Robust H_∞ FIR Sampled-Data Filtering for Uncertain Time-Varying Systems with Lipschitz Nonlinearity

Hee-Seob Ryu, Kyung-Sang Yoo, and Oh-Kyu Kwon

Abstract: This paper presents the results of the robust H_∞ FIR filtering for a class of nonlinear continuous time-varying systems subject to real norm-bounded parameter uncertainty and known Lipschitz nonlinearity under sampled measurements. We address the problem of designing filters, using sampled measurements, which guarantee a prescribed H_∞ performance in continuous time-varying context, irrespective of the parameter uncertainty and unknown initial states. The infinite horizon causal H_∞ FIR filter are investigated using the finite moving horizon in terms of two Riccati equations with finite discrete jumps.

Keywords: H_∞ FIR filter, Lipschitz nonlinearity, time-varying system, sampled-measurements

I. Introduction

There exists a vast literature on many new analysis tools that deal with the so-called H_∞ filtering [5]. The H_∞ filtering problem is concerned with designing estimators to minimize the H_∞ norm of the transfer function from the noise sources to the estimator error. However, the conventional H_∞ filters proposed so far are mainly limited to time-invariant systems. Therefore they can not be applied to general time-varying systems on the infinite horizon since one of two Riccati differential equations required to solve the problem can not be computed on the infinite horizon [4]. Therefore, the H_∞ FIR filter is investigated for a class of nonlinear continuous time-varying uncertain systems under sampled-data measurements on the infinite horizon.

Numerous industrial systems are continuous-time processes but monitored and measured by digital devices. The classical method of analyzing these systems is to develop a discrete-time method, based on the sampling frequency of the measurements. Digital filtering, smoothing and predicting devices built in this way tend to fail when the sampling frequency is too low and the system dynamics are relatively too fast. This is because the inter-sampling behavior of the system may be overlooked. So, in the continuous-time system, a filter design is required to produce a continuous-time estimate of an analogue signal based on sampled-data measurements. In this situation, the filtering performance measure should be defined directly in terms of the continuous-time signals, i.e. in the continuous-time context. We refer to this filtering approach as sampled-data filtering. As compared with the traditional discrete-time filter designs, the sampled-data filtering approach has the advantage of taking the inter-sampled behavior into consideration.

In this paper we consider the H_∞ FIR filtering problem for a class of uncertain nonlinear continuous time-varying systems under sampled-data measurements on the infinite horizon by the finite moving horizon. The class of uncertain systems is described by a nonlinear state-space model with real time-varying norm-bounded parameter uncertainty in the time-varying state and output matrices. Here, attention is focused on the design of causal filters, which guarantee a robust stability as well as a prescribed performance, irrespective of the uncertainty. The performance measure is defined directly in the continuous time-varying context and is of an H_∞ type. This filtering problem is referred to as *robust H_∞ FIR sampled-data filtering*. We show that the robust H_∞ FIR sampled-data filtering problem on the infinite horizon can be solved in terms of two Riccati equations. The basic idea of the current paper is to formulate the robust nonlinear H_∞ filtering problem on the continuous-time moving horizon $[t-T, t]$ and to adopt the FIR(Finite Impulse Response) filter structure. It is noted that this filter will works for the general time-varying systems under sampled-data measurements, and that this point will be one of the main contributions of the current paper.

II. Problem formulation and preliminaries

Consider the following class of nonlinear uncertain sampled-data time-varying systems:

$$\begin{aligned} \dot{x}(t) &= [A(t)+\triangle A(t)]x(t)+[G(t)+\triangle G(t)] \\ &\quad \cdot g[x(t)]+B(t)w(t),\ x(0)=x_0 \qquad (1) \\ z(t) &= L(t)x(t) \qquad (2) \\ z_d(i) &= L_d(i)x(i) \qquad (3) \\ y(i) &= [C(i)+\triangle C(i)]x(i)+[K(i)+\triangle K(i)] \\ &\quad \cdot k[x(i)]+D(i)v(i), \qquad (4) \end{aligned}$$

where $x(t) \in \Re^n$ is the state, x_0 is unknown initial state, $w(t) \in \Re^q$ is the process noise which belongs to $L_2[0,\infty)$, $y(i) \in \Re^m$ is the sampled measurement, $v(i) \in \Re^r$ is the measurement noise which belongs to $l_2(0,\infty)$, $z(t) \in \Re^p$ and $z(i) \in \Re^s$ are linear combinations of state variables to be estimated, i is an integer, $A(t)$, $B(t)$, $C(i)$, $D(i)$, $G(t)$, $K(i)$, $L(t)$ and $L_d(i)$ are known real time-varying bounded matrices of appropriate dimensions with $A(t)$, $B(t)$, $G(t)$ and $L(t)$

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being piecewise continuous, and $\triangle A(t)$, $\triangle C(i)$, $\triangle G(t)$ and $\triangle K(i)$ represent real time-varying parameter uncertainties in A, C, G and K respectively. These admissible uncertainties are assumed to be of the form

$$\triangle A(t) = HF(t)E, \quad \triangle G(t) = H_G F_G(t) E_G \quad (5)$$

$$\triangle C(i) = H_d F_d(i) E_d, \quad \triangle K(i) = H_K F_K(i) E_K, \quad (6)$$

where $F(t) \in \Re^{i \times j}$, $F_d(i) \in \Re^{i_\alpha \times i_\beta}$, $F_G(t) \in \Re^{i_G \times i_G}$ and $F_K(i) \in \Re^{i_K \times i_K}$ are unknown time-varying matrices satisfying

$$F^T(t)F(t) \le I, \quad F_G^T(t)F_G(t) \le I, \ \forall\, t \quad (7)$$

$$F_d^T(i)F_d(i) \le I, \quad F_K^T(i)F_K(i) \le I, \ \forall\, i \quad , \quad (8)$$

with the elements of F and F_G being Lebesgue measureable, and E, E_d, E_G, E_K, H, H_d, H_G and H_K are known real constant bounded matrices of appropriate dimensions with E, E_G, H and H_G being piecewise continuous. The matrices $A(t)$, $B(t)$, $C(i)$, $D(i)$, $G(t)$, $K(i)$, $L(t)$ and $L_d(i)$ describe the nominal model of system (1)-(4). For the sake of notation simplification, in the sequel the dependence on t or i for all matrices will be omitted.

