Impedance Control of Flexible Base Mobile Manipulator Using Singular Perturbation Method and Sliding Mode Control Law

Mahdi Salehi and Gholamreza Vossoughi

Abstract: In this paper, the general problem of impedance control for a robotic manipulator with a moving flexible base is addressed. Impedance control imposes a relation between force and displacement at the contact point with the environment. The concept of impedance control of flexible base mobile manipulator is rather new and is being considered for first time using singular perturbation and new sliding mode control methods by authors. Initially slow and fast dynamics of robot are decoupled using singular perturbation method. Slow dynamics represents the dynamics of the manipulator with rigid base. Fast dynamics is the equivalent effect of the flexibility in the base. Then, using sliding mode control method, an impedance control law is derived for the slow dynamics. The asymptotic stability of the overall system is guaranteed using a combined control law comprising the impedance control law and a feedback control law for the fast dynamics. As first time, base flexibility was analyzed accurately in this paper for flexible base moving manipulator (FBMM). General dynamic decoupling, whole system stability guarantee and new composed robust control method were proposed. This proposed Sliding Mode Impedance Control Method (SMIC) was simulated for two FBMM models. First model is a simple FBMM composed of a 2 DOFs planar manipulator and a single DOF moving base with flexibility in between. Second FBMM model is a complete advanced 10 DOF FBMM composed of a 4 DOF manipulator and a 6 DOF moving base with flexibility. This controller provides desired position/force control accurately with satisfactory damped vibrations especially at the point of contact. This is the first time that SMIC was addressed for FBMM.

Keywords: Flexible base mobile manipulator, sliding mode impedance control, singular perturbation.

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1. INTRODUCTION

Mobile manipulators have long been introduced as a way of expanding the effective workspace of robot manipulators. Robots with moving base such as macro-micro manipulators, space manipulators and URV's (underwater robotic vehicles) can be used for extending the workspace in repair and maintenance, inspection, welding, cleaning, and machining operations. The assumption of base rigidity in these systems however, is often unreal and compliance of the base in most cases results in the loss of accuracy and limitations in achievable speeds. The source for base flexibility can be for example the suspension system and/or the internal structural flexibility of the base platform or joint/link flexibility associated with a supporting manipulator/crane in a macro-micro type manipulator arrangement.

Mobile manipulators with flexible base can in general be land based, space or underwater type vehicles. In mobile manipulators, greater momentum and higher frequency vibrations produced at contact between end effecter and environment provides even more impetus for dealing with such flexibilities. Achieving high performance interactive or non-interactive manoeuvres in such applications is possible only when the flexibility and base motion are both considered for in the control synthesis procedure. Simultaneous base and manipulator control in the presence of such flexibilities may be essential in many cases where the base is a floating platform. In which, the manipulator and the base motions are coupled (as in underwater ROVs or macro /micro space manipulators, Fig. 1 and the base cannot be locked in position. In land based configurations, simultane-ous control of the base and the manipulator can enhance

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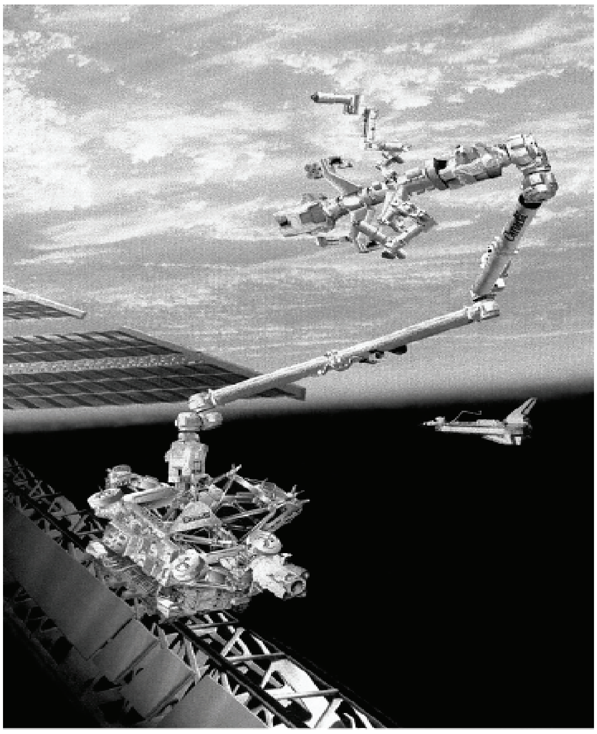


Fig. 1. Macro/Micro Mobile Servicing Manipulator on the International Space Station.

the application domain and improve the cycle time for both unconstrained and constrained ma-noeuvres.

Researchers have considered different control methods to improve the performance in flexible joint/link robotic systems. Modelling of flexible joint manipulators using singular perturbation method was first proposed by Khorosani and Kokotovic in 1985 [1]. Spong used perturbation method for dynamic modelling and control of manipulator with joint flexibility [2]. Singular perturbation is a unique systematic and mathematical tool for dealing with such flexibilities. This technique allows one to extract the slow and fast dynamics and formulate a separate control strategy for each subsystem. Tikhonov's Theorem [3] provides stability guarantees for the combined system. Among other methods, Lew introduced a simple robust control strategy for internal damping of mechanical vibrations for a manipulator with compliant (non-mobile) base [4].

As a space application, Finzi studied dynamic modeling and control strategies of mobile manipulator in space [5]. Hootsmans and Dubowsky addressed the joint motion control strategy of a macro-micro manipulator on a large mobile manipulator for improving the structural vibrations [6]. Torres and Dubwsky proposed a simple damping algorithm for errors associated with an elastically mounted space manipulator [7]. Mavroidis and Dubowsky proposed Inferred End-Point Control for long reach manipulator with base vibration [8]. These investigations are experimental and address error compensation of base vibrations without any stability and accuracy analysis.

The pioneering work in stiffness /impedance control is by Salisbury and Hogen [9,10]. Kazerooni presented a frequency domain interpretation and design method, and proposed an implementation more suitable for use with industrial robots [11]. The problem of Impedance Control and dynamic stability of mobile manipulators (without flexibility) has been addressed by Inoue [12]. The concept of virtual/ generalized impedance was proposed by Lao and Donath to avoid obstacles by redundant manipulators [13]. Modeling and Impedance Control of a two-manipulator system handling a flexible beam was addressed by Yan and Lin [14]. Multiple Impedance Control of cooperative manipulator in space was proposed by Mossavian, Papadaouplos and Poulakakis as an approach for handling large cargo in space [15]. To reduce contact forces in a mobile manipulators, simple damping-based posture control has been proposed by kang and his colleagues [16]. Flexibility hasn't been considered in any of the above investigations.

