# Estimation of a Mass Unbalance Under the Crack on the Rotating Shaft

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**Abstract:** The aim of this work is to present a new method of estimating the existence of a mass unbalance and mass unbalance under a crack on a rotating shaft. This is an advanced new method for the detection of a mass unbalance and a new way to estimate the position of it under crack influence. As the first step, the shaft is physically modelled with a finite element method and the dynamic mathematical model is derived by using the Hamilton principle; thus, the system is represented by various subsystems. The equation of motion of the shaft with a mass unbalance and a crack are established by adapting the local mass unbalance and the stiffness change. This is a reference system for the given system. Based on a model for transient behavior induced from vabrations measured at the bearings, an elementary Estimator is designed to detect mass unbalance on the shaft. Using the Estimator, a bank of the Estimator is established to estimate the position of the mass unbalance and arranged at a certain location on the shaft. The informations for the given system are the measurements of bearing displacements and velocity.

**Keywords:** dynamic behavior, mass unbalance, estimator, a bank of estimator, position of mass unbalance

## I. Introduction

Very often, a mass unbalance results in a bearing damage which is very dangerous in rotor system. This can lead to a catastrophe. Actually, there have been many reports to these disasters [1]-[3]. Especially, it happens with the generator in the power station and with the loading pump in a chemical plant. As an aspect to keep the stability of a system, to guarantee the safety for the men and to save the running cost, it is very important to estimate the mass unbalances and mass unbalances under a crack infulence on the shaft before meeting any demolition of the system. Here, a suitable step must be taken. But, up to now, there has been not any work in this area. Only, some of the classical methods for detecting a crack [4] and a new way to detect crack and to estimate position of a crack on the rotating shaft are mentioned [5]. Therefore, in this paper, a new method for estimating the mass unbalances during the process of operation and those under crack influence in running operation is presented. As an indicator for the existence of them, the linear dynamics due to mass unbalance and the nonlinear dynamic effects appeared by the change of the stiffness coefficients due to the rotation of the cracked shaft are going to be investigated. These effects related to the measurement on the bearing are one of the important hints to determine the existence of the mass unbalance on the rotating shaft. But it is not easy to set up the clear relation between mass unbalance under the crack and the caused phenomena in the time domain operation. This is the main problem in the area of a mass unbalance too.

As the first step, the basic Estimator is established in the way to modify the given system into the extended system with a fictitious model for the nonlinear system behavior. In this consideration, the effects of the extended system which may be nonlinear are interpreted as internal or external disturbance which is unknown at the initial stage.

The unknown linear and nonlinear effects are going to be approximated by the additional time signals yielded by an elementary Estimator. Because of using the finite element model, it is not necessary to calculate the relative compliance of the crack. Normally, the elementary stiffness matrix for an undamaged rotor is given in the stage of the construction. The stiffness corresponding to the crack can be calculated [6]-[8].

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Fig. 1. Mechanical model of the rotor.

As an example of the physical model, the shaft is modelled into N(=7) finite sub-shafts [5][9]. Each one is called a subsystem. At both ends of the shaft there exist dynamics of the bearings. For the initial data needed in the operating system, the displacements of the jounals are measured up on the bearings at the left and the right side of the shaft. It is assumed that the mass unbalance and the crack have the same direction on the radius of the shaft diameter [5][10]. The material properties are homogenous. The geometrical data and other detailed information are given in the appendix.

## II. Mathematical Model

Assuming that there is only small deviation from motion and non redundant coordinate [11][12], the system (Fig. 1) includes 3 harmonic unbalances in the 3rd, 4th and 5th subsystems in the middle of the shaft. Then the following equation can be accepted as linear system.

$$\begin{aligned} &M_g\, \ddot{q}(t) + (D_{dg} + G_g)\, \dot{q}(t) + K_g\, q(t) = \\ &\quad f_g(t) + Ls(j)\, n_R(t) + Lu(j)\, n_u(q(t), t), \end{aligned} \tag{1}$$

