Double Faults Isolation Based on the Reduced-Order Parity Vectors in Redundant Sensor Configuration

Cheol-Kwan Yang and Duk-Sun Shim*

Abstract: A fault detection and isolation (FDI) problem is considered for inertial sensors, such as gyroscopes and accelerometers and a new FDI method for double faults is proposed using reduced-order parity vector. The reduced-order parity vector (RPV) algorithm enables us to isolate double faults with 7 sensors. Averaged parity vector is used to reduce false alarm and wrong isolation, and to improve correct isolation. The RPV algorithm is analyzed by Monte-Carlo simulation and the performance is given through fault detection probability, correct isolation probability, and wrong isolation probability.

Keywords: Fault detection and isolation, inertial sensors, reduced-order parity vector, redundant sensors.

1. INTRODUCTION

Many systems such as control, navigation, and communication systems consist of various subsystems, and thus the hardware and software structure of those systems are complicated. Therefore, the importance of reliability of the whole systems has been increased. The reliability of the whole systems can be enhanced by using the fault detection and isolation (FDI) method. FDI methods have been studied using two approaches; hardware redundancy [1-5] and analytical redundancy [6-8]. Hardware redundancy means that multiple sensors of similar kind are used and measurements of same variables are compared, and thus any discrepancy is an indication of fault. For analytical redundancy, additional information is obtained from the mathematical model of a system. This type of redundancy is based on the idea that inherent redundancy exists in the dynamic relationship between inputs and outputs of the system model. Hardware redundancy is considered in this paper.

Inertial navigation systems (INS) use three accelerometers and gyroscopes to calculate navigation information such as position, velocity and attitude. To obtain reliability and to enhance navigation accuracy, INS use redundant sensors. A lot of studies on FDI for the redundant sensors have been performed so far such as look-up table [1], squared error method [1], generalized likelihood test [2], and optimal parity-vector test [3], sequential FDI [4], and singular value decomposition method [5] for hardware redundancy.

It is known that double faults cannot be isolated by using 6 sensors for vector variables. Most of the previous FDI methods focused on single fault and only a few FDI papers considered double fault [9-11]. Yoo [10] and Kim [11] proposed some methods for the double fault detection and isolation problem with 7 sensors using the parity space approach. Yoo [10] considered the RAIM (receiver autonomous integrity monitoring) problem in the GPS receiver and used 7 satellite measurements and parity space reconfigure-tion to detect double fault. However, 8 satellite measurements are necessary to detect double fault since for GPS, four variables should be estimated such as x, y, z positions and receiver clock bias. Thus, the performance of FDI may be poor with 7 satellite measurements for some combinations of double faults. And due to parity space reconfiguration and the fact that even though there is no fault, all FDI procedures should be done before no fault is declared, and the computation time is greatly increased. Kim [11] considered the same problem to detect and isolate double fault using 7 inertial sensors. However, the isolation performance is not good for some combinations of double fault as shown in Table 4 in [11].

In this paper a reduced-order parity vector (RPV) method with 7 inertial sensors is proposed to detect and isolate double fault. The proposed FDI method uses reduced-order parity vector. To isolate the fault, one or two measurements are omitted and then the

Manuscript received August 3, 2005; revised June 29, 2006; accepted November 17, 2006. Recommended by Editor Jae Weon Choi under the direction of past Editor Keum-Shik Hong. This work was supported by Grant No. (R01-2003-000-10430-0) from the Basic Research Program of the Korea Science & Engineering Foundation.

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corresponding parity vector is generated and tested. A similar idea is used in [10] and partially in [11]. However, the RPV method is the most simple and the performance is best. The RPV algorithm is analyzed by Monte-Carlo simulation and the performance is given through fault detection probability, correct isolation probability, and wrong isolation probability.

Section 2 defines the reduced-order parity vector and averaged parity vector, and Section 3 introduces the reduced-order parity vector algorithm. Section 4 shows the simulation results and the conclusions are given in Section 5.