Position/force control of flexible joint robots using singular perturbation method has been proposed by Hu [17]. Roy and Whitcomb used adaptive coefficients for force control law and he achieved better response using this control law [18]. A research group at DLR Aerospace Research Centre have studied impedance control of light link manipulators with fixed base and joint flexibility. They proposed a new approach based on decupled dynamics of torque and position errors [19,23]. Subudhi addressed dynamic modeling and control of manipulators with combined joints and links flexibilities using singular perturbation method [20]. Impedance control of rigid mobile manipulator was studied by Tan and his colleagues [21] and experimental results were presented with a mobile PUMA 560. Hang proposed a fuzzy control law for impedance control and was able to achieve a better response when impedance parameters were selected based on fuzzy rule base [22]. Vossoughi and Karimzadeh addressed the general impedance control of a flexible link manipulator using singular perturbation method and they presented simulation results of impedance control for a 2 DOF manipulator with fixed.

Vibration of flexible base is according to the situations of Perturbation Theorem, because base flexibility of FBMM for all applications including suspension system, tyre or structural flexibility is less than 0.001 totally ($K \geq 1000$N/m). Therefore, proposed SMIC can be used for the all applications of FBMM using singular perturbation method.

In this paper, a general form for the dynamic model of a mobile manipulator with base flexibility is assumed. Singular perturbation method is then used to decouple slow and fast dynamics. A new formulation for the Impedance Control based on Sliding Mode control theory is then presented for achieving the desired impedance on the slow dynamics subsystem. An appropriate control law is also proposed for the fast dynamic; one which guarantees asymptotic vibration damping. The proposed control algorithm is simulated for two simple and advanced FBMM models. First, Flexible Base Moving Manipulator (FBMM) composed of a 2 DOF's planar manipulator mounted on a flexible one DOF base. Second model is a advanced 10 DOF's Flexible Base Mobile Manipulator (FBMM) composed of a 4 DOF's manipulator and a 6 DOF's moving base with flexibility. The performance of the proposed controller during impact between the manipulator end effecter and the environment is also simulated.

2. DYNAMIC MODEL

Consider following general dynamics of FBMM by (1);

$$\begin{aligned}\tau &= M(X)\ddot{X} + C(X,\dot{X})\dot{X} + K(X)X + G(X)\\ &\quad + N(u_r,\dot{u}_r),\\ X &= [y, x_1, x_2, ..., \theta_1, \theta_2, ...]^T,\\ \tau &= [0, F_{x1}, F_{x2}, ..., \tau_1, \tau_2, ...]^T,\end{aligned} \tag{1}$$

where y is base flexibility vector, $x_1, x_2, ...$ are base DOF's, $\theta_1, \theta_2, ...$ are angular movements of manipulator links, $Fx_1, Fx_2, ...$ are applied force to base and $\tau_1, \tau_2, ...$ are applied torques to links. The matrices *M*, *C*, *K*, *G*, *N* represent inertia matrix, damping and centrifugal and Coriolis_terms matrix, stiffness matrix, gravity matrix and matrix of road input to base or so on.

Motion equations are decoupled as below:

$$\begin{bmatrix}0\\ \tau_\Theta\end{bmatrix} = \begin{bmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{bmatrix}\begin{bmatrix}\ddot{y}\\ \ddot{\Theta}\end{bmatrix} + \begin{bmatrix}C_{11} & C_{12}\\ C_{21} & C_{22}\end{bmatrix}\begin{bmatrix}\dot{y}\\ \dot{\Theta}\end{bmatrix} + \begin{bmatrix}Ky\\ 0\end{bmatrix} + \begin{bmatrix}G_1\\ G_2\end{bmatrix} + \begin{bmatrix}-Ku_r - c\dot{u}_r\\ 0\end{bmatrix},$$

where

$$\Theta = [x_1, x_2, ..., \theta_1, \theta_2, ...]^T \tag{2}$$

and, u_r is road input or each kind of input to the base and K represents the stiffness matrix associated with the base flexibility. Now, new parameters μ (base compliance), Q and ζ (quasi-steady state) will be defined as follow:

$$\begin{aligned}&\mu = 1/K(X) \qquad as \quad \mu, K \quad are \quad Scaler\\ &\mu = K(X)^{-1} \qquad as \quad \mu, K \quad are \quad Matrix\\ &Q = y = u_r = u.h^{-1}\zeta\end{aligned} \tag{3}$$

where h is a scaling factor. Assuming the following petitioning for M^{-1}.

$$M^{-1} = \begin{bmatrix}H_{11} & H_{12}\\ H_{21} & H_{22}\end{bmatrix} \tag{4}$$

The decoupled relations are as follow:

$$\begin{aligned}\ddot{\Theta} &= H_{22}\tau_\Theta - H_{21}[C_{11}(\mu h^{-1}\dot{\zeta} + \dot{u}_r) + C_{12}\dot{\Theta}]\\ &\quad - H_{22}[C_{21}(\mu h^{-1}\dot{\zeta} + \dot{u}_r) + C_{22}\dot{\Theta}]\\ &\quad - H_{21}h^{-1}\zeta + H_{21}c\dot{u}_r - H_{21}G_1 - H_{22}G_2,\end{aligned} \tag{5}$$

$$\begin{aligned}\mu h^{-1}\ddot{\zeta} &= H_{12}\tau_\Theta - H_{11}[C_{11}(\mu h^{-1}\dot{\zeta} + \dot{u}_r) + C_{12}\dot{\Theta}]\\ &\quad - H_{12}[C_{21}(\mu h^{-1}\dot{\zeta} + \dot{u}_r) + C_{22}\dot{\Theta}]\\ &\quad - H_{11}h^{-1}\zeta + H_{11}c\dot{u}_r\\ &\quad - H_{11}G_1 - H_{12}G_2 - \ddot{u}_r.\end{aligned} \tag{6}$$

3. SLOW DYNAMICS

Slow dynamics is equivalent dynamics with rigid base. In this case, flexibility coefficient of base is considered infinite parameter and in result, μ is zero. $\mu = 0$ is substituted in relation (6). We defined a new equivalent static parameter $\bar{\zeta}$ in this case. Also, $\bar{\tau}_\Theta$ is equivalent torque vector for slow dynamics.

$$\begin{aligned}&\mu = 0 \Rightarrow\\ &\bar{\zeta} = hH_{11}^{-1}[H_{12}\bar{\tau}_\Theta - H_{11}(C_{11}\dot{u}_r + C_{12}\dot{\Theta}) - H_{12}\\ &(C_{21}\dot{u}_r + C_{22}\dot{\Theta}) + H_{11}c\dot{u}_r - H_{11}G_1 - H_{12}G_2 - \ddot{u}_r].\end{aligned} \tag{7}$$

Finally, slow dynamics of FBMM is specified with substituting (7) into (5): (the motion equations of moving manipulator without base flexibility)

$$\bar{\tau}_\Theta = \hat{M}.\ddot{\Theta} + \hat{C}.\dot{\Theta} + \hat{G}, \tag{8}$$

where $\hat{M}, \hat{C}, \hat{G}$ are corresponding matrices for slow dynamics.

