Adaptive Control of a Class of Nonlinear Systems Using Multiple Parameter Models

Choon-Young Lee

Abstract: Many physical systems are hybrid in the sense that they have continuous behaviors and discrete phenomena. In control system with multiple models, switching strategy and stability of the closed-loop system under switching are very important issues. In this paper, a novel adaptive control scheme based on multiple parameter models is proposed to cope with a change in parameters. Switching strategy guarantees the non-increase in the global control Lyapunov function if the estimation of Lyapunov function value converges. Least-square estimation is used to find the estimated value of the Lyapunov function. Switching and adaptation law guarantees the stability of closed-loop system in the sense of Lyapunov. Simulation results on anti-lock brake system are shown to verify the effectiveness of the proposed controller in view of a large change in system parameters.

Keywords: Adaptive control, anti-lock bake system, multiple model, stability.

1. INTRODUCTION

A common approach to control complex dynamic systems is to design a set of different controllers, each of which for a particular operating region or performance objective, and then to switch them in real time to achieve the overall control objective. This is the philosophy behind gain-scheduled controller [1,2]. Many physical systems are hybrid in the sense that they have continuous behaviors and discrete phenomena. A good example of a complex hybrid system is an automobile. Discrete signals are gear ratios, load and road characteristics, driver inputs, and control signals and warnings of the Anti-lock Brake System(ABS). The continuous parts are often nonlinear dynamics of motion, motor characteristics, sensor signals, and so on. Continuous dynamic characteristics vary according to the state of discrete signals. Dynamics can be changed by the operator input or due to a change in the environment. Therefore, it is required to implement a different controller for each operating condition.

An intelligent control system may have the ability to operate in multiple environments by understanding the current operating condition and achieving the various tasks appropriately. The ability to adapt to unknown operating conditions is an important attribute of intelligent systems. Adaptive control is a promising technique to obtain a model of the plant and its environment from experimental data and to design a controller. Adaptive control for a feedback linearizable nonlinear system has attracted much interest among control system designers over several decades. If the exact knowledge of the system is available, it is possible to transform a nonlinear adaptive control problem into a linear control problem by using a feedback linearization technique [5].

However, in many cases, the plant to be controlled is too complex to obtain the exact system dynamics, and the operating conditions in dynamic environments may be unexpected. Therefore, recently, an adaptive control technique has been combined with function approximators such as neural networks, fuzzy inference systems, and series expansion. These types of controllers take the capability of learning unknown nonlinear functions by universal approximation theorem [6,7] and massive parallel computation [8]. Based on the fact that universal approximators are capable of uniformly approximating a given nonlinear function over a compact set to any degree of accuracy, a globally stable adaptive controller has been developed with an adaptation algorithm [9-19]. An adaptive control scheme was used with a neural network to obtain a stable controller [4,10,12,14,17]. An observer-based controller was derived for nonlinear systems with state estimation without measuring all the states [13,15]. Off-line training of the robot manipulator was used in [11]. Although all these methodologies showed good performance in

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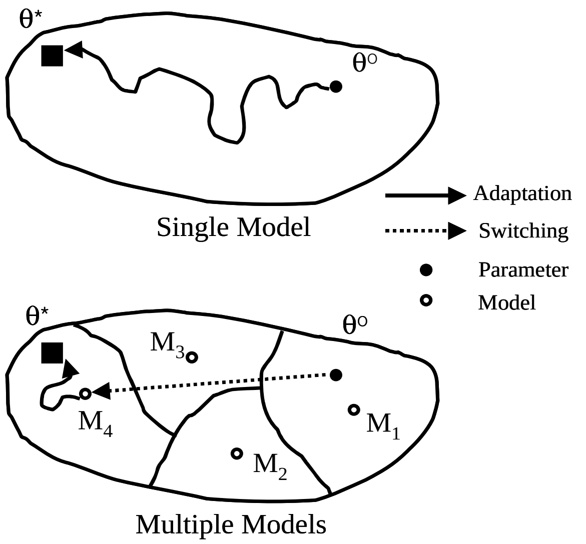

Fig. 1. Concept of the controller with multiple models.

controlling uncertain dynamic systems, there are unavoidable large transient errors at the time of task variation. For example, if a robot manipulator has to perform task 1, task 2, task 1, task 2, repeatedly in this order, an adaptive controller will always adapt itself to the new task, repeatedly, causing the system to forget the control skill acquired previously. Although task 1 is encountered for the second time, the adaptive controller recognizes it as a new task as the controller has already been adapted to task 2. However, if the dynamic parameters and control skills are stored for each task, this information can be utilized to recognize the tasks when the tasks encounter repeatedly at a later time. It also makes the system be able to cope with the repeating tasks quickly.

For this reason, a multiple-model-based reconfigurable control strategy was suggested [3]. The objective of multiple models is to improve the transient performance of linear adaptive control systems with large parametric uncertainties. As shown in Fig. 1, system parameters may change abruptly from a nominal point θ^o in the parametric space, to the point $\theta*$ due to a system failure. In some robotic applications, this situation includes the case that load at the end-effector affects parametric space of the system. The top figure illustrates the case of an adaptive controller with a single model. This system may be stable and ultimately find the optimal parameter vector to control the system after change, however, it is too slow to identify the new operating condition, and it may cause large transient errors during the adaptation. The bottom figure illustrates the case of an adaptive controller with multiple models. This architecture uses switching of multiple models. With some switching strategy, system parameters switch to the closest model, and the adaptive law finds the optimal parameters to control the system after change. This scheme can make fast and accurate response compared to that with a single model.

In control system with multiple models, switching strategy and stability of the closed-loop system under switching are very important issues. Most switching strategies aim to use identification errors between models and the real plant. In this paper, a new switching strategy is derived to select the most appropriate one among multiple models using the estimation of Lyapunov function values. Switching strategy guarantees the non-increase in the global control Lyapunov function if the estimation of Lyapunov function value converges. Least-square estimation was used to find the estimated value of the Lyapunov function. Switching and adaptation law guarantees the stability of closed-loop system in the sense of Lyapunov. This method is simply intuitive and performance-improving.

