Nonlinear FES Control of Knee Joint by Inversely Compensated Feedback System

Gwang-Moon Eom, Jae-Kwan Lee, Kyeong-Seop Kim*, Takashi Watanabe, and Ryoko Futami

Abstract: The aim of applying Functional Electrical Stimulation (FES) is to restore a person's motor function by directly supplying the controlled electrical currents to the site of the paralyzed muscles. However, most clinically utilized FES systems have adapted an open-loop control scheme. Recently the closed-loop control scheme has been considered for setting up the FES system, but due to the inherent nonlinearities in the musculoskeletal system, the nonlinearities were not fully compensated and it caused the oscillatory responses for tracking the output variables. In this study, a nonlinear controller model that has two inverse compensation units is proposed with the compromising feedback linearization method and this will eventually be used to design the FES control system for stimulating a knee joint musculoskeletal system.

Keywords: Feedback linearization, FES, inverse compensation, knee joint musculoskeletal system, nonlinear control.

1. INTRODUCTION

Functional Electrical Stimulation (FES) is defined as "the electrical stimulation for assistance or reconstruction of biological functions, with clear purpose and understanding of the mechanism" [1-3]. It is an effective method for restoring motor functions to the limbs paralyzed by spinal cord injury (SCI) or cerebral apoplexy. It utilizes the controlled electrical currents to evoke a certain skeletal muscle contraction for the paralyzed patients by supplying the proper electrical pulse trains to the intact muscles. However,

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due to the highly nonlinear nature of the musculoskeletal system, most clinical FES systems have been considered as only open-loop control schemes to stimulate a specific pattern predetermined by the relevant medical experts. [4,5]. Also, the clinical oriented FES systems have employed the open-loop control scheme because the closed-loop feedback type is difficult to implement when attaching the proper sensors so that good reproducibility at every attachment is guaranteed [6]. Indeed, the closed-loop EES clinical system induces rapid muscle fatigue [7], spinal reflexes and spasticity [8]. Also, due to the time-varying and unstable characteristics of the muscle [9], more difficulties are imposed in identifying the musculoskeletal system [10,11]. For these reasons, the closed-loop feedback control approaches have been applied to the FES clinical system only recently [12,13]. However, these efforts were incapable of compensating the inherent nonlinearities contained in the musculoskeletal system and consequently the controllers often caused the unstable oscillatory responses because they were not suitable for the overall range of a patient's motions such as FES standing, walking or cycling. In [14,15], a neural network based inverse model system trained with the complex nonlinear mapping for the feed-forward control was proposed for establishing a PID feedback controller and yielded the better performance. However, the stability issue remained unresolved due to the black-box nature of the neural network. Moreover, its nature does not provide any intermediate variable or clue about the physiological process. Effort was also put forth to apply the

physiologically based model for modeling an inverse system [12]. However, only the partial aspects of the nonlinearities were compensated and consequently the output system still had some oscillatory responses if the extent of a certain posture of FES motions such as standing or walking exceeds the range of the compensated linearity. Thus, we try to completely compensate the nonlinearities of a knee joint musculoskeletal model through feedback linearization with inverse compensation scheme and to suggest a new linear control scheme for tracking the reference trajectory of FES posture related with knee joint movements.

2. KNEE JOINT MUSCULOSKELETAL MODEL

The musculoskeletal system model of a knee joint with quadriceps muscle, which has an important role in FES standing, walking and cycling, is depicted as three blocks as shown in Fig. 1.

In Fig. 1, the first block contains the recruitment feature of muscle fibers, $r(s)$ stimulated by the intensity s, which is formulated in (1).

$$ r(s) = s_c \tanh(s_h(s - x_c)) + y_c. \quad (1) $$

Here, the parameters of a rising and a falling path are not identical and they represent the different hysteretic characteristics depending on the path, i.e.,

i) On a rising path: $s_c = s_{rc}, s_h = s_{rh}, x_c = x_{rc}, y_c = y_{rc}$,

ii) On a falling path: $s_c = s_{fc}, s_h = s_{fh}, x_c = x_{fc}, y_c = y_{fc}$.