4. FAST DYNAMICS

Fast dynamics is the equivalent dynamics associated with the flexibility in the system. Perturbation parameter, ε and new state variables are defined as follow.

$$\begin{cases} \mu \ is\ scalar \Rightarrow \varepsilon = \sqrt{\mu} \\ \mu \ is\ matrix \Rightarrow \mu = \varepsilon\varepsilon^T = L_\mu D_\mu L_\mu^T; \\ \qquad\qquad D_\mu diagonal \Rightarrow \varepsilon = L_\mu \sqrt{D_\mu} \end{cases} \tag{9}$$

$$Z_1 = h^{-1}\zeta,\ Z_2 = \varepsilon h^{-1}\dot{\zeta}$$

If μ is matrix, we can use Cholesky decomposition for calculating ε. Dynamic equations of slow and fast subsystems can be rewritten as following forms using singular perturbation method:

$$\begin{cases} \dot{X}_1 = X_2 \\ \dot{X}_2 = H_{22}\tau_\Theta - H_{21}[C_{11}(\varepsilon.Z_2 + \dot{u}_r) + C_{12}X_2] \\ \quad - H_{22}.[C_{21}(\varepsilon.Z_2 + \dot{u}_r) + C_{22}X_2] \\ \quad - H_{21}Z_1 + H_{21}C\dot{u}_r - H_{21}G_1 - H_{22}G_2 \end{cases} \tag{10}$$

$$\begin{cases} \varepsilon\dot{Z}_1 = Z_2 \\ \varepsilon\dot{Z}_2 = H_{12}\tau_\Theta - H_{11}[C_{11}(\varepsilon.Z_2 + \dot{u}_r) + C_{12}X_2] \\ \quad - H_{12}.[C_{21}(\varepsilon.Z_2 + \dot{u}_r) + C_{22}X_2] \\ \quad - H_{11}Z_1 + H_{11}C\dot{u}_r - H_{11}G_1 - H_{12}G_2 - \ddot{u}_r \end{cases} \tag{11}$$

Let $s = t/\varepsilon$ be the fast timescale, $\eta_1 = Z_1 - h^{-1}\overline{\zeta}$ and $\eta_2 = Z_2$ be the standard fast state variables.

Setting $\varepsilon \to 0$ in (11) and we will have following relation for fast manifold.

$$\begin{cases} \dfrac{d\eta_1}{ds} = \eta_2 \\ \dfrac{d\eta_2}{ds} = H_{12}\tau_f - H_{11}\eta_1 \end{cases} \tag{12}$$

$$\tau_f = \tau_\Theta - \overline{\tau}_\Theta$$

This is a linear state space system for fast dynamics. Unforced system is stable because H_{11} is a positive definite matrix. τ_f can be considered as control input for fast dynamics.

5. IMPEDANCE CONTROL

General dynamics of FBMM were decomposed using singular perturbation method. Singular perturbation method is the most important method for decoupling general systems including small perturbation parameters. Slow and fast dynamics will be controlled and then combined control law is proposed. We propose new impedance controlling method for slow dynamics using sliding method control law. Also, feedback torque control law is considered for asymptotic stability guarantee of fast dynamics.

Impedance control is a dynamic relation between position and force. Impedance relation indicates desired impedance by matrices M_m, B_m, K_m, K_f:

$$\begin{aligned} &M_m\ddot{e} + B_m\dot{e} + K_m e = -K_f e_f, \\ &e = x(t) - x_d(t), \\ &e_f = F(t) - F_d(t). \end{aligned} \tag{13}$$

M_m, B_m, K_m, K_f are impedance positive definite matrices and x_d, F_d are desired position and force vectors. Now, we consider combined sliding surface as following form:

$$s_c = \dot{e} + F_1 e + F_2 Z_c. \tag{14}$$

So, following relation indicates compensating dynamics for combined sliding surface:

$$\dot{Z}_c = AZ_c + K_1 e + K_1\dot{e} + K_3 e_f, \tag{15}$$

where K_1, K_2, K_3 are compensating positive matrices. A is semi-negative definite matrix .

It must be considered $s_c = \dot{s}_c = 0$ for reaching to desired sliding mode:

$$\begin{aligned} &\dot{Z}_c = -F_2^{-1}(\ddot{e} + F_1\dot{e}), \\ &Z_c = -F_2^{-1}(\dot{e} + F_1 e). \end{aligned} \tag{16}$$

We will have following relation by substituting (18) into (17);

$$\begin{aligned} &\ddot{e} + (F_1 - F_2AF_2^{-1} + F_2K_2)\dot{e} \\ &\quad + (F_2K_1 - F_2AF_2^{-1}F_1)e = -F_2K_3e_f, \end{aligned} \tag{17}$$

where K_1, K_2, K_3 are specified by comparison between two relations (17) and (13) as desired impedance relations;

$$\begin{aligned} &K_1 = F_2^{-1}M_m^{-1}K_m + AF_2^{-1}F_1, \\ &K_2 = F_2^{-1}M_m^{-1}B_m - F_2^{-1}F_1 + AF_2^{-1}, \\ &K_3 = F_2^{-1}M_m^{-1}K_f. \end{aligned} \tag{18}$$

Sliding mode law was defined as following relation:

$$\dot{s}_c = -F(s_c) = -k.sat(s_c) - \beta\int_0^t s_c dt \tag{19}$$

where α, β, k are positive definite and diagonal matrices. Function sat is as below:

$$sat(s_C) = \begin{cases} sign(s_C) & |s_C/\phi| > 1 \\ s_C/\phi & |s_C/\phi| \le 1. \end{cases} \tag{20}$$

Saturation function prevents the chattering associated with the use of sign function. ϕ determinates the boundary layer thickness around the sliding mode surfaces.

6. IMPEDANCE CONTROL OF SLOW DYNAMICS

Now, we obtain the sliding mode impedance control law for the slow dynamics. Given the tracking error definition, we have:

$$\begin{aligned} & e = x - x_d \\ & \Rightarrow \dot{e} = J(\Theta).\dot{\Theta} - \dot{x}_d \\ & \Rightarrow \ddot{e} = \dot{J}\dot{\Theta} + J\ddot{\Theta} - \ddot{x}_d . \end{aligned} \tag{21}$$

Using sliding mode control law, desired acceleration vector is as follow:

$$\begin{aligned} & \dot{s}_c = -F(s_c) = -k.sat(s_c) - \beta \int_0^t s_c dt \\ & \quad = \ddot{e} + F_1\dot{e} + F_2\dot{Z}_c \\ & \Rightarrow \dot{J}\Theta + J\ddot{\Theta} - \ddot{x}_d + F_1(J(\Theta).\dot{\Theta} - \dot{x}_d) + F_2\dot{Z}_c \\ & \qquad = -F(s_c) \\ & \Rightarrow \ddot{\Theta} = -J^{-1}(L_s + \dot{J}\dot{\Theta}), \end{aligned} \tag{22}$$

where

$$\begin{aligned} L_s = & F_2AZ_c + FL_2K_1e + (F_1 + F_2K_2)\dot{e} \\ & + F_2K_3e_f - \ddot{x}_d + F(s_c). \end{aligned} \tag{23}$$

J is the Jacobian matrix of slow dynamics and $\dot{J} = \partial J / \partial t$. J^* is robust pseudo-inverse Jacobian matrix for redundant manipulator and is considered by following relation. This relation provides the robust configuration of FBMM.

$$J^{-1} = J^T(JJ^T + k_m E)^{-1}, \tag{24}$$

where J^T is matrix transpose of J and k_m and E are identity scale factor and matrix for redundancy management of FBMM.