To make the system respond fast, general direct adaptive control design method is used with indirect estimation of the Lyapunov function value by least-square method. In this paper, we used the Lyapunov function of two arguments. One is tracking error which can be measured from the state values. The other is parameter estimation error which is basically unknown. Therefore the estimation of the Lyapunov function is done by calculating parameter estimation error. Under the assumption on the system structure, an identifier estimates parameter estimation error. By comparing the parameter estimation errors among the fixed model, we can infer the most closest nominal model from the multiple models. By switching the current adaptive model to the selected model under Lyapunov stability, we can achieve performance-improving switching. With the proposed scheme, the transient response of the system is improved despite the system parameter varies abruptly.

2. MODELING OF NONLINEAR SYSTEMS

Consider the *n*-th order nonlinear dynamic system of the form

$$\begin{aligned} &\dot{x}_1 = x_2, \\ &\dot{x}_2 = x_3, \\ &\cdots \\ &\dot{x}_n = f(x_1, x_2, \ldots, x_n) + bu + d, \\ &y = x_1, \end{aligned} \tag{1}$$

or equivalently

$$\begin{aligned} &x^{(n)} = f(x, \dot{x}, \ldots, x^{(n-1)}) + bu + d, \\ &y = x, \end{aligned} \tag{2}$$

where *f* is unknown but bounded functions, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the control input and output of the system respectively, and d is an external bounded disturbance.

If (2) is represented in the state space, we obtain

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}(f(\mathbf{x}) + bu + d),\\ y &= \mathbf{C}^{\mathrm{T}}\mathbf{x},\end{aligned} \tag{3}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \tag{4}$$

and $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}^T \in \mathbb{R}^n$ is a state vector where all x_i are assumed to be available for measurement. In order for (2) to be controllable, it is required that $b \neq 0$ for $\mathbf{x}$ in a certain controllability region $U_c \subset \mathbb{R}^n$.

Assumption 1: Without loss of generality, we can assume that $|f(\mathbf{x})| \leq f^U(\mathbf{x}) < \infty$ for $\mathbf{x} \in U_c$, and. Furthermore, external disturbance is bounded, i.e. $|d| \leq d_m$ where d_m is the upper bound of noise d.

Assumption 2: We assumed that $f(\mathbf{x}) = \sum_{i=1}^{p} \phi_i \theta_i$ for $\mathbf{x} \in U_c$. We also know that the regressor $\phi(\mathbf{x}) = \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_p \end{bmatrix}^T$.

Let the reference signal vector $\mathbf{y}_d$ and the tracking error vector $\mathbf{e}$ be defined as

$$\mathbf{y}_d = \left[y_d,\ \dot{y}_d, \cdots y_d^{(n-1)} \right]^T \in R^n, \tag{5}$$

$$\mathbf{e} = \left[e,\ \dot{e}, \cdots e^{(n-1)} \right]^T \in R^n, \tag{6}$$

where $e = y_d - y = y_d - x_1 \in R$. Then the control objective is to generate an appropriate control signal such that the system output y follows a given bounded reference signal y_d under the stability constraint that all signals involved in the system must be bounded.

3. DESIGN OF CONTROL SYSTEMS

If the functions $f(\mathbf{x})$ and b are known and there is no external disturbance d, then we can choose the following controller canceling the nonlinearity of the system,

$$\begin{aligned} u^* &= \frac{1}{b}\left[-f(\mathbf{x}) + y_d^{(n)} + \lambda^T \mathbf{e} \right] \\ &= \frac{1}{b}\left[-\phi^T \theta + y_d^{(n)} + \lambda^T \mathbf{e} \right]. \end{aligned} \tag{7}$$

If we apply the feedback linearizing controller (7) into the system (3), we obtain

$$\dot{\mathbf{e}} = \Lambda \mathbf{e}, \tag{8}$$

where

$$\Lambda = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ -\lambda_1 & -\lambda_2 & \cdots & -\lambda_n \end{bmatrix}. \tag{9}$$

In particular, let $\lambda = [\lambda_1, \lambda_2, \cdots, \lambda_n]^T \in \mathbb{R}^n$ be chosen such that Λ is Hurwitz, then $\lim_{t\to\infty} e(t) = 0$.

Suppose that the control u is the summation of an adaptive control $u_a(\mathbf{x} \mid \hat{\theta})$ and a supervisory control $u_s(\mathbf{x})$:

$$u = u_a(\mathbf{x} \mid \hat{\theta}) + u_s(\mathbf{x}). \tag{10}$$

If we select a Lyapunov function candidate as

$$V = \frac{1}{2}\mathbf{e}^T \mathbf{P} \mathbf{e} \tag{11}$$

and let the overall control law be defined as follows,

$$u = u_a + u_s, \tag{12}$$

$$\begin{aligned} u_a &= \frac{1}{b}\left[-\hat{f}(\mathbf{x}) + y_d^{(n)} + \lambda^T \mathbf{e} \right] \\ &= \frac{1}{b}\left[-\phi(\mathbf{x})^T \hat{\theta} + y_d^{(n)} + \lambda^T \mathbf{e} \right], \end{aligned} \tag{13}$$

$$\begin{aligned} u_s &= I^* \operatorname{sgn}(\mathbf{e}^T \mathbf{P} \mathbf{B}_c) \\ &\cdot \left[|u_a| + \frac{1}{b}\left(f^U(\mathbf{x}) + |y_d^{(n)}| + |\lambda^T \mathbf{e}| + d_m \right) \right], \end{aligned} \tag{14}$$

where $\mathbf{B}_c = [0, 0, \cdots 1]^T$, $\hat{f}$ is the estimate of f, $f^U(\mathbf{x})$ is the upper bound of f, $I^* = 1$, if $V_1 > V_c$ and $I^* = 0$ if $V_1 \leq V_c$, and $\operatorname{sgn}(a) = 1$ if $a \geq 0$ and $\operatorname{sgn}(a) = 0$ if $a < 0$, and $V_c > 0$ is a constant assigned by the user.