The first block also includes the 1st order activation dynamics of the muscle's normalized active state corresponding to the calcium release. The activation dynamics, $a(t)$ is described by
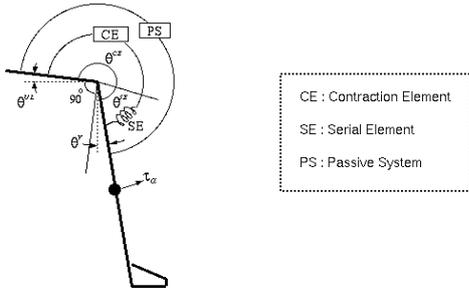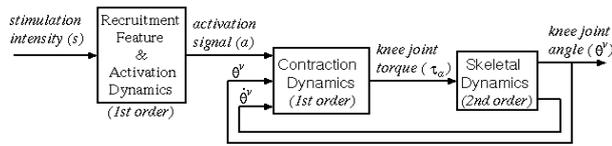

Fig. 1. A musculoskeletal system model of a knee joint.

$$ \dot{a} = (r - a)r / \tau_r + (r - (a - a_{\min}) - (r - a)r) / \tau_f. \quad (2) $$

Here, the time constants of τ_r and τ_f are the different rates of calcium release to and uptake from muscle fibers, respectively and $a_{\min}$ is the minimal active state.

The second block in Fig. 1 includes the 1st order muscle contraction dynamics. We adopted a lumped model of musculotendon and a moment arm as indicated in the lower schematics of Fig. 1 to refer the muscle to a torque generator [13]. The contraction dynamics of the musculotendon was stated as

$$ \dot{\tau}_a = (k_2\tau_a + k_1k_2\tau_{\max})\bullet $$
$$ \left(-\dot{\theta}^V - \dot{\theta}^{CE}_{\max} g^{-1}_{CE}\left(\tau_a / \left(\tau_{\max}k_{CE}(\theta^{CE})a\right)\right)\right), \quad (3) $$

where

$$ \theta^{CE} = 3\pi/2 - (\theta^V + \theta^{UL}) - \ln\left(\tau_a / \left(\tau_{\max}k_1\right) + 1\right)/k_2, $$

$$ k_1 = 1/\left(\exp(sh^{SE}) - 1\right), \quad k_2 = sh^{SE}/\theta^{SE}_{\max}, $$

$$ g^{-1}_{CE}(X) = \begin{cases} \dfrac{sh^{CEg}_{low}(X-1)}{X + sh^{CEg}_{low}} & (0 \le X < 1) \\[2mm] \dfrac{x_s sh^{CEg}_{up}(X-1)}{-X + y_s sh^{CEg}_{up} + y_s + 1} & (1 \le X < 1.3), \end{cases} $$

$$ k_{CE}(Y) = \begin{cases} \exp(-(\dfrac{Y - \theta^{CE_L}_{\max}}{sh^{CEk_L}})^2) & (Y < \theta^{CE}_{\max}) \\[3mm] (1 - c_1)\exp(-(\dfrac{Y - \theta^{CE_R}_{\max}}{sh^{CEk_R}})^2) \\[2mm] \quad + sl^{CEk}(Y - \theta^{CE_R}_{\max}) + c_1 & (\theta^{CE}_{\max} \le Y). \end{cases} $$

Here, τ_a is the active muscle torque, k_1 and k_2 are the parameters of series elastic element (SE). $\dot{\theta}^{CE}_{\max}$ is the maximum contraction velocity of the contractile element (CE), $g^{-1}_{CE}()$ is the inverse function of torque-angular velocity relationship of the CE, $\tau_{\max}$ is the maximum muscle torque and $k_{CE}()$ is the torque-angle relationship of the CE.