Control torque/force vector is obtained by substituting (22) into (8);

$$\begin{aligned} \bar{\tau}_\Theta = \tau_{slow} = & -\hat{M}J^{-1}L_s \\ & + (\hat{C} - \hat{M}J^{-1}\dot{J})\dot{\bar{\Theta}} + \hat{G}. \end{aligned} \tag{25}$$

7. CONTROL LAW OF FAST DYNAMICS

We have two reduced order subsystems in (8) and (12). Feedback control law can be used to achieve stabilization and vibration damping of the fast dynamics defined with relation (12) [14]:

$$\begin{aligned} & \tau_f = \tau_{fast} = -K_p\eta, \\ & \eta = [\eta_1, \eta_2]^T . \end{aligned} \tag{26}$$

We can consider nonlinear observer for indicating the vector η by measurable state variables, y and $\dot{y}$ using strain gauges and accelerometer. Other simple linear control law is as following form:

$$\tau_{fast} = -K_p H_{12}^T \dot{y}, \tag{27}$$

where K_p is positive feedback control coefficient or matrix and H_{12}^T is positive definite matrix related to state variables of slow dynamics.

8. COMPOSITE CONTROL METHOD, SMIC

Combined control method is considered as following relation using singular perturbation theorem.

$$\tau = \tau_{slow} + \tau_{fast} \tag{28}$$

This relation provides desired impedance and vibration damping and stability guarantee of FBMM. According to Tikhonov's Theorem, real state variables converge to the slow/fast state variables with the order of ε as following relations:

$$\begin{aligned} & X_1 = \bar{X}_1 + O(\varepsilon), X_2 = \bar{X}_2 + O(\varepsilon), \\ & Z_1 = h^{-1}\bar{\zeta} + \eta_1 + O(\varepsilon), Z_2 = \eta_2 + O(\varepsilon). \end{aligned} \tag{29}$$

As a result of singular perturbation method and Tikhonov's Theorem, if slow and fast dynamics are stable, stability of combined dynamics will be proved. So, slow dynamics (relation 8) and proposed sliding mode impedance control law (relation 25) guarantee dynamic stability. Also, unforced fast dynamics (relation 12) is stable and defined feedback control law (relations 26 and 27) guarantees asymptotic stability. Whole system stability is depended to stability of two slow and fast dynamics. If one of them isn't stable, whole system isn't stable. [3] (Chapter 11, pp 434)

9. SMIC FOR SIMPLE AND ADVANCED FBMM MODELS

9.1. Model (1), Simple FBMM model

First, a FBMM model (model 1) is considered with 2 DOF's manipulator and single DOF base with flexibility in between. Model has been shown in Fig. 2.

FBMM model (1) specification:

m_v=5kg ; m_b=2.5kg ; m_1=1kg ; m_2=1kg ; L_1=0.5m ; L_2=0.5m ; I_1=0.125kg.m^2 ; I_2=0.125kg.m^2 ; k=2000

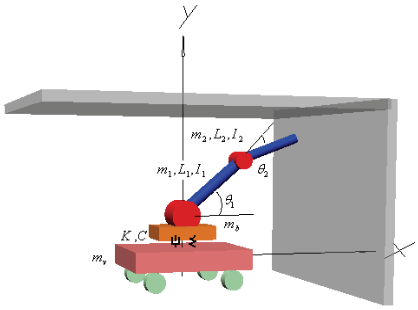


Fig. 2. FBMM model 1: $(x, y, \theta_1, \theta_2)$.

N/m ; c=100N.sec/m ; g=9.81m/sec^2 .

Initial conditions: $\theta_1(t=0) = 60°, \theta_2(t=0) = 30°$

Desired path is the motion of FBMM in direction x with 0.1 m/sec without any manipulator motion. Then end effector of manipulator will contact to the wall with stiffness K=1000 N/m in direction x. End effector towards wall surface with velocity 0.01m/sec. Desired position and force on the wall were selected as x_d =0.7 m and F_d = 2.25 N.

X and Y motions of the end effector, base motion and contact force have been shown in Figs. 3-6. Figs. 3, 4 show x_d =0.7 m. Vertical position of the end effector didn't change before contact moment. Fig. 6 shows that contact force is F_d = 2.25 N after contact moment according to defined and desired impedance by SMIC. This force was damped rapidly by combined slow/fast dynamics control method after contact and impact moment (damping time is about 4.1 sec to 4.4 sec). Also, Fig. 7 shows that vibration of manipulator's base was damped rapidly and obviously with minimum domain and frequency after the contact moment, as a result of composite slow and fast control method (Maximum domain is 1.5 mm and time is about 0.5 sec).

Meanwhile, Slow/fast dynamics control method guarantees stability totally with any impact value. Results show that SMIC provides desired path and contact force accurately as defined impedance parameters. Also, it expects that impact effect on the base motion. As an interesting result, composite slow and fast controller, SMIC causes to eliminate any effect on the motion in X direction even at the contact point. It is proved by Fig. 5 as we see simulation result after contact moment about 4 sec.

Figs. 8 ,9 indicate DOF and angular movements of manipulator links. Control force and torques of the base and links were indicated in Figs. 10-12. These figures show that shape and value of composite fast/slow control force and torques were changed suddenly by SMIC for reaching to both desired contact force and position. Interesting result is shown in Fig. 10 that control force for base was fixed on 2.5 N (is equal to contact force but in opposite direction) by SMIC.

Figs. 13-17 indicate same results as the motion of the end effector towards horizontal roof. Robotic specifications, defined impedance and control coefficients are similar to previous case. In this case, direction of vibration is same direction of contact force. Again, SMIC provides desired position/force and stability of the composite slow and fast dynamics. Of course, Figs. 16 and 17 shows that the domain of contact and vibration is less than same values for previous case, because gravity direction is in apposite direction of contact force in spite of previous case. But damping time is a little more, because same direction (Y) for contact force and vibration. In case 1, angular difference of the contact force and vibration direction is 90 degree and this causes lower damping time. These results show that SMIC provides task accuracy and stability guarantee of FBMM applications.

Case 1 : (SMIC for Model 1 , Vertical Environment)

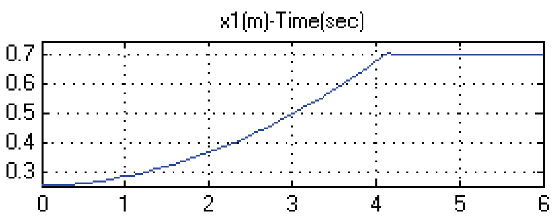


Fig. 3. Horizontal motion of the end effecter.

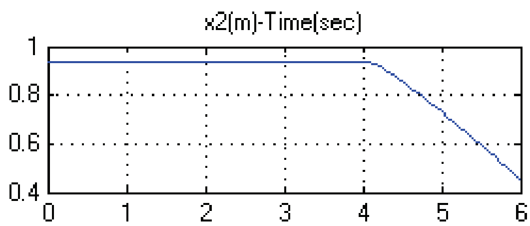


Fig. 4. Vertical motion of the end effecter.