Theorem 1: Consider the nonlinear system described by (3) satisfying Assumption 1, and subject to the controller given in (12)-(14). Then $V_1 < V_c$ as $t \to \infty$, where V_c is a positive constant.

Proof: Applying (12) to the plant (3) we obtain

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}(f(\mathbf{x}) + b(u_a + u_s) + d). \quad (15)$$

After simple manipulations, we can obtain the error dynamic equation

$$\begin{aligned}\dot{\mathbf{e}} &= \mathbf{A}\mathbf{e} - \mathbf{B}\lambda^T\mathbf{e} + \mathbf{B}_c\left\{u^* - u_a - u_s - \frac{1}{b}d\right\} \\ &= \Lambda\mathbf{e} + \mathbf{B}_c\left\{u^* - u_a - u_s - \frac{1}{b}d\right\},\end{aligned} \quad (16)$$

$$e_1 = \mathbf{C}^T\mathbf{e}, \quad (17)$$

where $\Lambda = \mathbf{A} - \mathbf{B}\lambda^T$, and $e_1 = y_d - x_1$.

Since Λ is Hurwitz, there exists a positive definite symmetric matrix $\mathbf{P}$ which satisfies the Lyapunov equation

$$\Lambda^T\mathbf{P} + \mathbf{P}\Lambda = -\mathbf{Q}, \quad (18)$$

where $\mathbf{Q}$ is an arbitrary positive definite matrix.

From the Lyapunov function candidate of (11), if we take the time derivative of (11) along the trajectories of error dynamics (16), we obtain

$$\dot{V} = \frac{1}{2}\dot{\mathbf{e}}^T\mathbf{P}\mathbf{e} + \frac{1}{2}\mathbf{e}^T\mathbf{P}\dot{\mathbf{e}} \quad (19)$$

$$\begin{aligned}&= \frac{1}{2}\mathbf{e}^T(\Lambda^T\mathbf{P} + \mathbf{P}\Lambda)\mathbf{e} \\ &\quad + \mathbf{e}^T\mathbf{P}\mathbf{B}_c\left\{u^* - u_a - u_s - \frac{1}{b}d\right\}\end{aligned} \quad (20)$$

$$= -\frac{1}{2}\mathbf{e}^T\mathbf{Q}\mathbf{e} + \mathbf{e}^T\mathbf{P}\mathbf{B}_c\left\{u^* - u_a - u_s - \frac{1}{b}d\right\}. \quad (21)$$

If we rewrite the above equation,

$$\begin{aligned}\dot{V}_1 \le &-\frac{1}{2}\mathbf{e}^T\mathbf{Q}\mathbf{e} + |\mathbf{e}^T\mathbf{P}\mathbf{B}_c|\left\{|u^*| + |u_a| + \frac{1}{b}|d|\right\} \\ &- \mathbf{e}^T\mathbf{P}\mathbf{B}_c u_s.\end{aligned} \quad (22)$$

Considering the case of $V_1 > V_c$ and substituting the supervisory controller (14) into (22), we obtain

$$\dot{V}_1 \le -\frac{1}{2}\mathbf{e}^T\mathbf{Q}\mathbf{e}. \quad (23)$$

Therefore, we always have $V_1 \le V_c$ by using the supervisory controller. □

The bound of V_1 implies the bound of the magnitude of the error vector $\mathbf{e}$. Moreover, it also means that the state vector $\mathbf{x}$ is bounded. Therefore, the closed-loop system with the controller (12) operates well to stabilize the given system in the sense that the error is not diverged.

Next, we develop an adaptive law to adjust the parameter vector $\hat{\theta}$.

Define the optimal parameter vector:

$$\theta^* = \arg\min_{|\hat{\theta}\le M_\theta}\left[\sup_{|\mathbf{x}|\le M_{\mathbf{x}}} |u_a(\mathbf{x}|\hat{\theta}) - u^*|\right] \quad (24)$$

and the minimum approximation error:

$$w = u_a(\mathbf{x}|\hat{\theta}) - u^*, \quad (25)$$

where $u_a(\mathbf{x}|\hat{\theta})$ is the approximated controller of the ideal adaptive controller and is different from u_a in (16). Ideal adaptive controller u_a in (16) will converge to u^*, however, the approximated controller in (25) will have approximation error, w.

If the approximated adaptive controller is used, the error equation (16) can be rewritten as

$$\dot{\mathbf{e}} = \Lambda\mathbf{e} + \mathbf{B}_c\left\{u^* - u_a\right\} - \mathbf{B}_c u_s - \mathbf{B}_c\left(w + \frac{1}{b}d\right), \quad (26)$$

$$\begin{aligned}\dot{\mathbf{e}} = &\Lambda\mathbf{e} + \mathbf{B}_c\left\{\frac{1}{b}\left(-\phi^T(\mathbf{x})\theta^*\right) - \frac{1}{b}\left(-\phi^T(\mathbf{x})\hat{\theta}\right)\right\} \\ &- \mathbf{B}_c u_s - \mathbf{B}_c\left(w + \frac{1}{b}d\right),\end{aligned} \quad (27)$$

$$\dot{\mathbf{e}} = \Lambda\mathbf{e} - \mathbf{B}_c\frac{1}{b}\left(\phi^T(\mathbf{x})\tilde{\theta}\right) - \mathbf{B}_c u_s - \mathbf{B}_c\left(w + \frac{1}{b}d\right), \quad (28)$$

where $\tilde{\theta} = \theta^* - \hat{\theta}$.