2. REDUCED-ORDER PARITY VECTOR

Consider inertial navigation systems that use more than three gyroscopes and three accelerometers. Then a typical measurement equation for the redundant inertial sensors of one kind can be described as follows.

$$m(t) = Hx(t) + f(t) + \varepsilon(t), \tag{1}$$

where

$m = [m_1\ m_2 \cdots m_n]^{\mathrm{T}} \in R^n$: inertial sensor measurement,

$H = [h_1 \cdots h_n]^T$: n×3 measurement matrix with rank (H^T)=3,

$x(t) \in R^3$: triad-solution (acceleration or angular rate),

$f(t) = [f_1 \quad f_2 \quad \cdots \quad f_n]^T \in R^n$: fault vector,

$\varepsilon(t) = [\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n]^T \in R^n$: measurement noise vector.

A parity vector is obtained using a matrix V as follows:

$$p(t) = Vm(t) = Vf(t) + V\varepsilon(t), \tag{2}$$

where V satisfies the following conditions,

$$VH = 0,\ VV^T = I_n,$$
$$V = [v_1 \quad v_2 \quad \cdots \quad v_n],\ |v_i| = 1 (i = 1, \cdots, n).$$

The matrix V can be obtained by singular value decomposition [5]. Measurement matrix H can be decomposed according to singular value decomposition as follows:

$$H = \mathrm{U}\Lambda\Gamma^*,$$

where $(\ )^*$ denotes complex conjugate transpose, and $U = [U_1 \vdots U_2]$, $\Lambda = \begin{bmatrix} \Sigma \\ 0_{(n-3)\times 3} \end{bmatrix}$, $\Gamma = I_3$, $U_1 \in R^{n\times 3}$, $U_2 \in R^{n\times(n-3)}$, $\Sigma \in R^{3\times 3}$, and U is a unitary matrix.

We can notice that $U_2{}^T H = 0$ and thus a matrix V satisfying $VH = 0$ is $V = U_2{}^T (\in R^{(n-3)\times n})$.

One reduced-order parity vector and two reduced-order parity vector are defined as follows.

2.1. One reduced-order parity vector

Denote the measurement vector excluding i-th sensor output as m_{-i}.

$$m_{-i} = [m_1 \quad m_2 \quad \cdots \quad m_{i-1} \quad m_{i+1} \quad \cdots \quad m_n]^T$$

The reduced-order parity vector is obtained from m_{-i} as follows:

$$p_{-i} = V_{-i} m_{-i}, \tag{3}$$

where

V_{-i} : (n-4)×(n-1) parity matrix corresponding to H_{-i},

H_{-i} : (n-1)×3 measurement matrix corresponding to m_{-i}, and V_{-i} and H_{-i} satisfies $V_{-i}V_{-i}{}^T = I$, $V_{-i}\mathrm{H}_{-i} = 0$.

2.2. Two reduced-order parity vector

Denote the measurement vector excluding i-th component (m_i) and j-th component (m_j) as $m_{-i,-j}$.

$$m_{-i,-j} = \begin{bmatrix} m_1\ m_2 \cdots m_{i-1}\ \ m_{i+1} \cdots\ m_{j-1}\ \ m_{j+1} \cdots\ m_n \end{bmatrix}^T$$

The reduced-order parity vector is obtained from $m_{-i,-j}$ as follows:

$$p_{-i,-j} = V_{-i,-j} m_{-i,-j}, \tag{4}$$

where

$V_{-i,-j}$: (n-5)×(n-2) parity matrix corresponding to $H_{-i,-j}$,

$H_{-i,-j}$: (n-2)×3 measurement matrix corresponding to $m_{-i,-j}$, and $V_{-i,-j}$ and $H_{-i,-j}$ satisfies $V_{-i,-j} V_{-i,-j}{}^T = I$, $V_{-i,-j} H_{-i,-j} = 0$.

We use the following averaged parity vector to reduce the effect of the measurement noise $\varepsilon(t)$.

The averaged parity vector for q samples from $t = t_{k-q+1}$ to $t = t_k$ is defined as follows:

$$\bar{p} \equiv \frac{1}{q}\{p(t_{k-q+1}) + p(t_{k-q+2}) + \cdots + p(t_k)\}. \tag{5}$$

Fault detection means the indication that something is going wrong in the system and fault isolation means the determination of the exact location of the fault. In this paper we propose a new FDI algorithm to detect and isolate double faults.