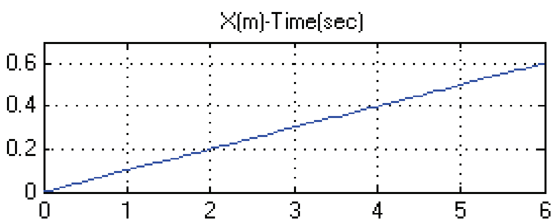


Fig. 5. Base motion in direction X.

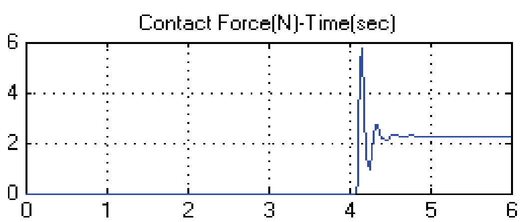


Fig. 6. Contact force of the end effecter and wall.

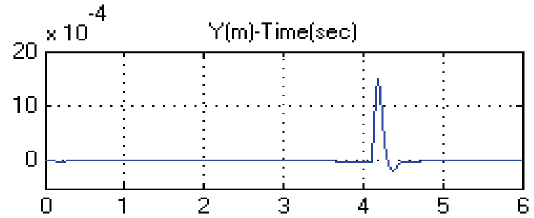


Fig. 7. Base vibration in direction Y.

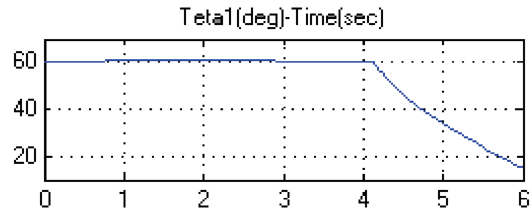


Fig. 8. Angular motion of the link 1.

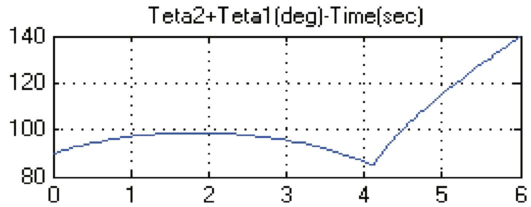


Fig. 9. Net angular motion of the link 2.

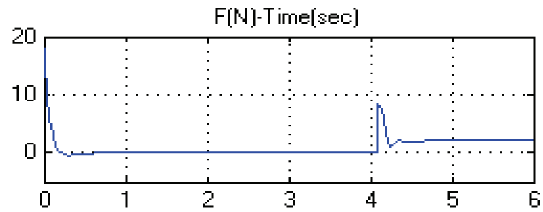


Fig. 10. Control force of the base in direction X.

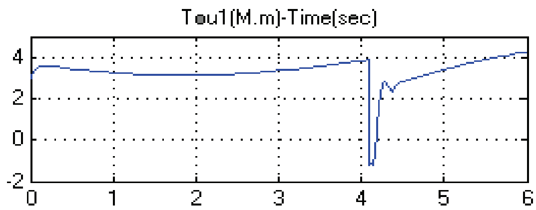


Fig. 11. Control torque of the link 1.

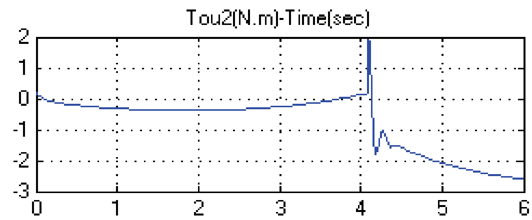


Fig. 12. Control torque of the link 2.

Case 2 : (SMIC for Model 1, Horizontal Environment)

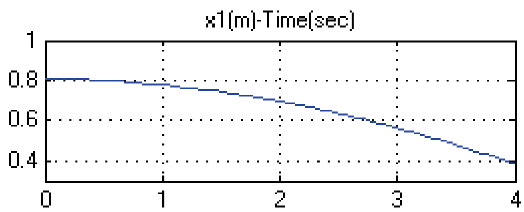


Fig. 13. Horizontal motion of the end effecter.

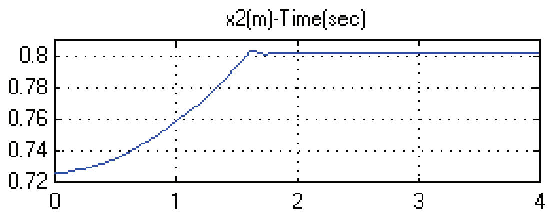


Fig. 14. Vertical motion of the end effecter.

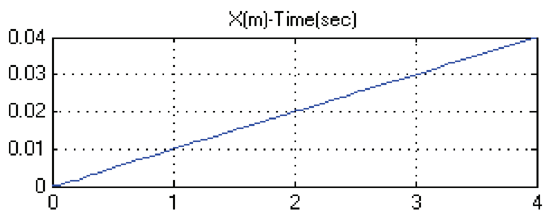


Fig. 15. Base movement in direction X.

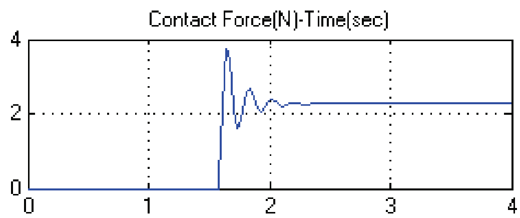


Fig. 16. Contact force of the end effecter and wall.

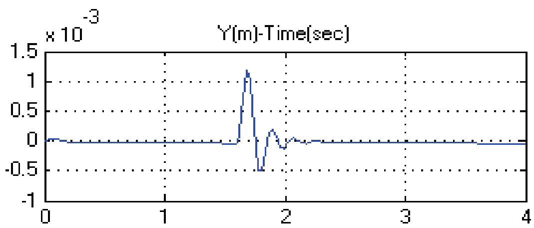


Fig. 17. Vibration of the base in direction Y.

9.2. Model (2), Advanced FBMM model

Complete and advanced model of FBMM is considered with application of welding, cleaning or so on. FBMM model is 10 DOF Flexible Base Mobile Manipulator (FBMM) composed of a 4 DOF manipulator and 6 DOF moving and flexible base. Non-holonomic constraint was considered for manipulator's base. Steering is possible by torque control of rear wheels. Model has shown in Fig. 18. Also, $[\theta_l \;\; \theta_r \;\; \psi_b]$ are dependent state variables. Where, state variables including fast and slow variables are;

Slow DOF's:

$$[x_b \;\; y_b \;\; \psi_b \;\; \theta_1 \;\; \theta_2 \;\; \theta_3 \;\; \theta_4]$$
$$or \quad [\theta_l \;\; \theta_r \;\; \theta_1 \;\; \theta_2 \;\; \theta_3 \;\; \theta_4]$$

Fast DOF's:

$$[z_b \quad \theta_b \quad \varphi_b] \tag{30}$$

A) Motion Equations of 10 DOF FBMM

Motion equations of this model were derived using two dynamic methods, Lagrange Method and Direct Path Method (DPM). Models were compared, simulated and confirmed. We provided two general packages; FBMMLAG and FBMMDPM using MAPLE software for driving motion equations and exporting to the SIMULINK, MATLAB for dynamic and control simulation.