Here, the index g denotes the whole system. Equation (1) is able to be discretized into N(=7) sub - finite systems. Its equation of motion with crack at a subsystem $i_e$ is described by

$$i_e = 1, ..., N \tag{2}$$

$$j_k(i_e) = \left[(i_e - 1)\, \frac{n}{2} + 1\right]_{(i_e=1,\ldots,N)} \tag{3}$$

$$i = j_k, ..., j_k + n - 1 \quad (4)$$
$$j = j_k, ..., j_k + n - 1. \quad (5)$$

With $i_e$, $j_k$, i, and j the vector in explicit form and the equation of motion can be given as follows:

$$q_{(i_e+1)(i)_{(i=1,..,\frac{n}{2}+1)}} = q_{(i_e-1)(\frac{n}{2}+i)} \quad (6)$$

$$\sum_{i_e=1}^{N} \sum_{j_k=j_k(i_e)}^{j_k(i_e)+n-1} [M_e \ddot{q}_{j_k(i_e)}(t) + (D_e + G_e)\dot{q}_{j_k(i_e)}(t) + K_e q_{j_k(i_e)}(t)] = f_g(t)(i_e)_{(i_e=1,...,N)} + Lu(n_u, i_e)[n_u(q_{(t)}(t), t)]_{(i_e=1,...,N)} + Ls(n_f, i_e)[n_R(t)]_{(i_e=1,...,N)}, \quad (7)$$

where, the index of e represents the elementary subsystem. The elementary notations in the equations denote as follows:

- $q(t)$, $\dot{q}(t)$, $\ddot{q}(t)$: displacement vector, velocity vector, and acceleration of the system
- $M_g, K_g$ : mass matrix, stiffness matrix of undamaged section
- $D_{dg},\ G_g = -G_g^T$: matrix of the damping and gyroscopic matrix
- $q_e(t)$, $\dot{q}_e(t)$, $\ddot{q}_e(t)$: displacement vector, velocity vector, and acceleration of the elementary subsystems. $q_e(t) \in \Re^n$, n(=8), and nn(=32) are degrees of freedom of the considered elementary subsystem and total system. The $q_e(t)$ consists of $q_e(t) = (x_l, y_l, \theta_{xl}, \theta_{yl};\ x_r, y_r, \theta_{xr}, \theta_{yr})$, the indices l and r denote the left and the right node and $x_r, y_r, \theta_{xr}, \theta_{yr}$ are the coordinates at the subsystem
- $f_g(t)$, n(q(t),t) : vector of gravitation input vector, and vector of the nonlinearities caused by unexpected influence(crack)
- $M_e, K_e$ : elelmentary mass matrix, stiffness matrix of undamaged secton with respect to a subsystem $i_e$
- $D_{de},\ G_e = -G_e^T$ : matrix of the damping , gyroscopic effects
- $n_{(R)}(t), n_{(u)}(t)$: nonlinear vector with regard to the crack at subshaft number $i_e$ and linear vector acording to the mass unbalance

All system matrices are constant in terms of time t and the distribute matrix for the crack, and the mass unbalance [5][9] is given in the following way:

$$Ls(j) = \underbrace{\begin{bmatrix} 000 & ,..., & \overbrace{1000}^{i_e.th\ position} & ,..., & 000 \\ 000 & ,..., & \underbrace{0100}_{i_e.th\ position} & ,..., & 000 \end{bmatrix}^T}_{(2\times N)} \quad (8)$$

$$Lu(j) = \begin{bmatrix} 000 & ,..., & \underbrace{1100}_{i_e.th\ position} & ,..., & 000 \end{bmatrix}^T. \quad (9)$$

From now on, the index j will be left out with respect to the whole dynamic system. It is normally convenient for further operation to write the above equation via state space notation with $x(t) = [q(t)^T, \dot{q}(t)^T]$ including the nonlinearities of the motion created by a crack.