Note that nonlinear models of the form (1)-(4) can be used to represent many important physical systems. A typical example is a power system modelled in the form of a single machine-infinite bus [1]. The parameter uncertainty structure as in (5)-(8) has been widely used in the problems of robust control and robust filtering of uncertain systems [2][3][8][9] and many practical system possess parameter uncertainties which can be either exactly modelled, or overbounded by (5)-(6).

The admissible known nonlinearity functions $g(\cdot)$ and $k(\cdot)$ are assumed to satisfy the following assumptions.

Assumption 1:

(a) $g(0) = 0$;

(b) There exist known constant matrices W_g and W_k such that for any x_1 and $x_2 \in \Re^n$,

$$\|g(x_1) - g(x_2)\| \le \|W_g(x_1 - x_2)\|$$
$$\|k(x_1) - k(x_2)\| \le \|W_k(x_1 - x_2)\|;$$

(c) $[D(i) \quad H_d(i) \quad K(i) \quad H_k(i)]$ is of full the row rank for all $i \in (0, T)$.

Assumption 1(c) means that the robust filtering problem is 'non-singular'. We observe that when there is no parameter uncertainty in the output matrix of system (1)-(4), Assumption 1(c) reduces to $D(i)D^T(i) \ge 0$, which corresponds to a standard nonsingularity condition in the H_∞ filtering problem for the nominal system (1)-(4).

In the current paper, the FIR filter is defined by the form

$$\hat{x}(t \mid T) = \int_{t-T}^{t} M(t, \tau; T) y(\tau) d\tau$$
$$\hat{z}(i \mid T) = L(i)\hat{x}(i \mid T),$$

where $M(t, \cdot; T)$ is the finite impulse response with the finite duration T. The estimation error is defined by

$$e(i) = z(i) - \hat{z}(i \mid T). \quad (9)$$

The H_∞ FIR filter is obtained by constructing its impulse response from that of the H_∞ filter on the finite moving horizon $[t - T, t]$.

It is noted that the problem does not need the assumption of stabilizability or detectability of the system since it is formulated on the finite moving horizon. In the sequel, the bounded real lemma for linear time-varying systems with finite discrete jumps which will be used throughout the paper, is reviewed.

Consider the following linear time-varying system with finite discrete jumps:

$$(\Sigma_1): \dot{x}(t) = Ax(t) + Bw(t), \ t \ne i \quad (10)$$
$$x(i) = A_d x(i^-) + B_d v(i), \ \forall\, i \in (0, T) \quad (11)$$
$$z(t) = Cx(t) \quad (12)$$
$$z_d(i) = C_d x(i^-), \quad (13)$$

where $x \in \Re^n$, $w \in \Re^q$ and $v \in \Re^r$ belongs to $L_2[0, T]$ and $l_2(0, T)$, respectively, $z \in \Re^p$, $z_d \in \Re^s$, and A, A_d, B, B_d and C are known real time-varying bounded matrices with A, B and C being piecewise continuous. Next, introduce the following worst-case performance index for (Σ_1):

$$J(\Sigma_1) = \sup \left[\frac{\|z\|^2 + \|z_d\|^2}{\|w\|^2_{[0,T]} + \|v\|^2_{(0,T)} + x_0^T R x_0} \right]^{1/2}, \quad (14)$$

where $R = R^T > 0$ is given weighting matrix for x_0 and the supremum is taken over all $(w, v, x_0) \in L_2[0, T] \oplus l_2(0, T) \oplus \Re^n$ such that $\|w\|^2_{[0,T]} + \|v\|^2_{(0,T)} + x_0^T R x_0 \ne 0$.

We now present a version of the bounded real lemma on finite horizon for interested filtering problem formulation of the system (Σ_1).

Lemma 1: [6]: Consider the system (Σ_1) and let $\gamma > 0$ be a given scalar. Then, the following statements are equivalent:

(a) $J(\Sigma_1, R, T) < \gamma$;

(b) There exists a bounded matrix function $P(t) = P^T(t) \ge 0$, $\forall\, t \in [0, T]$, such that

$$-\dot{P} = A^T P + PA + \gamma^{-2} PBB^T P + C^T C, \ t \ne i,$$
$$P(T) = 0 \quad (15)$$
$$\gamma^2 I - B_d^T P(i^+) B_d > 0 \quad (16)$$
$$P(i) = A_d^T P(i^+) A_d + A_d^T P(i^+) B_d [\gamma^2 I - B_d^T \cdot P(i^+) B_d]^{-1} B_d^T P(i^+) A_d + C_d^T C_d \quad (17)$$
$$P(0^+) < \gamma^2 R; \quad (18)$$