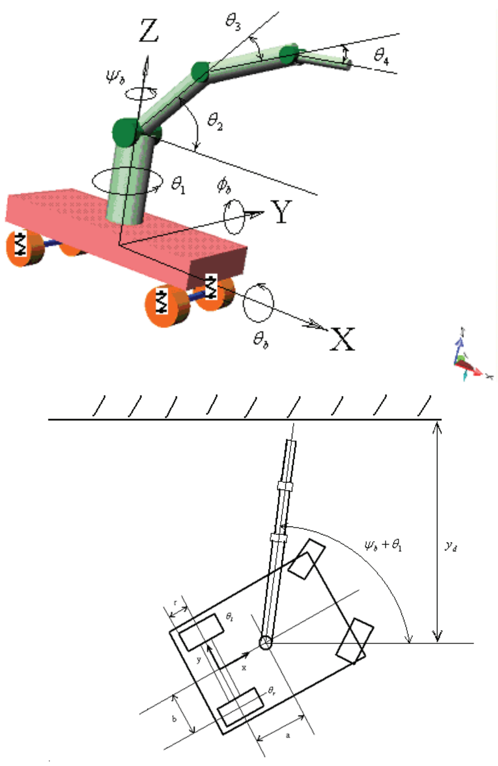


Fig. 18. FBMM model:
$[x_b \;\; y_b \;\; z_b \;\; \theta_b \;\; \phi_b \;\; \psi_b \;\; \theta_1 \;\; \theta_2 \;\; \theta_3 \;\; \theta_4]$
$or \quad [\theta_l \;\; \theta_r \;\; z_b \;\; \theta_b \;\; \phi_b \;\; \psi_b \;\; \theta_1 \;\; \theta_2 \;\; \theta_3 \;\; \theta_4]$.

B) FBMM specifications

$$\begin{aligned} & m_b = 20\text{kg}, \;\; a = 1\text{m}, \;\; b = 0.5\text{m} \\ & m_1 = 5\text{kg}, \;\; m_2 = 5\text{kg}, \;\; m_3 = 3\text{kg}, \; m_4 = 2\text{kg} \\ & L_1 = 2\text{m}, \; L_2 = 2\text{m}, \;\; L_3 = 1.5\text{m}, \;\; L_4 = 1\text{m} \\ & Ib_x = 1.67\text{kg}\cdot\text{m}^2, \;\; Ib_y = 6.67\text{kg}\cdot\text{m}^2, \\ & Ib_z = 8.3\text{kg}\cdot\text{m}^2 \\ & I_{L1} = 1.67\text{kg}\cdot\text{m}^2, \;\; I_{L2} = 1.67\text{kg}\cdot\text{m}^2, \\ & I_{L3} = 0.56\text{kg}\cdot\text{m}^2, \;\; I_{L4} = 0.17\text{kg}\cdot\text{m}^2 \end{aligned} \tag{31}$$

C) Initial conditions

$$\begin{aligned} & [x_b \;\; y_b \;\; z_b \;\; \theta_b \;\; \varphi_b \;\; \psi_b \;\; \theta_1 \;\; \theta_2 \;\; \theta_3 \;\; \theta_4]_{(t=0)} \\ & = \left[0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \frac{\pi}{3} \;\; \frac{\pi}{9} \;\; \frac{\pi}{9}\right]. \end{aligned} \tag{32}$$

D) Equivalent Stiffness of suspension system

$$\begin{aligned} & K(z_b) = K_{11} z_b + K_2, \\ & K_1 = 1000 \text{ N/m}^2, \;\; K_2 = 1000 \text{ N/m} \end{aligned} \tag{33}$$

E) Desired Position and Force

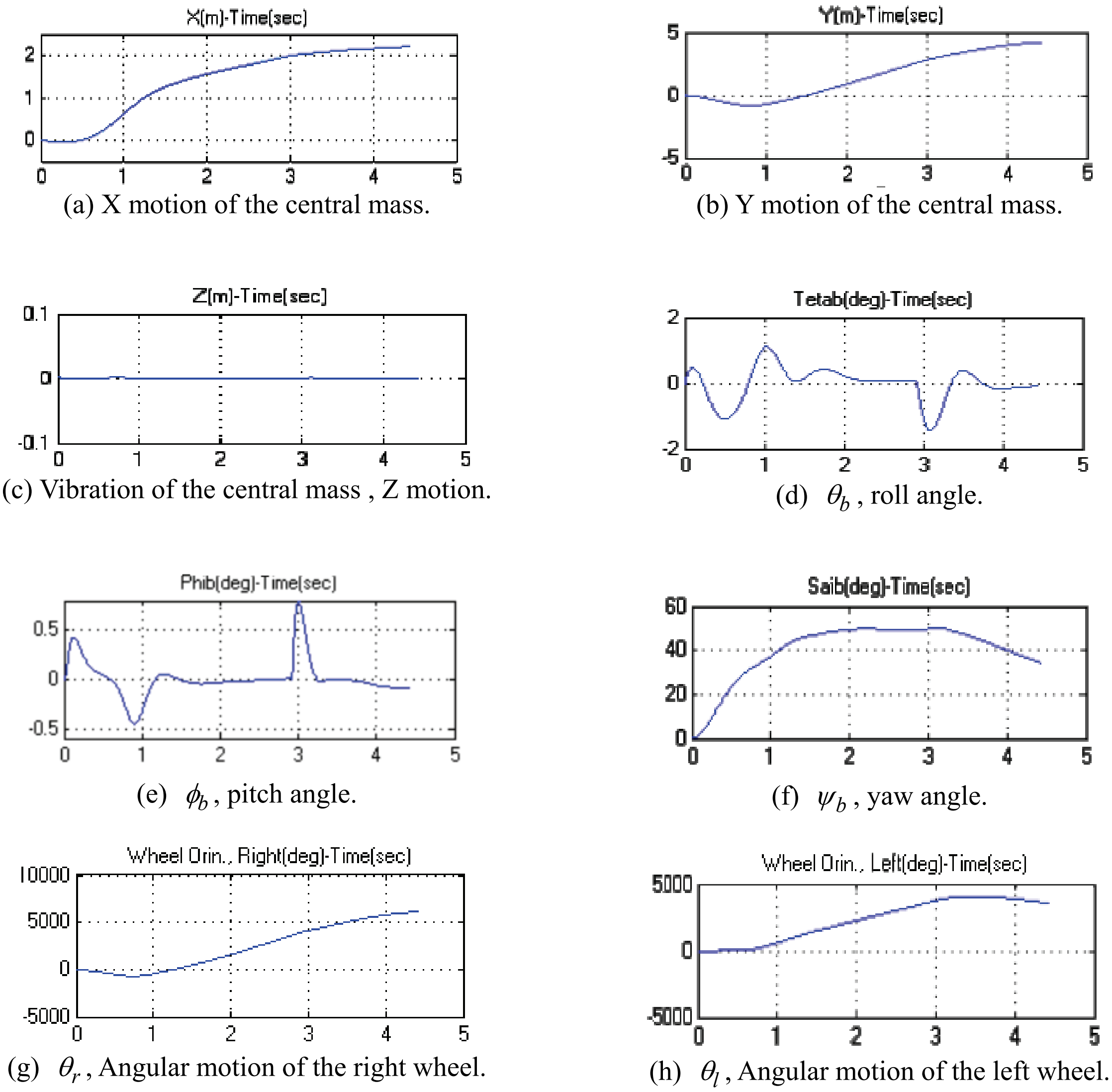
Desired path is the motion of FBMM towards the wall. Then the end effecter of manipulator will contact the wall with stiffness K=1000N/m normal of surface . Desired end effecter trajectory towards the wall is a linear path as welding or cleaning process. Also, principle angles (orientation) of the end effector (fourth link), $\psi_b + \theta_1$ and $\theta_2 + \theta_3 + \theta_4$, must be fixed 85 and 0 degree as uniform application. Desired y position and force on the wall were selected as y_d =4.5 m and F_{dy} = 2.5 N. In this simulation, it is assumed that tangential and friction forces on the wall's surface are negligible.