Let the Lyapunov function candidate be

$$V_2 = \frac{1}{2}\mathbf{e}^T\mathbf{P}\mathbf{e} + \frac{1}{2b}\tilde{\theta}\Gamma^{-1}\tilde{\theta}, \quad (29)$$

where Γ is a positive gain matrix.

Taking the time derivative of the above Lyapunov function candidate, we obtain

$$\begin{aligned}\dot{V}_2 = &-\frac{1}{2}\mathbf{e}^T\mathbf{Q}\mathbf{e} \\ &+ \mathbf{e}^T\mathbf{P}\mathbf{B}_c\left\{-\frac{1}{b}\phi^T\tilde{\theta} - u_s - w - \frac{1}{b}d\right\} \\ &+ \frac{1}{b}\tilde{\theta}\Gamma^{-1}\dot{\tilde{\theta}},\end{aligned} \quad (30)$$

$$\begin{aligned}\dot{V}_2 = &-\frac{1}{2}\mathbf{e}^T\mathbf{Q}\mathbf{e} \\ &- \frac{1}{b}\tilde{\theta}\left(\mathbf{e}^T\mathbf{P}\mathbf{B}_c\phi - \Gamma^{-1}\dot{\tilde{\theta}}\right) \\ &- \mathbf{e}^T\mathbf{P}\mathbf{B}_c\left\{u_s + w + \frac{1}{b}d\right\}.\end{aligned} \quad (31)$$

If we choose the adaptive law:

$$\dot{\hat{\theta}} = -\Gamma\mathbf{e}^T\mathbf{P}\mathbf{B}_c\phi(\mathbf{x}) \quad (32)$$

and the fact $\mathbf{e}^T \mathbf{PB}_c u_s \geq 0$, then, we obtain

$$\dot{V}_2 \leq -\frac{1}{2}\mathbf{e}^T \mathbf{Qe} - \mathbf{e}^T \mathbf{PB}_c \left\{ w + \frac{1}{b} d \right\}. \quad (33)$$

In order to guarantee $|\hat{\theta}| \leq M_\theta$, we will use a projection algorithm to modify the basic adaptive law [23,24].

If $|\hat{\theta}| < M_\theta$ or ($|\hat{\theta}| = M_\theta$ and $\mathbf{e}^T \mathbf{PB}_c \phi^T(\mathbf{x})\hat{\theta} \geq 0$),

$$\dot{\hat{\theta}} = -\Gamma \mathbf{e}^T \mathbf{PB}_c \phi(\mathbf{x}). \quad (34)$$

If $|\hat{\theta}| = M_\theta$ and $\mathbf{e}^T \mathbf{PB}_c \phi^T(\mathbf{x})\hat{\theta} < 0$,

$$\dot{\hat{\theta}} = -\Gamma \mathbf{e}^T \mathbf{PB}_c \phi(\mathbf{x}) + \Gamma \mathbf{e}^T \mathbf{PB}_c \frac{\hat{\theta}\hat{\theta}^T \phi(\mathbf{x})}{|\hat{\theta}|^2}, \quad (35)$$

where Γ is the adaptation gain.

Suppose the parameter estimate is reset instant-taneously from $\hat{\theta}(t)$ to $\hat{\theta}_i(t+)$ at any time instant t, the jump in the Lyapunov function is given by

$$\Delta V_2 = V_2(t+) - V_2(t), \quad (36)$$

where $t+$ denotes an infinitely small time increment of t. At this time of $t+$, the parameter update law (34) and (35) does not apply, and the Lyapunov function may be discontinuous. We also assumed constant error signals at time $t+$.

If we allow parameter estimate to be changed into the most possible model in the set of models, the stability of the closed loop system under switching can be considered using multiple lyapunov functions. The stability preserving switching is to select at each time the minimum of several Lyapunov functions [20]. A stability preserving and performance-improving reset condition for parameter estimate is now $\Delta V_2 < 0$.

We cannot evaluate the Lyapunov function (29) as we do not know the exact parameter θ^* to calculate the term $\tilde{\theta}$.

We assume a finite number of fixed parameter hypotheses $\hat{\theta}_1, \hat{\theta}_2, \cdots, \hat{\theta}_p$ that we compare at each time instant to see which one gives the largest guaranteed decrease in ΔV_2.

From the plant model $x^{(n)} = \phi^T(\mathbf{x})\theta + bu$, applying a low-pass filter $H(s) = \frac{a}{s+a}$, we obtain the following linear parametric model of the form

$$z(t) = \zeta^T(t)\theta, \quad (37)$$

where $z(t) = sH(s)x^{(n)} - bH(s)u(t)$ and $\zeta(t) = H(s)\phi(\mathbf{x})$. The main purpose of this filter is to replace differentiation operations by appropriate high-pass filters, and to reduce the effect of high-frequency noise.

The finite number of fixed parameter hypotheses $\hat{\theta}_i$ is used to make prediction using the following relations:

$$\hat{z}_i(t) = \zeta^T(t)\hat{\theta}_i \quad (38)$$

for all i=0,1,2, ... , p. Index 0 is for the parameter which is currently adapted. Then, prediction errors are defined as

$$\varepsilon_i(t) = z(t) - \hat{z}_i(t) = \zeta^T(t)(\theta^* - \hat{\theta}_i). \quad (39)$$

The signal $\varepsilon_i(t)$ is available, and contains information about θ^*. In order to achieve invertibility, this equation is premultiplied by the matrix $\zeta(t)$ and integrated on the time interval from t-T to t, where T>0 is a finite time window

$$\theta^* - \hat{\theta}_i = W(t)^{-1} p_i(t). \quad (40)$$

where $W(t) = \int_{t-T}^{t} \zeta(\tau)\zeta^T(\tau)d\tau$ and $p_i(t) = \int_{t-T}^{t} \zeta(\tau)\varepsilon_i(t)d\tau$.