(c) There exists a bounded matrix function $Q(t) = Q^T(t) > 0$, $\forall\, t \in [0, T]$, such that

$$-\dot{Q} > A^T Q + QA + \gamma^{-2} QBB^T Q + C^T C, \ t \ne i,$$
$$Q(T) > 0 \quad (19)$$
$$\gamma^2 I - B_d^T Q(i^+) B_d > 0 \quad (20)$$
$$Q(i) > A_d^T Q(i^+) A_d + A_d^T Q(i^+) B_d [\gamma^2 I - B_d^T \cdot Q(i^+) B_d]^{-1} B_d^T Q(i^+) A_d + C_d^T C_d \quad (21)$$
$$Q(0^+) < \gamma^2 R. \quad (22)$$

Lemma 2: [8]: Let A, E, F, H and M be real matrices of appropriate dimensions with M being symmetric. Then,

(a) For any scalar $\epsilon > 0$ and for all matrices F satisfying $F^TF \leq I$,

$$HFE + E^TF^TH^T \leq \frac{1}{\epsilon}HH^T + \epsilon E^TE;$$

(b) There exists a matrix $P = P^T > 0$ such that

$$[A + HFE]^T P[A + HFE] + M < 0$$

for all matrices F satisfying $F^TF \leq I$, if there exists some $\epsilon > 0$ such that the following conditions are satisfied

(i) $\epsilon^{1/2}H^TPH < I$

(ii) $A^TPA + A^TPH[\epsilon I - H^TPH]^{-1}H^TPA + \epsilon E^TE + M < 0$;

(c) For any scalar $\epsilon > 0$ such that $\epsilon^2E^TE \leq I$ and for all matrices F satisfying $F^TF \leq I$,

$$[A + HFE][A + HFE]^T \leq A(I - \epsilon E^TE)^{-1}A^T + \frac{1}{\epsilon}HH^T.$$

III. Robust H_∞ Fir sampled-data filters with known lipschitz nonlinearity

In this section, the robust H_∞ FIR filtering problem for the system (1)-(4) is considered.

First, a related performance analysis problem for a class of uncertain systems with finite discrete jumps is considered. The motivation for this is that, as will be shown later, the robust performance analysis for the estimation error associated with (1)-(4) and a linear sampled-data filter can be recast into the robust performance analysis of an uncertain system with finite discrete jumps of the form

$$\begin{aligned}(\Sigma_2) : \dot{x}(t) &= [A(t) + \triangle A(t)]x(t) + [G(t) + \triangle G(t)] \\ &\quad \cdot g[x(t)] + B(t)w(t), \quad t \neq i \qquad (23)\\ x(i) &= [A_d(i) + \triangle A_d(i)]x(i^-) + [K(i) \\ &\quad + \triangle K(i)]k[x(i^-)] + B_d(i)v(i) \qquad (24)\\ z(t) &= C(t)x(t) \qquad (25)\\ z_d(i) &= C_d(i)x(i), \qquad (26)\end{aligned}$$

where $x(t) \in \Re^n$ is the state, $w(t) \in \Re^p$ and $v(i) \in \Re^q$ are the continuous and discrete inputs, respectively, $z(t) \in \Re^r$ and $z_d(i) \in \Re^s$ are continuous and discrete outputs, respectively, $A(t)$, $A_d(i)$, $B(t)$, $B_d(i)$, $C(t)$, $C_d(i)$, $G(t)$ and $K(i)$ are known real time-varying bounded matrices with $A(t)$, $B(t)$, $C(t)$ and $G(t)$ being piecewise continuous, $g(\cdot) : \Re^n \rightarrow \Re^{n_g}$ and $k(\cdot) : \Re^m \rightarrow \Re^{m_k}$ are known Lipschitz nonlinear functions satisfying Assumption 1., and $\triangle A_d(i)$ is of the form

$$\triangle A_d = H_{d1}F_{d1}E_{d1}, \qquad (27)$$

where $E_{d1} \in \Re^{\beta \times n}$ and $H_{d1} \in \Re^{n \times \alpha}$ are known real time-varying bounded matrices and $F_{d1} \in \Re^{\alpha \times \beta}$ is an unknown matrix function satisfying

$$F_{d1}^TF_{d1} \leq I, \quad \forall i = 0, 1, 2, ... \qquad (28)$$

The robust performance analysis problem for the system (23)-(26), is as follows:

Given a scalar $\gamma > 0$, find condition which guarantee that

$$\{\|z\|^2 + \|z_d\|^2\} < \gamma^2\{\|w\|^2_{[t-T,t]} + \|v\|^2_{(i-T,i)}\} \qquad (29)$$

for all non-zero $(w, v, x_0) \in L_2[0,\infty) \oplus l_2(0,\infty) \oplus \Re^n$ and for all admissible uncertainties, where $R = R^T > 0$ is a given weighting matrix for x_0.

In such a situation, the system (23)-(26) is said to have robust H_∞ performance γ over the moving horizon $[t - T, t]$. In order to solve the above robust performance analysis problem, the following system associated with (23)-(26) is considered.