$$\begin{aligned} & [\theta_{1end} \;\; \theta_{2ebd} \;\; x_{end} \;\; y_{end} \;\; z_{end}] \\ & = [\psi_b + \theta_1 \;\; \theta_2 + \theta_3 + \theta_4 \;\; x_e \;\; y_e \;\; z_e] \\ & [\theta_{1end} \;\; \theta_{2ebd} \;\; x_{end} \;\; y_{end} \;\; z_{end}]_{des.} \\ & = [85° \;\; 0 \;\; 0.1xtime \;\; 4.5\text{m} \;\; -0.25xtime] \\ & F_d y = 2.5\text{N} \end{aligned} \tag{34}$$

Simulation results are shown for Model (2) as 10 DOF FBMM under SMIC. New proposed impedance control (SMIC) is used for reaching to specified and desired impedance (position, orientation and force).

Fig. 19 shows all dependent and independent DOF of manipulator's base. This figure shows X, Y motions, vibration of central mass, roll , pitch and yaw angles and angular motion of the rear wheels. Figs. 19(d) and (e) shows small changes for roll and pitch angles that they produced by base flexibility. These small perturbations are according to situations of the Perturbation Theorem. Base flexibility of FBMM for all applications including suspension system, tyre or structural flexibility is less than 0.001 totally ($K \geq 1000$ N/m). Therefore, proposed SMIC using singular perturbation method can be used for all applications of FBMM. Vibration of sprung or central mass is shown in Fig. 19(c). Stability of the fast dynamics is guaranteed for all moments. Impact between manipulator's end effector and environment doesn't have any effect on the base motion and vibration. This is other interesting result of SMIC, even motion speed and momentum of base is great. (About 1 m/s before contact and 0.2 m/s after it)

DOF's of manipulator's links and principle angels of End Effector are shown in Fig. 20. Figs. 20(d) and (e) shows net angular movement of End Effector (Link 4) . Principle angles of End Effector ($\psi_b + \theta_1$, $\theta_2 + \theta_3 + \theta_4$) on the wall are fixed on desired value 85 and 0 degree by SMIC. They were provided before contact moment at 2.88 sec. Also, coordinates of End Effector and Contact Force are shown in Fig. 21. This figure shows that all desired position and force are provided by SMIC. Figs. 21(a) and (c) shows that three principle motions of End Effector are according to desired and specified X and Z linear path. Fig. 21(b) shows that Y motion of End Effector was provided at 4.5 meter on the wall after contact. Fig. 22 shows 3 Dimensional path of End Effector before contact and on the wall. So, SMIC provided all desired position, orientation and force (impedance) for a real advanced FBMM model for industrial applications. Results show that SMIC provides desired path and contact force accurately as defined impedance parameters. Also, it expects that impact effect on the base motion.



(a) X motion of the central mass.

(b) Y motion of the central mass.

(c) Vibration of the central mass , Z motion.

(d) θ_b, roll angle.

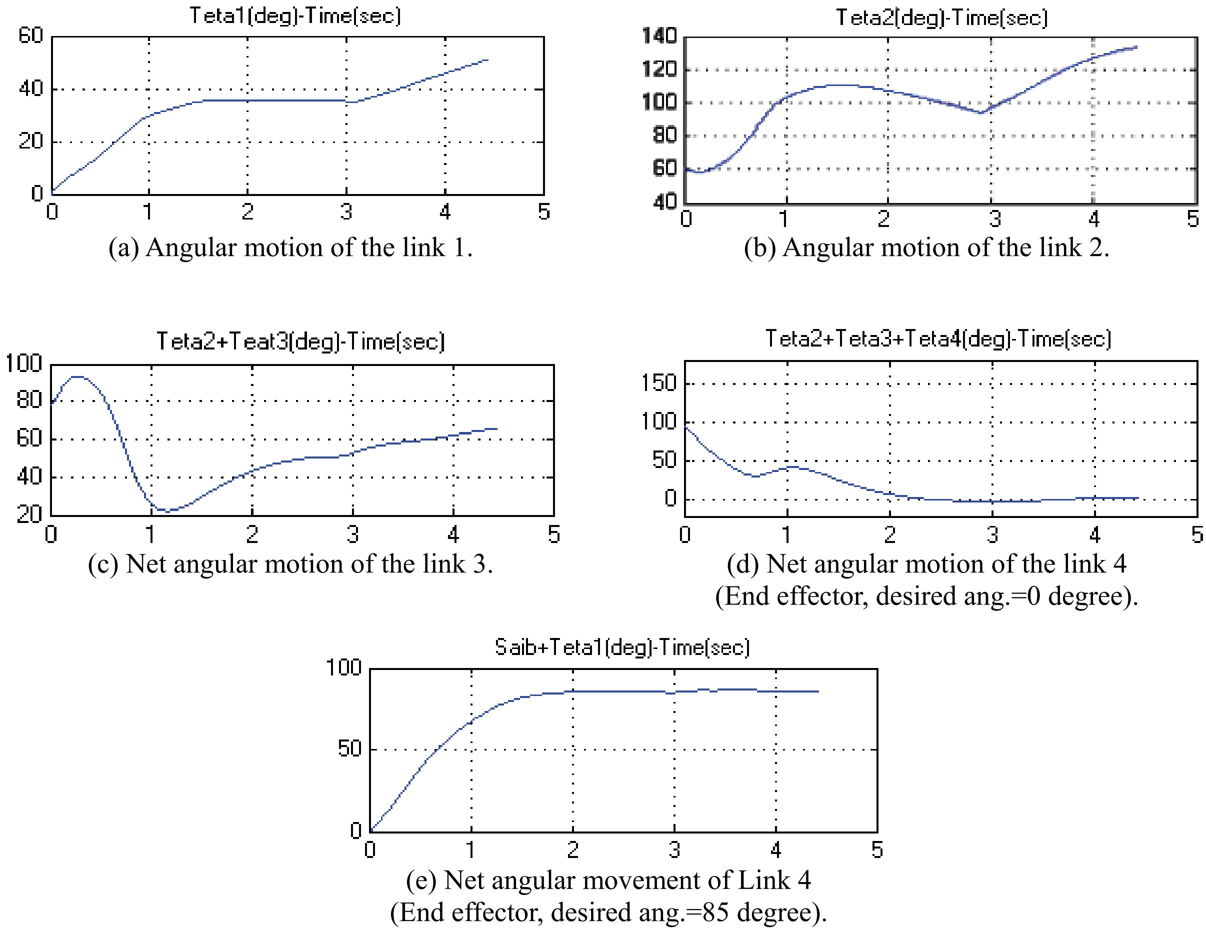
(e) ϕ_b, pitch angle.

