol{e}(t), \tilde{K}_j(t), \tilde{L}_j(t)) = V_\infty < \infty, \tag{45}$$

because V is bounded from below and is nonincreasing with time.

From (26) and (45), it follows that

$$\int_0^\infty \boldsymbol{e}^T\left(\frac{\sum w_i\mu_j Q_{ij}}{\sum w_i\mu_j}\right)\boldsymbol{e}\,d\tau = -\int_0^\infty \dot{V}\,d\tau = (V_0 - V_\infty) \tag{46}$$

where

$$V_0 = V(\boldsymbol{e}(0), \tilde{K}_j(0), \tilde{L}_j(0)).$$

On the other hand, from $0\le w_i\le 1$, $0\le\mu_j\le 1$, and $\lambda_{\min}(Q_{ij})\le \boldsymbol{e}^T Q_{ij}\boldsymbol{e}\le\lambda_{\max}(Q_{ij})$, which is obtained from the fact that $Q_{ij}=Q_{ij}^T>\boldsymbol{0}$ we have

$$\{\lambda_{\min}(Q_{ij})\}_{\min}\|\boldsymbol{e}\|^2 \le \boldsymbol{e}^T\left(\frac{\sum w_i\mu_j Q_{ij}}{\sum w_i\mu_j}\right)\boldsymbol{e}\le\{\lambda_{\max}(Q_{ij})\}_{\max}\|\boldsymbol{e}\|^2 \tag{47}$$

where

$$\{\lambda_{\min}(Q_{ij})\}_{\min} = \min\{\lambda_{\min}(Q_{11}),\cdots,\lambda_{\min}(Q_{ll})\}$$
$$\{\lambda_{\max}(Q_{ij})\}_{\max} = \max\{\lambda_{\max}(Q_{11}),\cdots,\lambda_{\max}(Q_{ll})\}.$$

After inserting (47) into (46), and straightforward manipulation, we have

$$(V_0-V_\infty)/\{\lambda_{\min}(Q_{ij})\}_{\min}\le\int_0^\infty\|\boldsymbol{e}\|^2 d\tau\le(V_0-V_\infty)/\{\lambda_{\max}(Q_{ij})\}_{\max}, \tag{48}$$

which implies that $\boldsymbol{e}\in\mathcal{L}_2$. Because $\boldsymbol{e}$, $\tilde{K}_j$, $\tilde{L}_j$, $\boldsymbol{r}\in\mathcal{L}_\infty$, it follows from (17) that $\dot{\boldsymbol{e}}\in\mathcal{L}_\infty$, which, together with $\boldsymbol{e}\in\mathcal{L}_2$, implies that $\boldsymbol{e}(t)\to 0$ as $t\to\infty$.

Appendix B

The proof of theorem 3: If the condition(40) is held, it is clear that

$$A_{m1}-BF_1=A_{mi}-BF_i=D_j-BG_j=H \quad (i=2,3,\cdots,l,\quad j=1,2,\cdots,l).$$

Then, the error system(39) can be

$$\dot{\boldsymbol{e}}_{\boldsymbol{m}}(t)=\sum_{i=1}^{l}\{h_i(\boldsymbol{x_m}(t))H\boldsymbol{x_m}(t)-v_i(\boldsymbol{x_R}(t))H\boldsymbol{x_R}(t)\}=$$
$$H\boldsymbol{x_m}(t)\sum_{i=1}^{l}h_i(\boldsymbol{x_m}(t))-H\boldsymbol{x_R}(t)\sum_{i=1}^{l}v_i(\boldsymbol{x_R}(t))$$
$$=H\boldsymbol{x_m}(t)-H\boldsymbol{x_R}(t)=H(\boldsymbol{x_m}(t)-\boldsymbol{x_R}(t))=H\boldsymbol{e_m}(t). \tag{49}$$

Chang-Woo Park
He was born in Seoul, Korea, in 1973. He received the B.S. degree in electronics from Korea University and M.S. degree in electronics from Yonsei University, Seoul, Korea, in 1997 and 1999, respectively. He is currently working toward the Ph.D. degree in electrical and electronic engineering at Yonsei University, Seoul, Korea. His current research interests include fuzzy control theory, nonlinear control theory and robot vision system.

Chang-Hoon Lee
He received the B.S. degree in electronics from the Yonsei University, Korea, and the Ph. D. degree in system science from Tokyo Institute of Technology, Japan. Currently he works at the Yonsei University, Seoul, Korea. His current research interests include stability of fuzzy control system, nonlinear control, robotics, and digital watermark.

Jung-Hwan Kim
He received the B.S. degree in electronics from Chungnam National Uniersity in 1984 and M.S. degree in electronics from Yonsei University, Seoul, Korea in 1996. He received Ph. D. degree in electrical and computer engineering from Yonsei University, Seoul, Korea, in 2000. Currently he works at the Hyupsung University, Korea. His current research interests include vision system and image processing, fuzzy theory and nonlinear control.

Mignon Park
He received the B.S. and M.S. degrees in electronics from Yonsei University, Seoul, Korea, in 1973 and 1977, respectively, and the Ph. D. degree from the University of Tokyo, Tokyo, Japan, in 1982. Since 1982, he has been a Professor in the Electrical and Electronic Department of Yonsei University. His research interests include fuzzy control and application engineering, robotics, and fuzzy biomedical system, etc.

Seungho Kim
He received the B.S., M.S. and Ph. D. degrees in mechanical Engineering from Yonsei University, Seoul, Korea, in 1979, 1982 and 1988 respectively. Since 1980, he has been a Lab. Manager of Advanced Robotics Lab. in Korea Atomic Energy Research Institute. His research interests include robot control, vibration control, flexible robot control, mobile robot control and tele-operated robotic system, etc.