At each time instant, we compare the following Lyapunov function values with the performance of the current adaptive controller.

$$\Delta V_{2i} = V_{2i} - V_{20}, \quad i = 1, 2, \cdots, p, \quad (41)$$

where V_{20} is the estimated Lyapunov function value for the current adaptive controller, and V_{2i}, $i = 1, 2, \cdots, p$ is the estimated Lyapunov function values for the fixed multiple models.

We switch controller with a new parameter of j-th model if the following condition holds:

$$\Delta V_{2j} < -\delta, \quad (42)$$

where $\delta > 0$ is a positive constant which prevents chattering during switching [21].

Theorem 2: Consider the nonlinear system described by (3) satisfying Assumption 1, and subject to the controller given in (12)-(14) and adaptation law in (34) and (35). If we apply switching of controller with the parameter of multiple models according to (42), then the overall control scheme after the convergence of switching guarantees the following properties:

(i) $|\hat{\theta}| \leq M_\theta$, state vector is bounded, and the control input is bounded.

(ii) $\int_0^t |\mathbf{e}|^2 d\tau \leq \alpha + \beta \int_0^t |w(\tau)|^2 d\tau + \frac{\beta}{b}\int_0^t |d(\tau)| d\tau$

for all $t \geq 0$, where α and β are constants, and w is the minimum approximation error defined by (25), and d is the disturbance input.

(iii) If w and d are squared integrable, i.e., $\int_0^\infty (|w(t)|^2 + |d(t)|^2)dt < \infty$, then $\lim_{t\to\infty} |\mathbf{e}(t)| = 0$.

Proof: Under any switching sequence, the adaptive law guarantees the bounded property of parameter estimation. Let $V_{\hat\theta} = \frac{1}{2}\hat\theta^T\hat\theta$. If (34) is true, we have either $|\hat\theta| < M_\theta$ or $\dot V_{\hat\theta} = -\Gamma \mathbf{e}^T \mathbf{P}\mathbf{B}_c \phi(\mathbf{x})\hat\theta \leq 0$ when $|\hat\theta| = M_\theta$, i.e., we always have $|\hat\theta| \leq M_\theta$. If (35) is true, we have $|\hat\theta| = M_\theta$ and $\dot V_{\hat\theta} = -\Gamma \mathbf{e}^T \mathbf{P}\mathbf{B}_c \phi(\mathbf{x})\hat\theta$ $+\Gamma \mathbf{e}^T \mathbf{P}\mathbf{B}_c \frac{|\hat\theta|^2 \phi(\mathbf{x})}{|\hat\theta|^2}\hat\theta = 0$ i.e., we always have $|\hat\theta| \leq M_\theta$. Therefore, we always have $|\hat\theta| \leq M_\theta$, for all $t \geq 0$.

In Theorem 1, we proved that $V_1 \leq V_c$, therefore, $\frac{1}{2}\lambda_{\min}(\mathbf{P})|\mathbf{e}|^2 \leq \frac{1}{2}\mathbf{e}^T\mathbf{P}\mathbf{e} \leq V_c$, i.e., $|\mathbf{e}| \leq \sqrt{\frac{2V_c}{\lambda_{\min}(\mathbf{P})}}$. Since $\mathbf{e} = \mathbf{y}_d - \mathbf{x}$, we have $|\mathbf{x}| \leq |\mathbf{y}_d| + |\mathbf{e}| \leq |\mathbf{y}_d| + \sqrt{\frac{2V_c}{\lambda_{\min}(\mathbf{P})}}$.

Supervisory control input has the following property

$$|u_s| = |u_a| + \frac{1}{b}\left(f^U(\mathbf{x}) + |y_d^{(n)}| + |\lambda| \sqrt{\frac{2V_c}{\lambda_{\min}(\mathbf{P})}} + d_m \right). \tag{43}$$

Adaptive control input has the following property

$$|u_a| = \frac{1}{b}\left[|\phi(\mathbf{x})| M_{\hat\theta} + |y_d^{(n)}| + |\lambda\} \sqrt{\frac{2V_c}{\lambda_{\min}(\mathbf{P})}} \right]. \tag{44}$$

Therefore, we have

$$\begin{aligned} |u| = |u_s| + |u_a| &\leq \frac{1}{b}(f^U(\mathbf{x}) + 2|\phi(\mathbf{x})| M_{\hat\theta} \\ &+ 3|y_d^{(n)}| + 3|\lambda| \sqrt{\frac{2V_c}{\lambda_{\min}(\mathbf{P})}} + d_m). \end{aligned} \tag{45}$$

Since switching occurs only if $\Delta V_{2j} < -\delta$,

$$\dot V \leq \dot V_2, \tag{46}$$

where $\dot V_2$ is a performance improvement without switching and V is the selected Lyapunov function among the multiple Lyapunov functions.

$$\begin{aligned} \dot V &\leq -\frac{1}{2}\mathbf{e}^T\mathbf{Q}\mathbf{e} - \mathbf{e}^T\mathbf{P}\mathbf{B}_c\left\{w + \frac{1}{b}d\right\} \\ &\leq -\frac{1}{2}\lambda_{\min}(\mathbf{Q})|\mathbf{e}|^2 - \mathbf{e}^T\mathbf{P}\mathbf{B}_c\left\{w + \frac{1}{b}d\right\} \\ &\leq -\frac{1}{2}(\lambda_{\min}(\mathbf{Q}) - 1)|\mathbf{e}|^2 + \frac{1}{2}|\mathbf{P}\mathbf{B}_c\left\{w + \frac{1}{b}d\right\}|^2, \\ \dot V &\leq -\frac{1}{2}(\lambda_{\min}(\mathbf{Q}) - 1)|\mathbf{e}|^2 + \frac{1}{2}|\mathbf{P}\mathbf{B}_c w|^2 + \frac{1}{2b}|\mathbf{P}\mathbf{B}_c d|^2. \end{aligned} \tag{47}$$