$$\begin{aligned}(\Sigma_2^a) : \dot{x}(t) &= Ax(t) + [\bar{B}(\epsilon_1, \epsilon_2) \quad \gamma^{-1}B]\tilde{w}(t) \qquad (30)\\ x(i) &= A_dx(i^-) + [\bar{B}_d(\epsilon_3, \epsilon_4) \quad \gamma^{-1}B_d]\tilde{v}(i) \qquad (31)\\ \tilde{z}(t) &= [\bar{C}^T(\epsilon_1) \quad C^T]^Tx(t) \qquad (32)\\ \tilde{z}_d(i) &= [\bar{C}_d^T(\epsilon_3) \quad C_d^T]^Tx(i), \qquad (33)\end{aligned}$$

where $x(t) \in \Re^n$ is the state, $\tilde{w}(t) \in \Re^{p+i+n_G}$ and $\tilde{v}(t) \in \Re^{q+j+n_K}$ are the continuous and discrete inputs, respectively, $\tilde{z} \in \Re^{r+j}$ and $\tilde{z}_d \in \Re^{s+j}$ are the continuous and discrete outputs, respectively, $\gamma > 0$ is a desired 'H_∞ performance' for system (23)-(26), A, A_d, B, B_d, C and C_d are the same as in (23)-(26), and $\bar{B}(\epsilon_1, \epsilon_2)$, $\bar{B}_d(\epsilon_3, \epsilon_4)$, $\bar{C}(\epsilon_1)$ and $\bar{C}_d(\epsilon_3)$ satisfy the following relations

$$\begin{aligned}\bar{B}(\epsilon_1, \epsilon_2)^T\bar{B}(\epsilon_1, \epsilon_2) &= \frac{1}{\epsilon_1^2}HH^T + \frac{1}{\epsilon_2^2}H_GH_G^T \\ &\quad + G(I - \epsilon_2^2E_G^TE_G)^{-1}G^T \qquad (34)\\ \bar{B}_d(\epsilon_3, \epsilon_4)^T\bar{B}_d(\epsilon_3, \epsilon_4) &= \frac{1}{\epsilon_3^2}H_{d1}H_{d1}^T + \frac{1}{\epsilon_4^2}H_KH_K^T \\ &\quad + K(I - \epsilon_4^2E_K^TE_K)^{-1}K^T \qquad (35)\\ \bar{C}(\epsilon_1)^T\bar{C}(\epsilon_1) &= \epsilon_1^2E_1^TE_1 + W_g^TW_g \qquad (36)\\ \bar{C}_d(\epsilon_3)^T\bar{C}_d(\epsilon_3) &= \epsilon_3^2E_{d1}^TE_{d1} + W_k^TW_k. \qquad (37)\end{aligned}$$

In the above, E, E_G, E_K, E_{d1}, G, H, K, H_G, H_K and H_{d1} are the same as in (23)-(26), W_g and W_k are as in (9), and ϵ_1, ϵ_2, ϵ_3 and ϵ_4 are positive scaling parameters to be chosen, with $I - \epsilon_2^2E_G^TE_G > 0$ and $I - \epsilon_4^2E_K^TE_K > 0$.

The first result deals with the strict bounded realness of (23)-(26) over the finite moving horizon $[t - T, t]$. For simplicity in notation, we will regard the finite moving horizon $[t - T, t]$ as $[0, T]$.

Theorem 1: Given a scalar $\gamma > 0$, the system (23)-(26) has a robust H_∞ performance γ over the finite moving horizon $[0, T]$ if there exist positive scalars ϵ_1, ϵ_2, ϵ_3 and ϵ_4 such that $\epsilon_2^2E_G^TE_G < I$, $\epsilon_4^2E_K^TE_K < I$ and the system (30)-(33) satisfies $J(\Sigma_2^a, \gamma^2R, T) < 1$.

Proof: First, by Lemma 1, $J(\Sigma_2^a, \gamma^2R, T) < 1$ implies that there exists a matrix function $Q(t) = Q^T(t) > 0, \forall t \in [0, T]$, such that

$$-\dot{Q} > A^TQ + QA + Q\tilde{B}\tilde{B}^TQ + \bar{C}(\epsilon_1)^T\bar{C}(\epsilon_1) + C^TC,$$
$$t \neq i;\ Q(T) = 0 \quad (38)$$
$$I - \tilde{B}_d^TQ(i^+)\tilde{B}_d > 0 \quad (39)$$
$$A_d^TQ(i^+)A_d - Q(i) + A_d^TQ(i^+)\tilde{B}_d[I - \tilde{B}_d^TQ(i^+)\tilde{B}_d]^{-1} \cdot \tilde{B}_d^TQ(i^+)A_d + \bar{C}_d(\epsilon_3)^T\bar{C}_d(\epsilon_3) + C_d^TC_d < 0 \quad (40)$$
$$Q(0) < \gamma^2R, \quad (41)$$

where

$$\tilde{B} = [\bar{B}(\epsilon_1, \epsilon_2) \quad \gamma^{-1}B], \quad \tilde{B}_d = [\bar{B}_d(\epsilon_3, \epsilon_4) \quad \gamma^{-1}B_d].$$

Next, in view of Lemma 2 (a) and (c), together with equalities (34) and (36), it results from Eq. (38) that

$$\dot{Q} + (A + \triangle A)^TQ + Q(A + \triangle A) + \gamma^{-2}QBB^TQ + Q(G + \triangle G)(G + \triangle G)^TQ + C^TC + W_g^TW_g < 0,$$
$$t \neq i;\ Q(T) = 0. \quad (42)$$