The third block of Fig. 1 can be described by the following 2nd order skeletal dynamics:

$$ \ddot{\theta}^V = 1/I\left(\tau_a - G\sin(\theta^v) - D\dot{\theta}^V + s_1(e^{-s_2(\theta^V + \theta^{UL})} - 1)\right). \quad (4) $$

Here, θ^v is the knee joint angle with reference to the vertical line, I and G are the moment of inertia and the gravity constant of the lower leg, respectively. The latter term of (4), $-D\dot{\theta}^V + s_1(e^{-s_2(\theta^V + \theta^{UL})} - 1)$ represents the damping and elastic torque induced by the

Table 1. Knee joint, vastus lateralis muscle parameters used in our computer simulation.

s_{rc}	1.661	τ_r	0.02	$sh_{low}^{CE_g}$	0.25	sh^{CEk_R}	0.2
s_{rh}	2.346	τ_f	0.2	$sh_{up}^{CE_g}$	0.25	sl^{CEk}	-0.18
x_{rc}	1.143	a_{min}	0.001	x_s	0.2	θ_{max}^{CE}	3.491
y_{rc}	1.538	sh^{SE}	2.7	y_s	0.3	I	0.44
s_{fc}	0.516	θ_{max}^{SE}	0.384	$\theta_{max}^{CE_L}$	3.491	G	16.5
s_{fh}	7.054	θ^{UL}	0.154	sh^{CEk_L}	0.2	D	0.22
x_{fc}	0.792	τ_{max}	11.32	c_1	0.75	s_1	0.092
y_{fc}	0.536	$\dot{\theta}_{max}^{CE}$	18	$\theta_{max}^{CE_R}$	3.456	s_2	5.09

passive system, such that D, s_1, s_2 are the constants and θ^{UL} is the angle of the upper leg with reference to the horizontal line. The muscle contraction and skeletal dynamics stated in (3) and (4) can be alternatively expressed by the state space representation form as stated in (5) if we define $x_1 \equiv \tau_a$, $x_2 \equiv \theta^V$, $x_3 \equiv \dot{\theta}^V$ and the output variable, $y \equiv \theta^V$.

$$\dot{x}_1 = (k_2 x_1 + k_1 k_2 \tau_{max}) \cdot$$
$$\left(-x_3 - \dot{\theta}_{max}^{CE} g_{CE}^{-1} \left(x_1 / (\tau_{max} k_{CE}(x_1, x_2) a) \right) \right),$$
$$\dot{x}_2 = x_3,$$
$$\dot{x}_3 = 1/I \left(x_1 - G\sin(x_2) - Dx_3 + s_1 \left(e^{-s_2(x_2 + \theta^{UL})} - 1 \right) \right),$$
$$y = x_2. \tag{5}$$

Thus the overall plant (musculoskeletal system) dynamics can be interpreted as a 4th order system composing of i) one activation dynamic unit, ii) one contraction dynamics unit, iii) two skeletal dynamics units. For our computer simulations, we adopted the estimated knee joint and vastus lateralis muscle parameters that are identified by the lump parameters for knee extensors for the voluntary contraction and for knee joints with the vastus lateralis muscle, respectively as stated in Table 1.

The identification procedures determining muscle parameters shown in Table 1 are listed in [13,16] and the accuracy of the model parameters are evaluated by tracking the model-predicted joint angle trajectories compared with experimental data [13].

3. NONLINEAR CONTROLLER DESIGN

The main control task of our study is to force a certain knee joint angle to track the reference trajectory. The overall control system structure is shown in Fig. 2.