Integrating both sides of (47) and assuming that $\lambda_{\min}(\mathbf{Q}) > 1$ since we can determine such a $\mathbf{Q}$, we have

$$\begin{aligned} \int_0^t |\mathbf{e}|^2 d\tau &\leq \frac{2}{\lambda_{\min}(\mathbf{Q}) - 1}(|V(0)| + |V(t)|) \\ &+ \frac{1}{\lambda_{\min}(\mathbf{Q}) - 1}|\mathbf{P}\mathbf{B}_c|^2 \int_0^t |w|^2 d\tau \\ &+ \frac{1}{b(\lambda_{\min}(\mathbf{Q}) - 1)}|\mathbf{P}\mathbf{B}_c|^2 \int_0^t |d|^2 d\tau \\ &\leq \alpha + \beta\int_0^t |w(\tau)|^2 d\tau + \frac{\beta}{b}\int_0^t |d(\tau)| d\tau, \end{aligned} \tag{48}$$

where $\beta = \frac{2}{\lambda_{\min}(\mathbf{Q})-1}(|V(0)| + \sup_{t\geq 0}|V(t)|)$ and $\beta = \frac{1}{\lambda_{\min}(\mathbf{Q})-1}|\mathbf{P}\mathbf{B}_c|^2$.

If $w \in L_2$ and $d \in L_2$, then $\mathbf{e} \in L_2$. From the error dynamics equation (16), $\dot{\mathbf{e}} \in L_\infty$ as it has been proved that all the variables on the right-hand side of (16) are bounded. Using Barbalat's lemma [25], we have $\lim_{t\to\infty} |\mathbf{e}(t)| = 0$. □

To prove the boundedness of all signals under switching scheme, we use the following definitions.

Definition 1 $[PC_{[0,\infty]}]$**:** the set of all real piecewise continuous functions.

Definition 2 [Large order]**:** $y(t) = O[x(t)]$ if there exist positive constants M_1, M_2, and t_0 such that $|y(t)| = M_1|x(t)| + M_2$, $t \geq t_0$.

Definition 3 [Small order]**:** $y(t) = o[x(t)]$ if there exist a function $\beta(t) \in PC_{[0,\infty]}$ and t_0 such that $|y(t)| = \beta(t)x(t)$, $\forall t \geq t_0$ and $\lim_{t\to\infty} \beta(t) = 0$.

Definition 4 [Equivalence]**:** If $y(t) = O[x(t)]$ and $x(t) = O[y(t)]$ then x and y are said to be *equivalent* and denoted by $x \sim y$.

From the above definition, we can easily find the following properties.

Property 1: If x is unbounded and y is bounded, then $y = o[x]$.

Property 2: If z is unbounded and $y = o[x]$ and $x = O[z]$ then $y = o[z]$.

The following lemma will be used in the proof of stability under switching.

Lemma 1: Consider a linear system

$$\dot{\mathbf{e}} = \Lambda\mathbf{e} + \mathbf{B}_c[\tilde u + v], \tag{49}$$

where Λ is an asymptotically stable matrix and $v \in L^{\infty}$. If $\tilde{u}$ is bounded over the interval $[0, t_0)$ and $|\tilde{u}| < \eta M(t)$ for $t \geq t_0$, where $M(t) = \sup_{\tau \leq t} \|e(\tau)\|$ and $\eta < \frac{1}{\|e^{\Lambda t}\mathbf{B}_c\|_1}$ with $\|e^{\Lambda t}\mathbf{B}_c\|_1$ denoting the L^1 norm of $e^{\Lambda t}\mathbf{B}_c$, then the error $\mathbf{e}$ is uniformly bounded, and $\tilde{u} = O[\mathbf{e}]$ [26].

Theorem 3: Consider the nonlinear system described by (3) satisfying Assumption 1, and subject to the controller given in (12)-(14) and adaptation law in (34) and (35). If we apply controller switching with the parameter of multiple models according to (42), then all signals involved are bounded under switching.

Proof: We prove the boundedness of the signals by contradiction. Assume that y is unbounded. From the definition of the error signal, we can say that $\mathbf{e} \sim y$. Since the error dynamics $\dot{\mathbf{e}} = \Lambda\mathbf{e} + \mathbf{B}_c[\tilde{u} - u_s]$ has an asymptotically stable dynamics, i.e., any input $\tilde{u}$ cannot grow faster than the error signal $\mathbf{e}$, we have $\tilde{u} = O[\mathbf{e}] = O[y]$. Since $\tilde{u} = \frac{1}{b}[-\phi^T\tilde{\theta}]$, we can say that $\phi = O[\mathbf{e}] = O[y]$, or ϕ does not grow faster than $\mathbf{e}$. In the switching scheme, there are two cases: The first case if where there is no switching when the estimate of the Lyapunov function is unbounded. The second case is where the parameter estimate is jumped to one of the multiple models when one of the estimate of Lyapunov functions is finite. In the first case, the system is stable from the proof of Theorem 2. In the second case, we can say that $\mathbf{e} = o[\phi]$ since we assumed that y is unbounded. That is, error signal grows at a lower rate than the regressor vector. From the above procedure, we have $\mathbf{e} = o[\mathbf{e}]$ and $y = o[y]$. That is, the signal y grows slower than itself. This cannot happen if y is assumed to be unbounded. Therefore, y is bounded, and the boundedness of other signals follows in the same way. □

4. APPLICATIONS

This section shows simulation studies to verify the effectiveness of the proposed control system. The system is an anti-lock brake system which is a safety device in an automobiles.

Anti-lock brake systems (ABS) were first introduced in railcars in the early 1900's. The original motivation is to avoid flat-spot of the steel wheels. It was soon noticed that stopping distance was also reduced by the ABS. Vehicle traction control, which includes anti-skid braking and anti-spin acceleration, can enhance vehicle performance and handling. The objective of this control is to maximize tire traction by preventing the wheels from locking during braking and from spinning during acceleration.