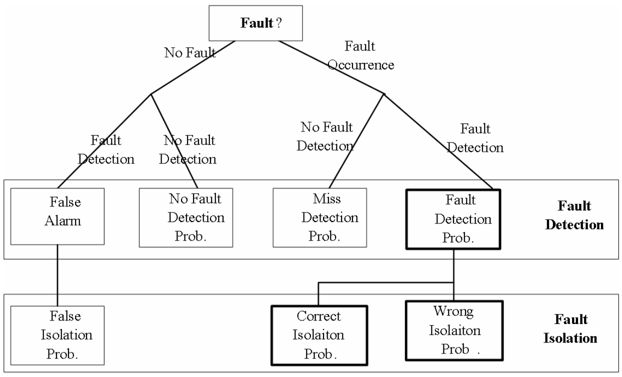


Fig. 1. FDI performance parameters.

The following assumptions are used for the sensor configuration.

Assumption 1: N sensors are used and the input axes of any 3 sensors are not on the same plane.

Assumption 2: Sensors of equivalent kind (accelerometers and gyroscopes) have identical noise characteristics, i.e., the measurements have white noise with normal distribution of same standard deviation, $\varepsilon(t)=[\varepsilon_1,\varepsilon_2,\cdots,\varepsilon_n]R^n$, $\varepsilon_i(t)\sim N(0,\sigma)$.

There are many parameters to determine the performance of FDI problems. The most significant parameters are fault detection probability, correct isolation probability, and wrong isolation probability, which will be considered for the performance analysis of the proposed FDI algorithm. Fig. 1 shows performance parameters related with FDI.

3. THE RPV (REDUCED-ORDER PARITY VECTOR) ALGORITHM FOR DOUBLE FAULTS

The feature of the proposed FDI method is that each sensor is omitted and the square of the magnitude of RPV (reduced-order parity vector) is checked to detect and isolate faults. When fault occurrence is decided, then the next step is to find whether it is a single fault or double fault and then to isolate it. The proposed FDI method is given in Fig. 2 as a flowchart.

The detail procedure of the RPV algorithm is as follows.

3.1. The procedure of the RPV algorithm

Step 1: Fault detection

Check whether there is a fault or not with the following decision function.

$$F=\max_i\{\overline{p}_{-i}^{\,T}\,\overline{p}_{-i}:i=1,\cdots,n\},$$
$$F\ \underset{H_0}{\overset{H_1}{\gtrless}}\ Th_1, \tag{6}$$

where H_1 denotes a fault hypothesis and H_0 a no-fault hypothesis, and Th_1 is a threshold determined from the probability of false alarm with χ^2 distribution.

Stop if H_0 results in a decision of no fault and go to Step 2 if H_1.

Step 2: Determination of a single fault or double fault

Calculate S as follows

$$S=\min_i\{\overline{p}_{-i}^{T}\overline{p}_{-i}:i=1,\cdots,n\}. \tag{7}$$

If $S<Th_2$, then a single fault occurs and go to Step 3. Otherwise, double fault occurs and go to Step 4, where Th_2 is a threshold determined from the probability of false alarm with χ^2 distribution.

Step 3: Single fault isolation

Calculate k as follows.

$$k=\arg\min_i\{\overline{p}_{-i}^{T}\overline{p}_{-i}:i=1,\cdots,n\} \tag{8}$$

The k-th sensor has a fault and should be isolated.

Step 4: Double fault isolation

Calculate (k,l) as follows.

$$(k,l)=\arg\min_{i,j}\{\overline{p}_{-i,-j}^{T}\overline{p}_{-i,-j}:i,j=1,\cdots,n(i\neq j)\} \tag{9}$$

The k-th and l-th sensors have faults and should be isolated.

Remark 1: The concept of the proposed RPV method is that when faults occur, the magnitude of the reduced-order parity vector obtained from omitting the faulty measurement should have minimum value.