$$\dot{x}(t) = A\, x(t) + Bu(t) + N_R n_R(t) + N_u n_u(x(t), t). \quad (10)$$

The equation of the measurement is given by

$$y = C\, x(t), \quad (11)$$

where, A is $(N_n \times N_n)$ dimensional system matrix which is responsible for the system dynamic with $N_n = 2nn$, u(t) denotes $r$ -dimensional vector of the excitation inputs due to gravitation and unbalances. B is $(N_n \times r)$ dimensional input matrix. The matrix C presents $(m_e \times N_n)$-dimensional measurement matrix. x(t) is $N_n$ -dimensional state vector, and y(t) is $m_e$ -dimensional vector of measurements, respectively. Here, the vector $n_R(t)$ and $n_u(x(t), t)$ characterize the $n_f$ -dimensional vector of nonlinear functions due to the mass unbalance and the crack, respectively. $N_u$ and $N_R$ are the input matrices of the linear and the nonlinearties, and the order of $N_R$ is of $(Nn \times n_f)$. It is presupposed that the matrices A, B, C, $N_R$ , the vector u(t), and y(t) are already known. Now it remains to reconstruct the unknown linear vector $n_u(x(t), t)$ and nonlinear vector $n_R(t)$ which mention the disturbance force caused by a mass unbalance and crack. The basic idea is to get the signals from $n_R(t)$ and $n_u(x(t), t)$ approximated by the fictitious model [14]

$$n_R(t) \approx H_1\, v_1 \quad (12)$$
$$\dot{v}_1(t) = V_1 v_1(t) \quad (13)$$
$$n_u(x(t), t) \approx H_2\, v_2 \quad (14)$$
$$\dot{v}_2 = V_2 v_2(t) \quad (15)$$
$$H_1 = I, \quad (16)$$
$$H_2 = [1\ \ 0] \quad (17)$$
$$V_1 = 0 \quad (18)$$
$$V_2 = \begin{bmatrix} 0 & 1 \\ -\Omega^2 & 0 \end{bmatrix}. \quad (19)$$

that describes the time behaviour of the nonlinearities due to the appearance of the crack approximately as follows:

$$n_R(t) \approx \hat{n}_R(t) = H_1 \hat{v}_1(t) \quad (20)$$
$$n_u(x(t), t) \approx \hat{n}_u(\hat{x}(t)) = H_2 \hat{v}_2(t) \quad (21)$$

At first, the given system (10) has to be extended with the fictitious model(12, 13, 16, 15, 18, 17) into the extended model

$$\underbrace{\begin{bmatrix} \dot{x}(t) \\ \dot{v}_1(t) \\ \dot{v}_2(t) \end{bmatrix}}_{\dot{x}_e(t)} = \underbrace{\begin{bmatrix} A & N_R H_1 & N_u H_2 \\ 0 & V_1 & 0 \\ 0 & 0 & v_2 \end{bmatrix}}_{A_e} \underbrace{\begin{bmatrix} x(t) \\ v_1(t) \\ V_2(t) \end{bmatrix}}_{x_e(t)} + \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix} \tilde{u}(t) \quad (22)$$

$$y(t) = \underbrace{\begin{bmatrix} C & 0 & 0 \end{bmatrix}}_{C_e} \begin{bmatrix} x(t) \\ v_1(t) \\ v_2(t) \end{bmatrix}, \quad (23)$$

here, $N_u H_2$ and $N_R H_1$ couple the fictitious model (12, 16, 18) to the whole system. To enable the successful estimation it is obligatory to pay attention to the condition $m_e \geq n_f$. i.e., the number of the measurements must be at least equal to or greater than the modelled nonlinearities. In the case the above requirements are satisfied, the elementary Estimator (Fig. 2) in terms of an identity observer [5] can be designed.

Fig. 2. Elementary estimator.

The Fig.2 shows the procedure of the establishment of an elementary Estimator which has the same input signal as a plant. It consists of plant, fictitious model and input signal. The matrices $H_1, H_2, V_1$, and $V_2$ have to be chosen according to the technical background considered in terms of oscillator model for mass unbalance and integrator model for crack [16][18]. In this way, the additional forces created by mass unbalance and by crack are going to be reconstructed through the estimation of the disturbance vector $v_1(t)$ and $v_2(t)$. To make signals $v_1(t)$ and $v_2(t)$ available, it is necessary to design the elementary Estimator.