(f) ψ_b, yaw angle.

(g) θ_r, Angular motion of the right wheel.

(h) θ_l, Angular motion of the left wheel.

Fig. 19. DOF of manipulator's base.



(a) Angular motion of the link 1.

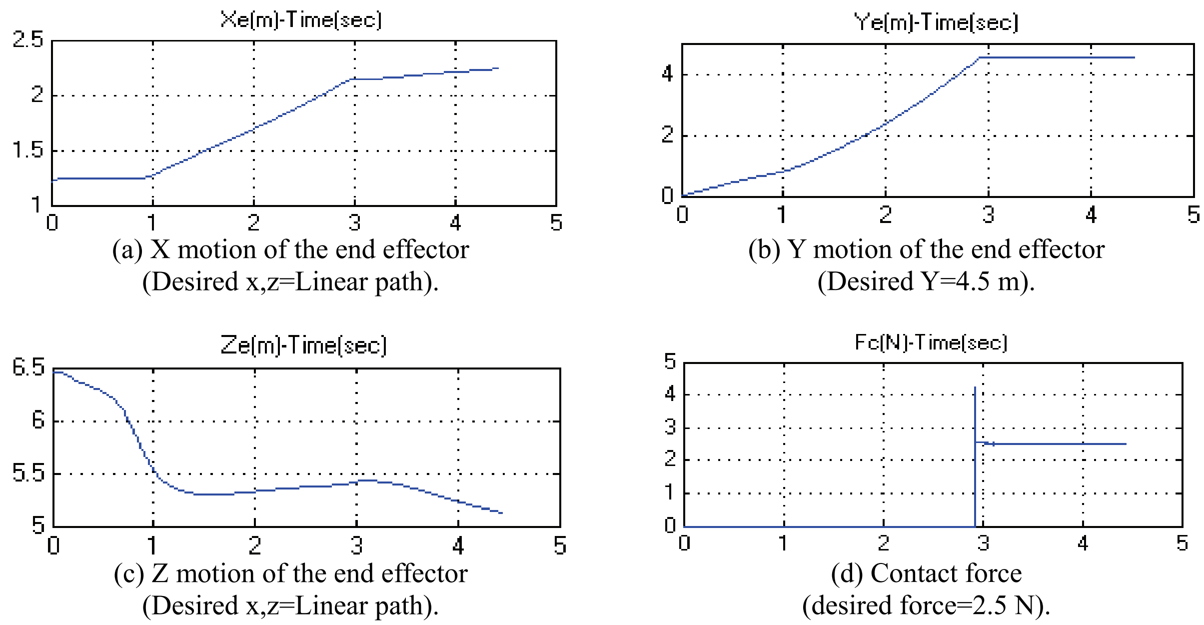
(b) Angular motion of the link 2.

(c) Net angular motion of the link 3.

(d) Net angular motion of the link 4 (End effector, desired ang.=0 degree).

(e) Net angular movement of Link 4 (End effector, desired ang.=85 degree).

Fig. 20. DOF of manipulator’s links and principle angels of the end effector.



(a) X motion of the end effector (Desired x,z=Linear path).

(b) Y motion of the end effector (Desired Y=4.5 m).

(c) Z motion of the end effector (Desired x,z=Linear path).

(d) Contact force (desired force=2.5 N).

Fig. 21. End effector motion and contact force.

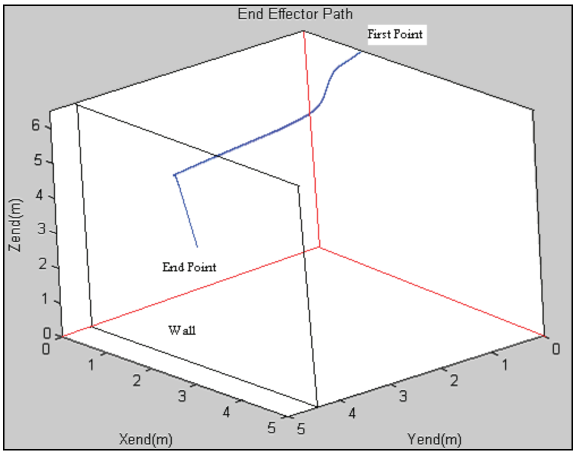


Fig. 22. 3D motion of the end effector

But as an interesting result, combined slow and fast controller, SMIC causes to omit any effect on the motion in x direction even at the contact point.

10. CONCLUSION

The demand of Flexible Base Mobile Manipulator (FBMM) has risen in recent years and the applications are many and varied. This research proposed new composite sliding mode impedance control (SMIC) for FBMM using singular perturbation method. FBMM applications include robotic manipulator mounted on the mobile vehicle on the land, in space under water. These applications are like welding, cleaning, machine tooling, construction, finishing and inspection. In the all applications, accurate position/ force control and stability guarantee shall be provided.

Of course, assumption of rigid base is unreal for all kind of FBMM. Impact value of mobile manipulators depends to both base motion and links masses which cause greater vibration on the flexible base at the contact point.

In this paper, general dynamic of FBMM was considered and dynamic equations were decoupled using singular perturbation method. Base flexibility of FBMM for all applications including suspension system, tyre or structural flexibility is less than 0.001 totally ($K \geq 1000$ N/m). These small perturbations are according to situations of the Perturbation Theorem. Therefore, proposed SMIC using singular perturbation method can be used for all applications of all kind of FBMM

As new concept, composite control method for slow and fast dynamics (SMIC) is proposed for impedance control using new application of sliding mode control law and singular perturbation method. Proposed SMIC guarantees asymptotic stability of FBMM. Of course, this new method of impedance control can be used as the general impedance control method for every kind of FBMM.

Two FBMM models were considered. A simple two DOF manipulator on the 1 DOF flexible base and an advanced 10 DOF FBMM model were considered including of a 4 DOF manipulator and a 6 DOF moving base with flexibility. SMIC provides desired path, orientation and contact force between the end effector and environment. It guaranties stability of slow and fast dynamics. Also it causes to damp high frequency and domain vibration at the contact point completely. Contact force was damped at the contact point rapidly and impact doesn't have any effect on the base motion as an interesting result of SMIC application. As first time, base flexibility of FBMM was analyzed accurately in this paper. Also, accurate and general dynamic decoupling, whole system stability guarantee and new composed robust control method (SMIC) were proposed for combined slow/fast dynamics.

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