Now, using the matrix inversion lemma, together with equalities (35) and (37), (40) leads to

$$Q(i) > A_d^T[Q^{-1}(i^+) - \gamma^{-2}B_{d1}B_{d1}^T - \epsilon_3^{-2}H_{d1}H_{d1}^T]^{-1}A_d + \epsilon_3^2E_{d1}^TE_{d1} + W_k^TW_k + C_d^TC_d, \quad (43)$$

where $B_{d1}B_{d1}^T = B_dB_d^T + \gamma^2K(I - \epsilon_4E_K^TE_K)^{-1}K^T + \frac{\gamma^2}{\epsilon_4^2}H_KH_K^T$. Also, note that from (39), it follows that

$$I - \gamma^{-2}B_{d1}^TQ(i^+)B_{d1} > 0. \quad (44)$$

By denoting $X^{-1} = Q^{-1}(i^+) - \gamma^{-2}B_{d1}B_{d1}^T$, (43) can be rewritten as

$$Q(i) > A_d^TXA_d + A_d^TXH_{d1}[\epsilon_3^2I - H_{d1}^TXH_{d1}]^{-1} \cdot H_{d1}^TXA_d + \epsilon_3^2E_{d1}^TE_{d1} + W_k^TW_k + C_d^TC_d. \quad (45)$$

First, by Lemma 2 (b), (45) leads to

$$Q(i) > (A_d + \triangle A_d)^TX(A_d + \triangle A_d) + W_k^TW_k + C_d^TC_d.$$

Next, by Lemma 2 (c), we have that

$$B_{d1}B_{d1}^T = B_dB_d^T + \gamma^2K(I - \epsilon_4^2E_K^TE_K)K^T + \frac{\gamma^2}{\epsilon_4^2}H_KH_K^T$$
$$\geq B_dB_d^T + \gamma^2(K + \triangle K)(K + \triangle K)^T \quad (46)$$
$$\equiv B_{d2}B_{d2}^T, \quad (47)$$

where $B_{d2} = [B_d \quad \gamma(K + \triangle K)]$.

From (44) and (47), we have that

$$I - \gamma^{-2}B_{d2}^TQ(i^+)B_{d2} > 0. \quad (48)$$

Hence

$$Q(i) > (A_d + \triangle A_d)^T(Q^{-1}(i^+) - \gamma^2B_{d2}B_{d2}^T)^{-1} \cdot (A_d + \triangle A_d) + W_k^TW_k + C_d^TC_d.$$

It then follows from the matrix inversion lemma that

$$(A_d + \triangle A_d)^TQ(i^+)(A_d + \triangle A_d) - Q(i) + (A_d + \triangle A_d)^T \cdot Q(i^+)B_{d2}[\gamma^2I - B_{d2}^TQ(i^+)B_{d2}]^{-1}B_{d2}^TQ(i^+) \cdot (A_d + \triangle A_d) + W_k^TW_k + C_d^TC_d < 0, \quad (49)$$

where $B_{d2}B_{d2}^T = B_dB_d^T + \gamma^2(K + \triangle K)(K + \triangle K)^T$. Now, in view of (23), (25) and (42), it is easily verified that for any $\tau \in (i, i+1)$,

$$\int_{i^+}^{\tau} \frac{d}{dt}(x^TQx)dt = \int_{i^+}^{\tau} x^TM(t)xdt + \gamma^2\|w\|_\tau^2 - \|z\|_\tau^2 - \|\delta\|_\tau^2 - \|\mu\|_\tau^2 - \|g(x)\|_\tau^2 - \|W_gx\|_\tau^2, \quad (50)$$

where $M(t)$ denotes the left-hand side of (42), $A_\triangle = A + HFE, G_\triangle = G + H_GF_GE_G$ and $\|\cdot\|_\tau$ means $\|\cdot\|_{[i,\tau]}$ and

$$\delta(t) = \gamma[w(t) - \gamma^{-2}B^TQx(t)]$$
$$\mu(t) = g[x(t)] - G_\triangle^TQx(t).$$

Next, considering (24) and (26) and by completing the squares, we have that

$$x^TQx\|_{i^-}^{i^+} = x^T(i^+)Q(i^+)x(i^+) - x^T(i^-)Q(i)x(i^-)$$
$$= x^T(i^-)M_d(i)x(i^-) - \|z_d(i)\|^2 + \gamma^2\|v(i)\|^2 - \|r(i)\|^2 + \|k[x(i^-)]\|^2 - \|W_kx(i^-)\|^2, \quad (51)$$

where $M_d(i)$ stands for the left-hand side of (49) and

$$r(i) = [\gamma^2I - B_{d2}^TQ(i^+)B_{d2}]^{1/2}\{\hat{v}(i) - [\gamma^2I - B_{d2}^T \cdot Q(i^+)B_{d2}]^{-1}B_{d2}^TQ(i^+)(A_d + \triangle A_d)x(i^-)\}$$
$$\hat{v}(i) = [v(i) \quad \gamma^{-1}k[x(i^-)]]^T.$$

By combining (50) and (51) over all possible i in $(0, T)$ and considering that $M(t) < 0, \forall t \neq i, M_d(i) < 0, \forall i \in (0, T)$, we obtain that

$$x(t)^TQ(t)x(t)\ |_0^T < \gamma^2[\|w\|^2 + \|v\|^2] - [\|z\|^2 + \|z_d\|^2] + \|g(x)\|^2 - \|W_gx\|^2 + \|k(x)\|^2 - \|W_kx\|^2. \quad (52)$$

By taking into account Assumption 1 and the fact that $Q(T) = 0$ and $Q(0) < \gamma^2R$, (52) implies that

$$\|z\|^2 + \|z_d\|^2 < \gamma^2[\|w\|^2 + \|v\|^2 + x_0^TRx_0],$$

whenever $\|w\|^2 + \|v\|^2 + x_0^TRx_0 \neq 0$ for all admissible uncertainties, which completes the proof. ■

Now, a solution to the robust H_∞ FIR filtering problem for system (1)-(4) over a moving horizon $[0, T]$ is provided.