$$\begin{bmatrix} \hat{x}_i(t) \\ \hat{v}_1(t) \\ \hat{v}_2(t) \end{bmatrix} = \underbrace{\begin{bmatrix} A - L_x\, C & N_R\, H_1 & N_u H_2 \\ -L_{v1}\, C & V_1 & 0 \\ -L_{v2}\, C & 0 & V_2 \end{bmatrix}}_{A_o} \tag{24}$$

$$\begin{bmatrix} \hat{x}_i(t) \\ \hat{v}_1(t) \\ \hat{v}_2(t) \end{bmatrix} + \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix} \tilde{u}(t) + \begin{bmatrix} L_x \\ L_{v1} \\ L_{v2} \end{bmatrix} y(t)$$

For the Estimator, the requirement

$$Rank \begin{bmatrix} \lambda I_{N_n} - A & -N_R H_1 & -N_u H_2 \\ 0 & \lambda I_{s1} - V_1 & 0 \\ 0 & 0 & \lambda I_{s2} - V_2 \\ C & 0 & 0 \end{bmatrix}$$

$$= dim(x(t)) + dim(v_1(t)) + dim(v_2(t))$$

$$= N_n + n_f + 2\, n_f \;\; \forall\, \lambda \in C^+,$$

and the requirement of the controlability

$$Rank \begin{bmatrix} \lambda I_{N_n} - A & B \end{bmatrix} = N_n, \tag{25}$$

must be satisfied. The output equation for the measurement is presented as follow:

$$\hat{y}(t) = \underbrace{\begin{bmatrix} C & 0 & 0 \end{bmatrix}}_{C_e} \begin{bmatrix} \hat{x}(t) \\ \hat{v}_1(t) \\ \hat{v}_2(t) \end{bmatrix}. \tag{26}$$

Where matrices $L_x$, $L_{v1}$ and $L_{v2}$ are the gain matrix of the observer. The above equation (24, 26) means that the observer consists of a simulated model with a correction feedback of the estimation error between real and simulated measurements. The matrix $A_o$ has $(N_n + n_f \times N_n + n_f)$ -dimensions and represents the dynamic behaviour of the elementary observer. The asymptotic stability of the elementary observer can be guaranteed by a suitable design of the gain matrices $L_x$ and $L_v$ which are possible under the conditions of detectability or observabilty of the extended system (22, 23). To achieve the successfull estimation under the asymptotic stability, and to make the dynamic of the observer faster than the dynamic of the system, the eigenvalue of the considerd Estimator $(A_o)$ must be settled on the left side of the eigenvalue of the given system $(A_e)$. The fictitious model of the behaviours of the mass unbalance is able to be designed using oscillator model [13]. The observer gain matrices $L_x$, $L_{v1}$ and $L_{v2}$can be calculated by pole assignment or by the Riccati equation [17][18] as follows:

$$A + P + P\, A^T - P\, C^T R_m^{-1} C\, P + Q = 0$$

$$\begin{bmatrix} L_x \\ \cdots \\ L_{v1} \\ \cdots \\ L_{v2} \end{bmatrix} = P\, C^T R_m^{-1}. \tag{27}$$

The weighting matrix Q and $R_m$ have to be suitably chosen by the trial and errors.

## III. Design of an estimator for the mass unbalance

In the above section it has been studied how to design the elementary Estimator for the detection at a given local position. It means that a certain place on the shaft is initially given as the position of a mass unballance. In the real running operation there is not any information about the position of the mass unbalance, so the elementary Estimator has to survey not only the assigned local position but also any other place on the shaft. It also give the signals whether a mass unballance exists or not. As it has been known, it is possible to detect the mass unbalance assigned certain place on the shaft. In the case a mass unbalance appears at any subsystem in running time, it must be detected as well. But in many cases it has been shown that it is impossible or very difficult to estimate the position of the mass unbalance at all subsystems on the shaft with one Estimator. Generally, it depends on the number of the subsystem and the number of Estimator. For the estimation of a position of mass unbalance, a Estimator bank based on an elementary Estimator is designed. The main idea is to reconstruct the related forces of a mass unbalance from certain local position to the arranged elementary Estimator. This is the main task in this section.