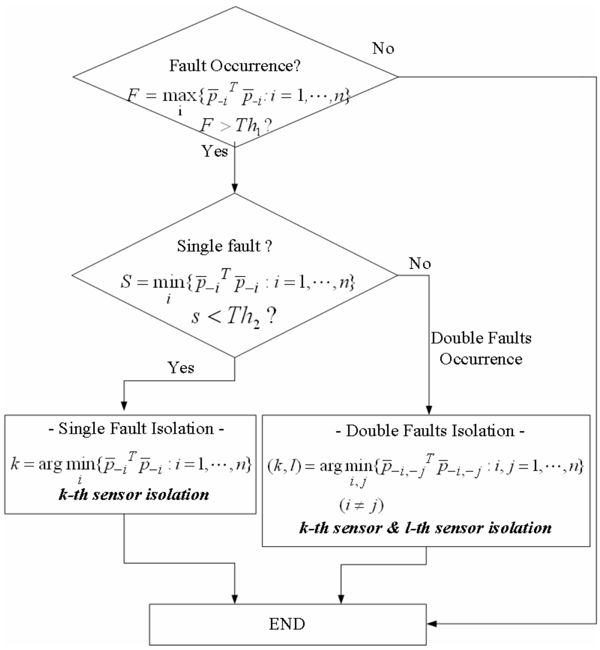


Fig. 2. The RPV algorithm.

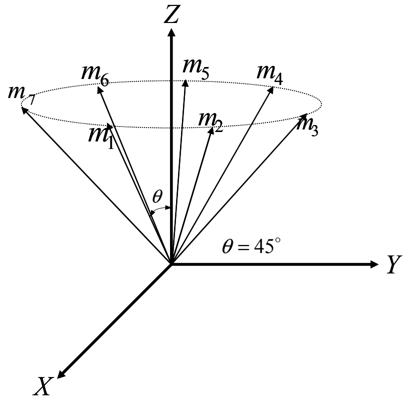


Fig. 3. Cone configuration with 7 sensors.

Remark 2: Suppose that *i*-th and *j*-th components have faults. When *n*=6, then $V_{-i,-j}$ is 1×4 matrix and $p_{-i,-j}$ is a scalar. Therefore $p_{-i,-j}$ may be zero for some particular faults. If $n \geq 7$, then $p_{-i,-j}$ is a non-zero vector, which results in $p_{-i,-j}^T p_{-i,-j} > 0$. This observation shows that the minimum number of sensors should be 7 to use RPV algorithm.

Remark 3: The proposed algorithm can handle the fault type 3 mentioned in Fig. 4 in [11]. Fault type 3 is the case that the magnitudes of two faults are bigger than the fault threshold, but the sum is smaller than the fault threshold. Consider fault type 4 in [11], which is the case that the magnitudes of two faults are smaller than the fault threshold, but the sum is greater than the fault threshold. In this case, the proposed algorithm detects the bigger one as a fault. If the directions of two faults were identical, then a fault-candidate with a magnitude half that of the fault threshold would be decided as a fault. In the case of our simulation with (10) and Fig. 3, the smallest angle between two fault directions is 66.6°. Thus for the worst case, a fault-candidate with a magnitude 0.598 times of the fault threshold may be decided as a fault.

4. SIMULATIONS

4.1. Sensor configuration

According to Remark 2 we use 7 sensors with cone configuration of symmetric structure as shown in Fig. 3. The measurement matrix *H* is described in (10).

$$H = \begin{bmatrix} 0.7071 & 0 & 0.7071 \\ 0.4409 & 0.5528 & 0.7071 \\ -0.1573 & 0.6894 & 0.7071 \\ -0.6371 & 0.3068 & 0.7071 \\ -0.6371 & -0.3068 & 0.7071 \\ -0.1573 & -0.6894 & 0.7071 \\ -0.4409 & -0.5528 & 0.7071 \end{bmatrix} \tag{10}$$

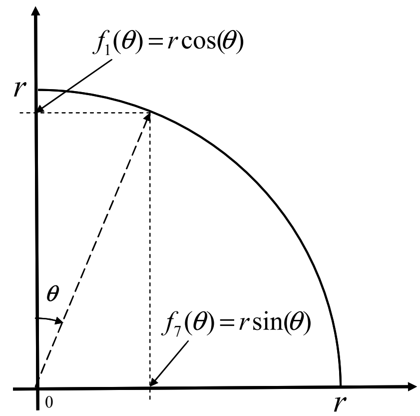


Fig. 4. Fault size of faulty sensors 1 and 7.