Corollary 1: Consider the system (1)-(4) satisfying Assumption 1. Given a scalar $\gamma > 0$ and an initial state weighting matrix $R = R^T > 0$, the robust H_∞ FIR sampled-data filtering problem over a moving horizon $[0, T]$ is solvable if there exist positive scalars ϵ_1, ϵ_2, ϵ_3 and ϵ_4 such that $\epsilon_2^2E_G^TE_G < I$, $\epsilon_4^2E_K^TE_K < I$ and the following conditions are satisfied:

(a) There exists a bounded solution $P(t) = P^T(t) \geq 0$ over the moving horizon $[0, T]$ to the Riccati differential equation with jumps

$$\dot{P}(t) + A^T P(t) + P(t)A + \gamma^{-2}P(t)[\hat{B}\hat{B}^T + BB^T]P(t) + \epsilon_1^2 E^T E + W_g^T W_g = 0, \quad t \neq i \tag{53}$$

$$P(i) = P(i^+) + \epsilon_3^2 E_d^T E_d + W_k^T W_k \tag{54}$$

with terminal condition $P(T) = 0$ and such that $P(0^+) < \gamma^2 R$, where

$$\hat{B} = \begin{bmatrix} \frac{\gamma}{\epsilon_1}H & \gamma G(I - \epsilon_2^2 E_G^T E_G)^{-1/2} & \frac{\gamma}{\epsilon_2}H_G \end{bmatrix}. \tag{55}$$

(b) There exists a bounded solution $S(t)$ over the moving horizon $[0, T]$ to the Riccati differential equation with jumps

$$\dot{S}(t) = \hat{A}S(t) + S(t)\hat{A}^T + \gamma^{-2}S(t)\hat{L}^T\hat{L}S(t) + \hat{B}\hat{B}^T + BB^T, \tag{56}$$

$$S(i) = [S^{-1}(i^-) - \gamma^{-2}\hat{L}_d^T\hat{L}_d + C^T V^{-1} C]^{-1} \tag{57}$$

with initial condition $S(0) = [R - \gamma^{-2}P(0)]^{-1}$, where

$$\hat{L}^T\hat{L} = L^T L + W_g^T W_g, \quad \hat{L}_d^T\hat{L}_d = L_d^T L_d + W_k^T W_k,$$

$$\hat{A}(t) = A + (\gamma^{-2}BB^T + \epsilon_1^{-2}HH^T)P(t) \tag{58}$$

$$V = \begin{bmatrix} -I & 0 \\ 0 & \hat{D}\hat{D}^T \end{bmatrix}$$

$$\hat{D} = \begin{bmatrix} D & \frac{\gamma}{\epsilon_3}H_d & \gamma K(I - \epsilon_4^2 E_K^T E_K)^{-1/2} & \frac{\gamma}{\epsilon_4}H_K \end{bmatrix}. \tag{59}$$

Moreover, if conditions (a) and (b) are satisfied, a suitable filter is given by

$$\dot{\hat{x}}(t) = \hat{A}\hat{x}(t) + Gg[\hat{x}(t)] \tag{60}$$

$$\hat{x}(i) = \hat{x}(i^-) + S(i)C^T V^{-1}\{y(i) - C\hat{x}(i^-) - Kk[\hat{x}(i^-)]\} \tag{61}$$

$$\hat{z}(t) = L\hat{x}(t) \tag{62}$$

$$\hat{z}_d = L_d\hat{x}(i). \tag{63}$$

Proof: First, associated with (1)-(4) and (60)-(63), we define $\tilde{x} \equiv x - \hat{x}$. Since $x(i) = x(i^-)$, from (1)-(4) and (60)-(63), we have that

$$\dot{\tilde{x}}(t) = [A + \triangle A_e]\tilde{x}(t) + [\triangle A - \triangle A_e]x(t) + Bw(t) + [G + \triangle G]g(x) - Gg(\hat{x})$$

$$\tilde{x}(i) = A_d\tilde{x}(i^-) + B_d \triangle Cx(i^-) + B_d K(k[x(i^-)] - k[\hat{x}(i^-)]) + B_d \triangle Kk[x(i^-)] + B_d Dv(i),$$

where

$$A_d = I - S(i)C^T V^{-1} C, B_d = -S(i)C^T V^{-1},$$

$$\triangle A_e(t) = (\gamma^{-2}BB^T + \epsilon_1^{-2}HH^T)P(t).$$

Hence, we have the following estimation error dynamics for the estimator error $z - \hat{z}$ and $z_d - \hat{z}_d$ is as follows

$$\dot{\eta}(t) = [A_e + H_e F(t) E_e]\eta(t) + B_e w(t) + [G_e + H_{ge}F_G E_{ge}]g_e(x, x_e) \tag{64}$$

$$\eta(i) = [A_{de} + H_{de}F_d E_{de}]\eta(i^-) + B_{de}v(i) + [K_e + H_{ke}F_K E_{ke}]k_e(x, x_e) \tag{65}$$

$$z(t) - \hat{z}(t) = L_e\eta(t) \tag{66}$$

$$z_d(i) - \hat{z}_d(i) = L_{de}\eta(i), \tag{67}$$

where $\eta = [x^T \quad \tilde{x}^T]^T$ and

$$A_e = \begin{bmatrix} A & 0 \\ -\triangle A_e & A + \triangle A_e \end{bmatrix}, \; A_{de} = \begin{bmatrix} I & 0 \\ 0 & A_d \end{bmatrix},$$

$$B_e = \begin{bmatrix} B \\ B \end{bmatrix}, \; B_{de} = \begin{bmatrix} 0 \\ B_d D \end{bmatrix}, \; H_{ke} = \begin{bmatrix} 0 \\ B_d H_K \end{bmatrix},$$