The structure of the Estimator considered is presented in the work [5][9]. It consists of a few elementary Estimators. The number of elementary Estimator depends on the number of the subsystems modelled. Every elementary Estimator which is distinguished from the distribution vector $Ls_{(i_e)}$ gets the same input(excitation) u(t) and the feedback of the measurements, and is going to be set up at a suitable place on the given system. For the appropriate arrangement of the elementary Estimator, the distribution matrix on the analogy of (9) has been applied. In this way the Estimator bank is established with the Estimator. To estimate the local place of the mass unbalance, there are two steps. First of all, the Estimator must be observable in the meaning of the asymptotical stability in the system. The requirement has been satisfied by the criteria from Hautus [14][16] (). This means that the Estimator has to be capable of estimating the mass unbalance at any location, where Estimator is situated on the given system.

To guarantee this, the requirement () is supposed to be fulfilled. In this work three Estimators are arranged at the 2nd, 4th subsystems and the 6th like this:

$$Ls(i_e) = \left[ \; 00 \quad ,...., \quad \overbrace{1100}^{i_e th.position} \quad ,...,0 \; \right]^T_{(i_e=2,4,6)}. \tag{28}$$

The unknown position of a mass unbalance is found by the Estimator according to the related forces of the mass unbalance resulting from the mass unbalance in some other location on the shaft. The Estimator bank can be presented as follow:

$$\begin{aligned} \begin{bmatrix} \hat{x}_i(t) \\ \hat{v}_{1_i}(t) \\ \hat{v}_{2_i}(t) \end{bmatrix} &= \underbrace{\begin{bmatrix} A - L_{x_i} C & N_R H_1 & N_u H_2 \\ -L_{v1_i} C & V_1 & 0 \\ -L_{v2_i} C & 0 & V_2 \end{bmatrix}}_{A_o} \\ \begin{bmatrix} \hat{x}_i(t) \\ \hat{v}_{1_i}(t) \\ \hat{v}_{2_i}(t) \end{bmatrix} &+ \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix} \tilde{u}(t) + \begin{bmatrix} L_{x_i} \\ L_{v1_i} \\ L_{v2_i} \end{bmatrix} y(t) \\ i_e &= 2,4,6 \end{aligned} \tag{29}$$

$$N_R = \begin{bmatrix} 0 \\ ----- \\ M^{-1}Ls(i_e) \end{bmatrix} \tag{30}$$

$$N_u = \begin{bmatrix} 0 \\ ----- \\ M^{-1}Lu(i_e) \end{bmatrix}. \tag{31}$$

Each elementary observer for the subsystems $i_e = 2,4,6$ yields estimates

$$j : max_{i_e}\{\|\hat{n}_{c,i_e}(t)\| = \|H_1\hat{v}_{1,i}(t)\|\} \quad i_e = 1,...,N+1 \tag{32}$$

$$l : max_{i_e}\{\|\hat{n}_{u,i_e}(t)\| = \|H_2\hat{v}_{2,i_e}(t)\|\} \quad i_e = 1,...,N+1, \tag{33}$$

Looking for the maximum values

$$\hat{n}_{max} = max_{i_e,t}\{\hat{n}_{u,i_e}(t)\}. \tag{34}$$

The corresponding index $i_e$ approximately defines the subsystem where the crack occured.

## IV. Examples

The Estimator bank consists of three elementary Estimator. The first Estimator A is situated at the 2nd subsystem, the 2nd Estimator B is at the 6th subsystem and the 3rd Estimator C is placed at the 4th subsystem. The criteria to detect a mass unbalance are the magnitude of the forces. In order to localize a mass unbalance position, it is necessary to choose the maximal magnitude of the force from all Estimator by the comparison among the forces turn out. In the case, the Estimator shows none of the force, there is not any mass unbalance in this system considered. If any one of the Estimator gives the signal of a force the system has a mass unbalance in a corresponding position. As the 1st example, the given mass unbalance is at the 1st and 4th of the node in the system considered.

Fig. 3. Estimator A: Mass unbalance in the 1st and 4th Subsystem, Y coordinate: force in N , X coordinate: time in sec.