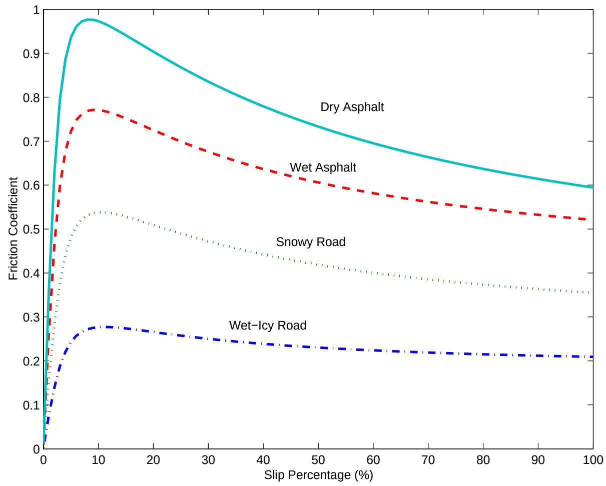

Fig. 2. $\mu - \lambda$ curve for different road surfaces.

Wheel slip is the difference between the vehicle speed and the wheel speed (normalized by the vehicle speed for braking and the wheel speed for acceleration), and it is chosen as the controlled variable for most of the traction control algorithm as it has a strong influence on the tractive force between the tire and the road. Typical relationships between the longitudinal adhesion coefficient (the ratio between the longitudinal tractive force and the normal load) and wheel slip ($\mu - \lambda$ curve) are shown in Fig. 2. Pacejka Magic Formula [22] was used to simulate friction coefficient between the road and tire under predefined scenario.

A quarter car model is given by the following motion equation:

$$m\dot{v} = -F_z, \tag{50}$$

$$J\dot{w} = rF_x - T_b sign(w), \tag{51}$$

where v is the longitudinal speed at which the car travels, w is the angular speed of the wheel, F_z is the vertical force, F_x is tire friction force, T_b is the brake torque, r is the wheel radius, and J is the wheel inertia. The tire friction force is given by $F_x = F_z\mu(\lambda, \mu_H, \alpha)$, where $\lambda = (v - wr)/v$ is the slip value, μ_H is the maximum value of friction coefficient for different road conditions, and α is the steering angle.

The dynamics of slip can be written as follows, assuming $w \geq 0$ and $v > 0$,

$$\dot{\lambda} = -\frac{1}{v}\left\{\frac{1}{m}(1-\lambda) + \frac{r^2}{J}\right\}F_z\mu(\lambda, \mu_H, \alpha) + \frac{1}{v}\frac{r}{J}T_b, \tag{52}$$

$$\dot{v} = -\frac{1}{m}F_z\mu(\lambda, \mu_H, \alpha). \tag{53}$$

The control objective is to maintain slip ratio $\lambda = 0.2$ during braking. The road surface is assumed to be wet-icy between 1 second and 4 second after braking has been applied. Four fixed models were used with $\mu^i = \{0.2,\ 0.5,\ 0.7,\ 0.9\}$.

Figs. 3-4 show the results of adaptive control with a single model on ABS control. Slip control error slowly decreases (Fig. 3) since it takes time to estimate accurate friction coefficients (Fig. 4). Figs. 5-8 shows the results of adaptive control with multiple models on ABS control. Slip control error converges rapidly to zero (Fig. 5) since the estimation of friction coefficient converges to the actual value quickly (Fig. 6). Fig. 7 shows the estimated Lyapunov function values for each model. According to the switching strategy, the most promising parameter is selected comparing the estimated Lyapunov function values. Fig. 8 shows the switching sequences. There are two switching between models, around 1 second and 4 second, as expected. Vertical axis indicates the index of parameter models. Table 1 presents the comparison of error measurements (IAE: Integral of Absolute Error) between adaptive control with single model (ACS) and adaptive control with multiple model (ACM) for ABS control. From the table, we achieved more than 50 percent of improvement in the integral of absolute errors during braking.

From the simulation results, we can see that adaptive control with multiple models shows improved transient response despite the change in

Table 1. Performance comparison between ACS and ACM for ABS system.

Interval	IAE_{ACS}	IAE_{ACM}	PI
$t \in [0, 6.85]$	0.1024	0.0430	58.0367

$$\left(\text{PI} = \frac{IAE_{ACS} - IAE_{ACM}}{IAE_{ACS}} \times 100\%\right)$$

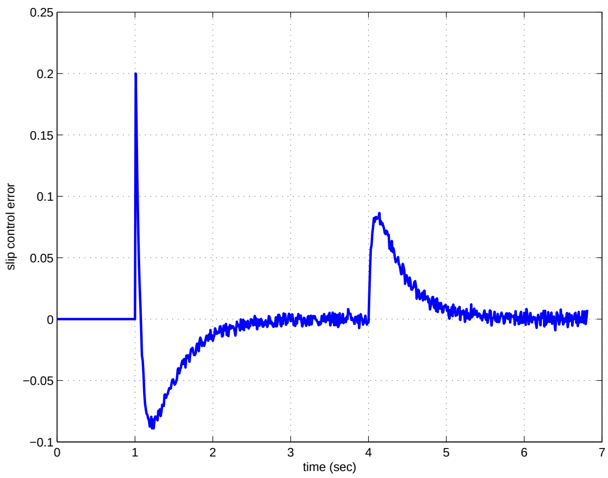

Fig. 3. Slip control error by adaptive control with a single model.