We use 100 samples to obtain averaged parity vectors. To take as many cases as possible into consideration we use the magnitude of fault vectors of 2σ, 4σ, and 6σ, respectively, where 1σ is the standard deviation of the sensor noise. Suppose that sensors 1 and 7, f_1 and f_7, have faults. To analyze the performance of the RPV algorithm, simulations are done for many different values of f_1 and f_7. For each fixed points on the circle of radius of *r*, Monte Carlo simulation is performed 300 times for each combination of f_1 and f_7. Fig. 4 shows the fault sizes of sensor 1 and 7 with respect to θ. As θ increases, the magnitude of f_1 decreases and that of f_7 increases.

4.2. Performance of a former double fault isolation algorithm

It is known that double fault detection and isolation cannot be performed with 6 sensors. This means that any algorithm cannot show good performance for all combinations of fault sizes between two faults. For example, Fig. 5 indicates the correct isolation probability when the singular value decomposition

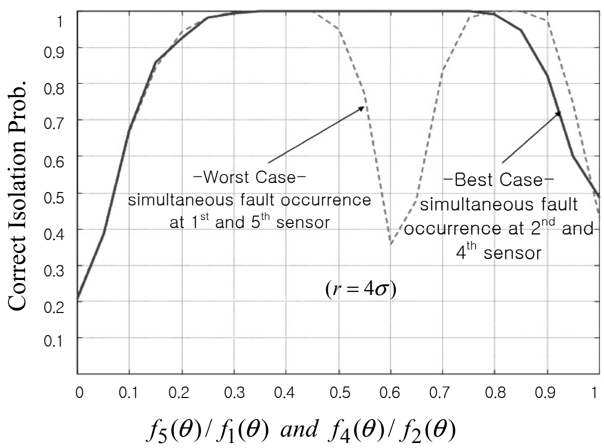
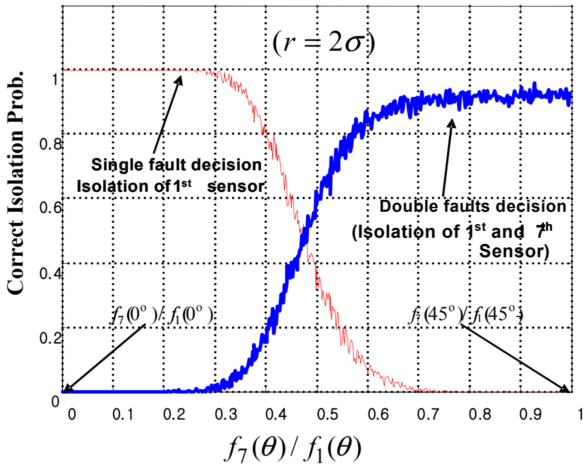
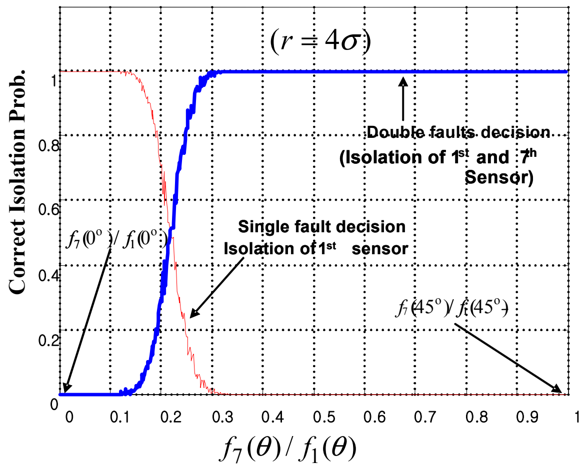