The central idea of our suggested controller is utilizing a feedback linearization. In other words, we approximate a nonlinear system dynamics into a linear one with applying the algebraic transformation. With this scheme, the inherent nonlinearities contained in a musculoskeletal system model of the knee joint are vanished, and consequently the relatively simple linear control scheme can be applied. In Fig. 2, two inverse compensation units for the feedback linearization are described [17]. The first inverse compensation unit as stated in the following equation is to eliminate the nonlinearity of a term, g_{CE}^{-1} $\left(x_1 / (\tau_{max} k_{CE}(x_1, x_2) a) \right)$, which represents the muscle contraction dynamics.

$$r_{in} =$$
$$\begin{cases} \dfrac{-x_1 u_{in} + sh_{low}^{CE_g} \dot{\theta}_{max}^{CE} x_1}{sh_{low}^{CE_g} \tau_{max} k_{CE}(x_1, x_2) u_{in} + sh_{low}^{CE_g} \dot{\theta}_{max}^{CE} \tau_{max} k_{CE}(x_1, x_2)} \\ \qquad \left(0 \le \dfrac{x_1}{\tau_{max} k_{CE}(x_1, x_2) a} < 1 \right) \\ \dfrac{x_1 u_{in} + x_s sh_{up}^{CE_g} \dot{\theta}_{max}^{CE} x_1}{((y_s sh_{up}^{CE_g} + y_s + 1)\tau_{max} u_{in} + x_s sh_{up}^{CE_g} \dot{\theta}_{max}^{CE} \tau_{max}) k_{CE}(x_1, x_2)} \\ \qquad \left(1 \le \dfrac{x_1}{\tau_{max} k_{CE}(x_1, x_2) a} < 1.3 \right) \end{cases}$$
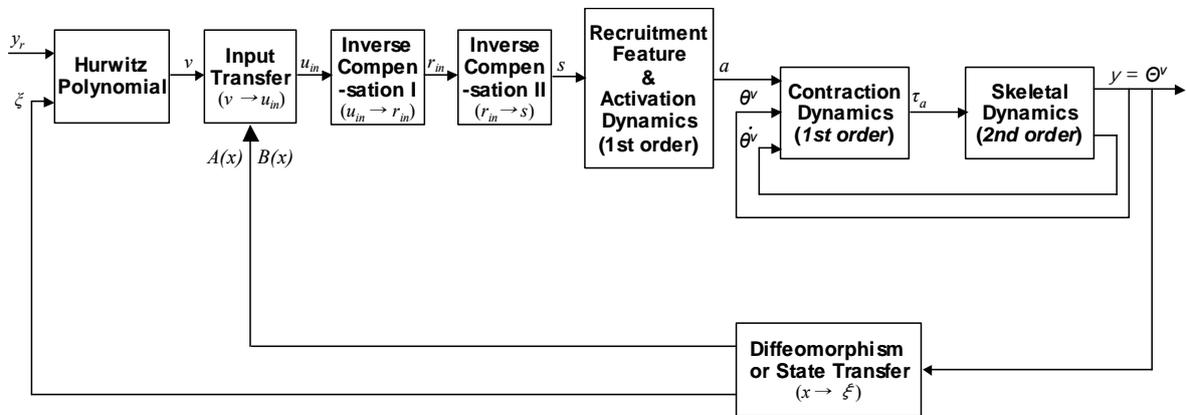$$\tag{6}$$

Fig. 2. The proposed control system structure.

In (6), u_{in} is the control input and required recruitment r_{in} is the output. The second inverse compensation unit as stated in the following equation is to remove the nonlinearity of the recruitment.

$$s = \ln\big(\big(r_{in} + (s_c - y_c)\big)/\big(-r_{in} + (s_c + y_c)\big)\big)/2s_h + x_c. \quad (7)$$

In (7), r_{in} is the input and the normalized stimulation intensity, s is the output. Since the normalized active state a in (2) is not measurable, it is regarded as a bounded and structured uncertainty. Then, the diffeomorphism (state transfer) [18] is defined as in (8) and the control law (input transfer) u_{in} is defined as in (9), respectively.