$$H_{dc} = \begin{bmatrix} 0 \\ B_d H_d \end{bmatrix}, \; H_{gc} = \begin{bmatrix} H_G \\ H_G \end{bmatrix}, \; H_c = \begin{bmatrix} H \\ H \end{bmatrix},$$

$$E_e = [E \quad 0], \; E_{de} = [E_d \quad 0], \; E_{ge} = [E_G \quad 0],$$

$$E_{ke} = [E_K \quad 0], \; L_e = [0 \quad L_e], \; L_{de} = [0 \quad L_d],$$

$$G_e = \begin{bmatrix} G & 0 \\ 0 & G \end{bmatrix}, \; K_e = \begin{bmatrix} 0 & 0 \\ 0 & B_d K \end{bmatrix},$$

$$g_e(x, x_e) = \begin{bmatrix} g(x) \\ g(x) - g(x_e) \end{bmatrix},$$

$$k_e(x, x_e) = \begin{bmatrix} k[x(i^-)] \\ k[x(i^-)] - k[x_e(i^-)] \end{bmatrix}.$$

Note that by Assumption 1,

$$\|g_e(x, x_e)\| \leq \|\hat{W}_g\eta\|, \forall\eta \in \Re^{2n}$$

$$\|k_e(x, x_e)\| \leq \|\hat{W}_k\eta\|, \forall\eta \in \Re^{2n}, \tag{68}$$

where

$$\hat{W}_g = \begin{bmatrix} W_g & 0 \\ 0 & W_g \end{bmatrix}, \; \hat{W}_k = \begin{bmatrix} W_k & 0 \\ 0 & W_k \end{bmatrix}. \tag{69}$$

From Theorem 3.1 in [7], condition (b) is necessary and sufficient for the solvability of the moving horizon H_∞ FIR filtering problem for the linear system with sampled measurements

$$\dot{\xi}(t) = \hat{A}\xi(t) + [\hat{B} \quad B]\hat{w}(t) \tag{70}$$

$$\hat{y}(i) = C\xi(i) + \hat{D}\hat{v}(i) \tag{71}$$

$$z_e(t) = L\xi(t) \tag{72}$$

$$z_{ed}(i) = L_d\xi(i), \tag{73}$$

where $\xi \in \Re^n$ is the state, ξ_0 is an unknown initial state, $\hat{w} \in \Re^{p+i+n_G}$ is the process noise, $\hat{y}(i) \in \Re^m$ is the sampled measurement, $\hat{v}(i) \in \Re^{q+\alpha+m_K}$ is the measurement noise, $z_e \in \Re^r$ and $z_{ed} \in \Re^s$ are linear combinations of the state variables to be estimated, and the filtering performance measure is given by

$$\sup\left\{\left[\frac{\|z_e - \hat{z}_e\|^2 + \|z_{ed} - \hat{z}_{ed}\|^2}{\|\hat{w}\|_{[0,T]}^2 + \|\hat{v}\|_{(0,T)}^2 + \xi_0^T[R - \gamma^{-2}P(0)]\xi_0}\right]^{1/2}\right\}, \tag{74}$$

where $\hat{z}_e$ and $\hat{z}_{ed}$ are the estimates of z_e and z_{ed}, respectively. In the above, the supremum is taken over all $(\hat{w}, \hat{v}, \xi_0) \in L_2[0,\infty) \oplus l_2(0,\infty) \oplus \Re^n$ such that $\|\hat{w}\|^2_{[0,T]} + \|\hat{v}\|^2_{(0,T)} + \xi_0^T[R - \gamma^{-2}P(0)]\xi_0 \neq 0$. Also, observe that suitable estimates $\hat{z}_e$ and $\hat{z}_{ed}$ are given by

$$\begin{aligned}(\Sigma_{e1}) : \dot{\hat{\xi}}(t) &= \hat{A}\hat{\xi}(t), \quad t \neq i; \quad \hat{\xi}(0) = 0\\ \hat{\xi}(i) &= \hat{\xi}(i^-) + S(i)C^T V^{-1}[\hat{y}(i) - C\hat{\xi}(i^-)]\\ \hat{z}_e(t) &= L\hat{\xi}(t)\\ \hat{z}_{ed}(i) &= L_d\hat{\xi}(i).\end{aligned}$$

Now, letting $\tilde{\xi} = \xi - \hat{\xi}$, it follows from the system (Σ_{e1}) and (70)-(73) that

$$\begin{aligned}\dot{\tilde{\xi}}(t) &= \hat{A}\tilde{\xi}(t) + [\hat{B} \quad B]\hat{w}(t), \quad \tilde{\xi}(0) = \xi_0\\ \tilde{\xi}(i) &= A_d\tilde{\xi}(i^-) + B_d\hat{D}\hat{v}(i)\\ z_e(t) - \hat{z}_e(t) &= L\tilde{\xi}(t)\\ z_{ed}(i) - \hat{z}_{ed}(i) &= L_d\tilde{\xi}(i).\end{aligned}$$