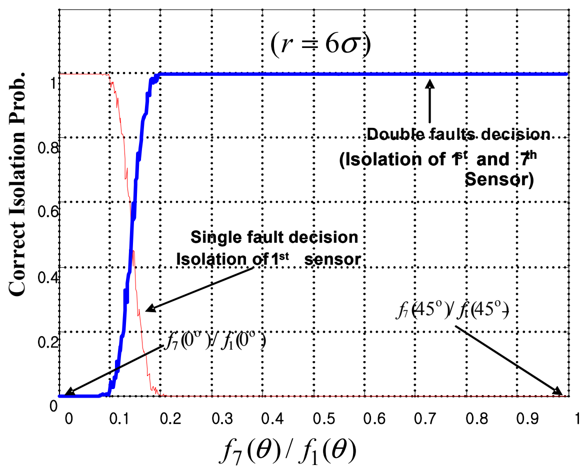
Fig. 5. Performance of SVD algorithm in [5].



(a) Radius of 2σ.



(b) Radius of 4σ.



(c) Radius of 6σ.

Fig. 6. Correct isolation probability in case of double fault occurrence.

(SVD) method in [5] is applied to the same problem as will be given in Fig. 6(b). Even for the best case as the bold line in Fig. 5, the correct isolation probability becomes 0.5 as the ratio of f_4/f_2 goes to 1.

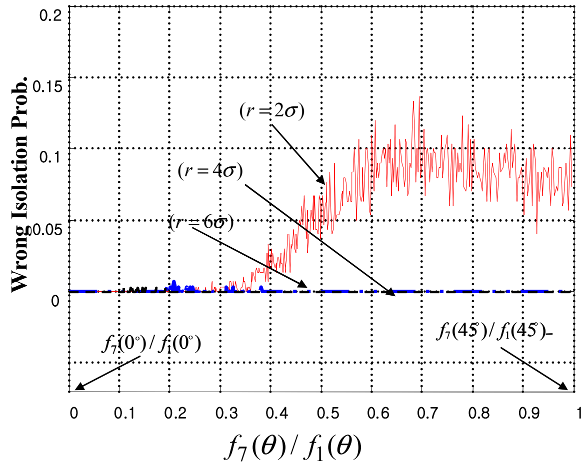


Fig. 7. Wrong isolation probability in case of double fault occurrence.

4.3. Performance of the RPV algorithm

The performance of the proposed isolation method is analyzed for $0° \le \theta \le 45°$ in Fig. 6. Fig. 6 shows the correct isolation probability of the RPV algorithm when the magnitude of fault vector is 2σ, 4σ and 6σ, respectively. It is supposed that the 1st and 7th sensors have faults. The thin line shows the case of isolating only one sensor after deciding that only one sensor has a fault. The bold line shows the case of isolating two sensors after deciding that two sensors have faults. When the magnitude of f_7 is much smaller than f_1, the RPV algorithm decides that there is only one fault f_1. As the magnitude of f_7 gets larger, the algorithm decides that two faults occur simultaneously. As the radius of fault size gets larger, the correct isolation probability becomes 1. Even though the ratio of f_7/f_1 lies in the transitive region, we can isolate at least one fault, which is shown in the wrong isolation probability in Fig. 7. We can notice that if the magnitude of the fault vector is greater than or equal to 4σ, then wrong isolation probability is almost 0 even for the transition region.

Remark 4: The simulation result shows that the fault detection probability of the RPV algorithm is almost 1. Therefore, the fault detection probability is not drawn in the figure.

5. CONCLUSIONS

A double fault detection and isolation method with 7 sensors is proposed based on the reduced-order and averaged parity vector. A decision rule to distinguish between a single fault and a double fault is also proposed. The proposed RPV algorithm is very simple and shows good performance compared with previous methods. It is analyzed by Monte-Carlo simulation.

The simulation results indicate that the performance of the RPV algorithm is satisfactory from the viewpoint of fault detection probability, correct isolation probability, and wrong isolation probability.

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