$$\begin{aligned}
\xi_1 &\equiv y = x_2, \\
\xi_2 &\equiv \dot{y} = x_3, \\
\xi_3 &\equiv \ddot{y} = x_1/I - G\sin(x_2)/I - Dx_3/I \\
&\quad + s_1(\exp(-s_2(x_2 + \theta^{UL})) - 1)/I,
\end{aligned} \quad (8)$$

$$u_{in} = (v - B(x))/A(x), \quad (9)$$

where

$$\begin{aligned}
A(x) &\equiv -(k_2 x_1 + k_1 k_2 \tau_{\max})/I, \\
B(x) &\equiv (k_2 x_1 + k_1 k_2 \tau_{\max})(-x_3 + 1)/I - G\cos(x_2)x_3/I \\
&\quad - D(x_1/I - G\sin(x_2)/I - Dx_3/I \\
&\quad + s_1\big(\exp(-s_2(x_2 + \theta^{UL})) - 1\big)/I)/I \\
&\quad + s_1(-s_2 \exp(-s_2(x_2 + \theta^{UL}))x_3)/I.
\end{aligned}$$

(9) is derived by combining the diffeomorphism of (8) and the control law. The input to the control law v is selected as stated in (10) and (11),

$$\ddot{y} = v, \quad (10)$$

$$v = \ddot{y}_r + \alpha_2(\ddot{y}_r - \xi_3) + \alpha_1(\dot{y}_r - \xi_2) + \alpha_0(y_r - \xi_1), \quad (11)$$

where $\alpha_2, \alpha_1, \alpha_0$ are the coefficients of the Hurwitz polynomial $s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0$. The Hurwitz polynomial coefficients are chosen with considering the behavior and response of the closed-loop system in terms of overshoot, rising time and settling time. Since the degree of the control system is three, we need to differentiate the output three times to get the control input.

The reference trajectory $y_r(t)$ was set as a sinusoidal function, which is useful in clinical practice because it is often used for paralyzed muscle training. The simulation result is shown in Fig. 3. With the inverse compensations of nonlinearities and the feedback linearization stated above, the stable output tracking of the overall closed-loop system was successfully achieved.
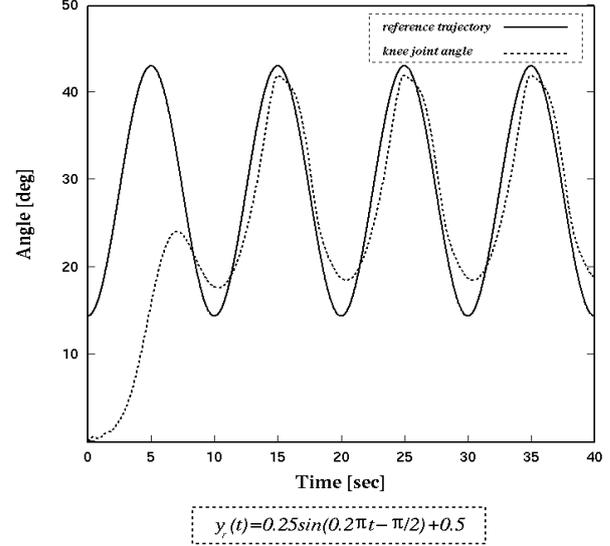

$$y(t) = 0.25\sin(0.2\pi t - \pi/2) + 0.5$$

Fig. 3. The simulation result of FES nonlinear control.

4. CONCLUSIONS

As we mentioned earlier, the recent closed-loop control schemes for FES do not completely compensate the nonlinearities of the plant and consequently often lead to the oscillatory responses. Our control plant shows better performance in terms of output tracking. Also, the output of the control system achieves the stable conditions by utilizing a feedback linearization with considering inverse compensations. Thus, we believe that we have suggested a new promising application especially for controlling the inherent nonlinearity of the FES system. In other words, we propose a new nonlinear control scheme to track a referenced angle trajectory of the paralyzed knee joint with implementing a controller composing two linearization units.

For the real clinical application of our control scheme, the following two issues must be considered. Firstly, the model parameters must be easily identified so that we can minimize the physical and psychological burdens of a patient. Secondly, the control scheme must be improved to be adaptive to time varying factors in the musculoskeletal system such as muscle fatigue.

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