Since the above system satisfies (74) by Lemma 1, this implies that there exists a bounded matrix $Z(t) = Z^T(t) \geq 0$, satisfying the following Riccati differential equation with jumps

$$\dot{Z}(t) + \hat{A}^T Z(t) + Z(t)\hat{A} + \gamma^{-2}Z(t)[\tilde{B}\tilde{B}^T + BB^T] \cdot Z(t) + L^T L = 0, \quad t \neq i; \quad Z(T) = 0 \tag{75}$$

$$\gamma^2 I - \hat{D}^T B_d^T Z(i^+) B_d \hat{D} > 0, \quad i \in (0,T) \tag{76}$$

$$Z(i) = A_d^T Z(i^+) A_d + A_d^T Z(i^+) B_d \hat{D}[\gamma^2 I - \hat{D}^T \cdot B_d^T Z(i^+) B_d \hat{D}]^{-1} \hat{D}^T B_d^T Z(i^+) A_d + L_d^T L_d \tag{77}$$

$$Z(0) < \gamma^2 R - P(0). \tag{78}$$

Next, let

$$X(t) = \begin{bmatrix} P(t) & 0 \\ 0 & Z(t) \end{bmatrix},$$

where $P(t)$ and $Z(t)$ are the non-negative definite solution of (53) and (54) and (75)-(78), respectively. Note that since $Z(0) < \gamma^2 R - P(0)$, there exists a sufficiently small scalar $\delta > 0$ such that

$$X(0) < X_0 = \begin{bmatrix} P(0^+) + \delta I & 0 \\ 0 & \gamma^2 R - P(0^+) - \delta I \end{bmatrix}.$$

It is straightforward to verify that there exists a matrix $X(t) = X^T(t) \geq 0, \forall t \in [0,T]$ satisfying the following Riccati differential equation with jumps

$$\dot{X}(t) + A_e^T X(t) + X(t)A_e + X(t)\hat{B}_e\hat{B}_e^T X(t) + \hat{C}_e^T C_e = 0, \quad t \neq i; \quad X(T) = 0 \tag{79}$$

$$I - \hat{B}_{de}^T X(i^+)\hat{B}_{de} > 0, \quad i \in (0,T) \tag{80}$$

$$X(i) = A_{de}^T X(i^+) A_{de} + A_{de}^T X(i^+)\hat{B}_{de}[I - \hat{B}_{de}^T \cdot X(i^+)\hat{B}_{de}]^{-1}\hat{B}_{de}^T X(i^+) A_{de} + \hat{C}_{de}^T \hat{C}_{de}, \tag{81}$$

$$X(0) < X_0, \tag{82}$$

where

$$\hat{B}_e = [\bar{B}_e \quad \gamma^{-1}B_e], \quad \hat{B}_{de} = [\bar{B}_{de} \quad \gamma^{-1}B_{de}],$$
$$\hat{C}_e = [\bar{C}_e \quad L_e]^T, \quad \hat{C}_{de} = [\bar{C}_{de} \quad L_{de}]^T,$$

and δ being a positive number with $\bar{B}_e$, $\bar{B}_{de}$, $\bar{C}_e$ and $\bar{C}_{de}$ such that

$$\bar{B}_e\bar{B}_e^T = G_e(I - \epsilon_2^2 E_{ge}^T E_{ge})^{-1}G_e^T + \frac{1}{\epsilon_1^2}H_e H_e^T + \frac{1}{\epsilon_2^2}H_{ge}H_{ge}^T \tag{83}$$

$$\bar{B}_{de}\bar{B}_{de}^T = K_e(I - \epsilon_4^2 E_{ke}^T E_{ke})^{-1}K_e^T + \frac{1}{\epsilon_3^2}H_{de}H_{de}^T + \frac{1}{\epsilon_4^2}H_{ke}H_{ke}^T \tag{84}$$

$$\bar{C}_e^T\bar{C}_e = \epsilon_1^2 E_e^T E_e + \hat{W}_g^T\hat{W}_g \tag{85}$$

$$\bar{C}_{de}^T\bar{C}_{de} = \epsilon_3^2 E_{de}^T E_{de} + \hat{W}_k^T\hat{W}_k, \tag{86}$$

where $\hat{W}_g$ and $\hat{W}_k$ are as in (69).

By Lemma 1, (79)-(82) implies that the system as below

$$(\Sigma_3) : \dot{\xi}(t) = A_e\xi(t) + \hat{B}_e\hat{w}(t) \tag{87}$$

$$\xi(i) = A_{de}\xi(i^-) + \hat{B}_{de}\hat{v}(i) \tag{88}$$

$$z_e(t) = \hat{C}_e\xi(t) \tag{89}$$

$$z_e(i) = \hat{C}_{de}\xi(i) \tag{90}$$

satisfies $J(\Sigma_3, \gamma^2, \hat{X}_0, T) < I$, where $\hat{X}_0 = \gamma^{-2}X_0$. Finally, by considering the system (64)-(67) and (87)-(90), and the fact that the initial state of (64) satisfies $\eta^T(0)\hat{X}_0\eta(0) = x_0^T R x_0$, we conclude that the estimation error dynamics (64)-(67) satisfy

$$\{\|z - \hat{z}\|^2 + \|z_d - \hat{z}_d\|^2\} < \gamma^2\{\|w\|^2_{[0,T]} + \|v\|^2_{(0,T)} + x_0^T R x_0\} \tag{91}$$

for all non-zero $(w, v, x_0) \in L_2[0,\infty) \oplus l_2(0,\infty) \oplus \Re^n$ and for all admissible uncertainties. ■

IV. Conclusions

In this paper, the problem of robust H_∞ FIR filtering problem under sampled data system with known Lipschitz nonlinearity is addressed. Attention is focused on the simultaneous estimation of a continuous and discrete time-varying signal using a performance measure which involves a mixed L_2/l_2 norm of the estimation error for the continuous and discrete time-varying signals. The causal filters on the moving horizon, which provide a guaranteed H_∞ performance, is developed. But the strict performance analysis and computation burden of the robust H_∞ FIR proposed requires further research.

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