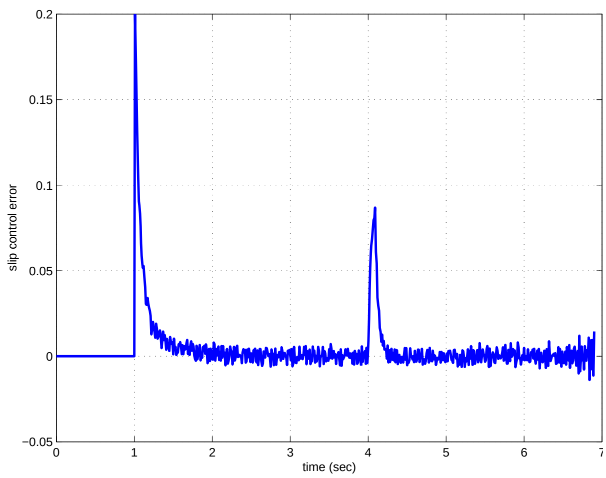

Fig. 5. Slip control error by adaptive control with multiple models.

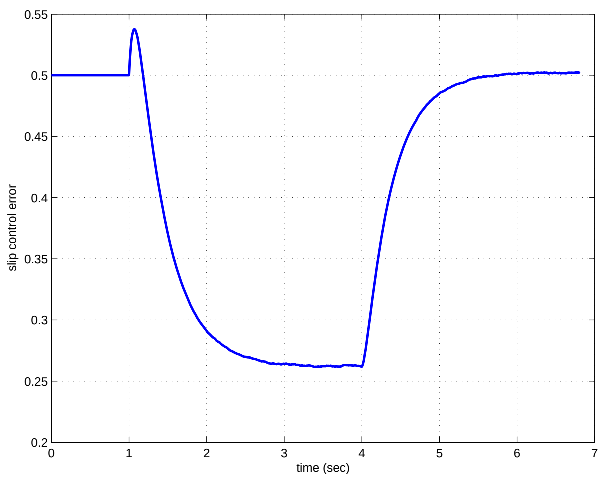

Fig. 4. Estimation of friction coefficient by adaptive control with a single model during braking.

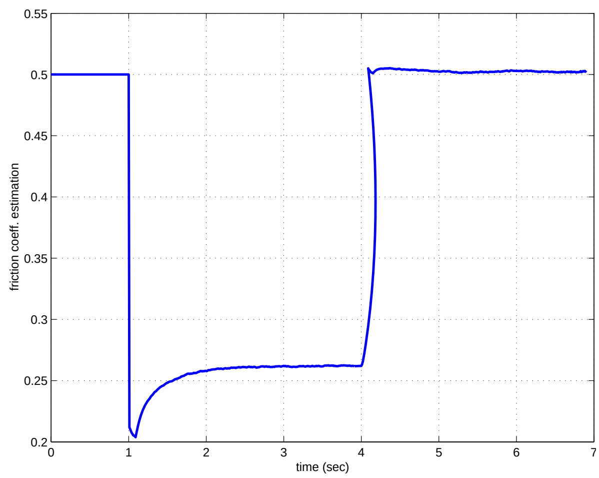

Fig. 6. Estimation of friction coefficient by adaptive control with multiple models during braking.

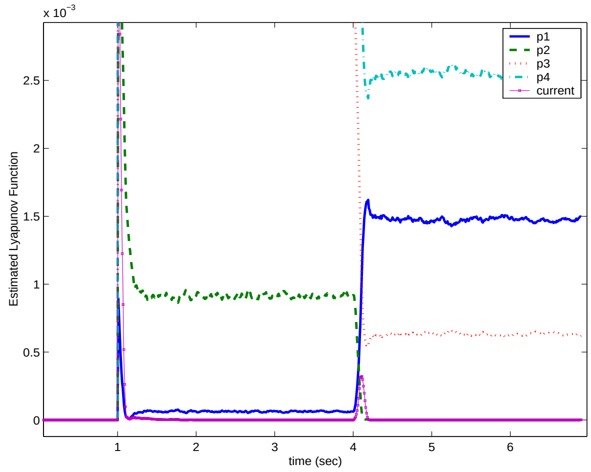

Fig. 7. Estimated Lyapunov function values of models by adaptive control with multiple models.

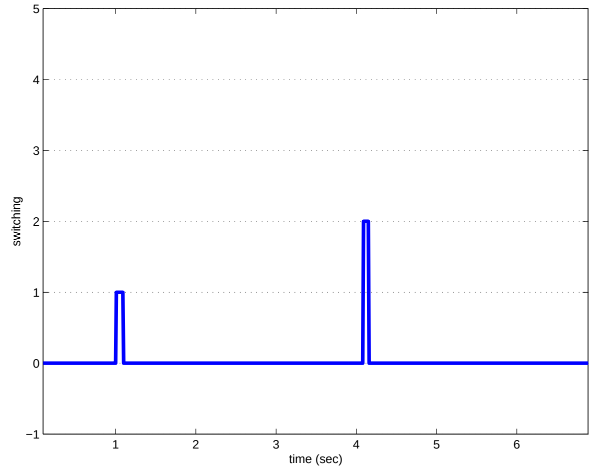

Fig. 8. Switching sequences by adaptive control with multiple models.

road surface characteristics. The key idea of this paper is similar to the literature [21] and this paper applied the estimator-resetting technique to an adaptive switching control, instead of the adaptive backstepping. This paper used a supervisory controller with adaptive switching control to compensate modeling error and noise. The advantage of the proposal is to easily implement adaptive switching control from the certainty equivalence principle for the considered form of nonlinear systems. Although the class of nonlinear systems considered in this paper is more restricted than the one in [21], the proposed algorithm in this paper is useful if we model an unknown nonlinear system as companion form with augmented nonlinearity practically.

5. CONCLUSIONS

General direct adaptive control design method was described with indirect estimation of the Lyapunov function value by least-square method. Under the assumption on the system structure, an identifier estimated parameter estimation error using filtering. By comparing the parameter estimation errors of the fixed models, we could infer the most closest nominal model from the multiple models. By switching the current adaptive model to the selected model under Lyapunov stability, we could achieve performance-improving switching. With the proposed scheme, the transient response of the system was improved in view of abrupt parameter variations. Simulation results on the control of anti-lock brake system showed the validity of the proposed approach.

The automatic generation of parameter models and the management of multiple models for control system will be further research topics.

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