The Fig. from 3 shows that the Estimator detects a mass unbalance at the 1st node. The first courve with '+' presents the simulated force due to the equation of the harmonic mass unbalance (see appendix). The force of a mass unbalance depends on the mass of the excentricity $m_{ex}$, excentricity $ex$, velocity of the angle $\Omega$, angle of the phase $\beta$ and time $t$. The 2nd courve with '-' is the estimated force come from the Estimator bank. The Fig. a and Fig. b from 3 tell the force of mass unbalance with rpm(=150) and rpm(107.5), respectively. By the comparison of the forces, there are some differences between estimated force and simulated force. For example, the magnitude of the force and shape of vabration. The reason for the difference can not be defined. The main task of the work is to detect the force of the mass unbalance and to estimate the position of the mass unbalance in terms of forces. The establishment of the relationship between the magnitude and caused bearing damage is not considered in this papaer. However, the Estimator bank can estimate the force of the mass unnbalance on the shaft.

Fig. 4. Estimator A: Mass unbalance in the 1st and 4th Subsystem, Y coordinate: force in N , X coordinate: time in sec.

The results in the Fig. 4 describe the forces of mass unbalance in the 1st node under the influence of a crack. The crack has been appeared at 3 [s] later after runtime operation. Up to 3[sec], the forces of the mass unbalance and crack have been overlapped. This denotes that the mass unbalance and crack exist in the same place(node) on the shaft. It has been already mentioned that the breathing derection of a crack and the position of the mass unbalance on the radius of the diameter of shaft are on the same line. By the comparison of the forces in a and b in Fig. 4, the elementary Estimator A sees the lagest magnitude of force. It means that the mass unbalance exists closer or near to the 1st of the node than to the 4th of the node.

Fig. 5. Estimator A and B: Mass unbalance in the 6th and 2nd Subsystem, respectivele, Y coordinate: force in N , X coordinate: time in sec.

The Fig. 5 (a) shows the estimated forces by the Estimator A according to the mass unbalance on the 6th subsystem The elementary Estimator A is hardly to know the existence of a mass unbalance and it is very difficult to differentiate the force of the mass unbalance and the effects of any other noise by measurement or system. The Fig.5 (b) illustrates also the force from the Estimator B concerning mass unbalance on the 6th subsystem The magnitude of the force by the Estimator A is almost the same as by the Estimator B. This says that the Estimator A almost can not estimate the mass unbalance at the 6th node and Estimator B can not detects the mass unbalance at the 2nd node.

Fig. 6. Estimator C: Mass unbalance in the 4th and 6th Subsystem, Y coordinate: force in N , X coordinate: time in sec.

The Estimator C shows the existence of a mass unbalance at the 4th and 1st node. But by the comparison of the magnitude of forces between Fig. 6 (a) and Fig. 6(b) the force by the Estimator C is greater than by 1st node C. This telles that the mass unbalance situates in the 4th node.

Fig. 7. Estimator B: Mass unbalance in the 4th Subsystem, Y coordinate: force in N , X coordinate: time in sec.

The results in the Fig. from 7 denote the forces of mass unbalance in the 4th node. The Fig. (a) illustrates the lageer magnitude of the force anyother node. It tells that the mass unbalance is placed at the 6th node. Like the examples, the Estimator gives the information where a mass unbalance exists.

In this way the Estimator estimates the existence of a mass unbalance under the crack influence of crack and none crack on the shaft, and localize its position according to the magnitude of force of mass unbalance. These forces related from certain position of a mass unbalance under crack to Estimator A, Estimator B and Estimator C are supposed to be interpreted as mechanical forces due to the harmonic balance from oscillator model [15]. The numerical value of the $\rho_q$ concerned with the weighting matrix Q is in the appendex. The factor $\rho_r$ of the weighting matrix $R_m$ is of 0.45 and $diagR_{(i,j)}$ is of 1. The matrices Q and $R_m$ have been chosen by the trial and errors. It has been noticed that the Estimator estimates the signals very well. If only the mass unbalance is situated at the position where the Estimator are located. Otherwise the position of the estimator plays a part in the values of the forces regarding to the excited inputs as well. However, the forces od the mass unbalance are a clear indicator for the existence of a mass unbalance in the shaft. The other figures which have been left out because of quantity of this paper, show that Estimator B which is arranged at the right bearing, is not able to estimate the crack in the 1st of the node in the system. The given magnitude of the mass unbalace $e_m$ is in the appendix.

## V. Summary and conclusions

Using FEM the mathematical model of the rotating shaft including a crack has been presented. Based on the mathematical model, the elementary Estimator and an Estimator bank have been developed. With this Estimator the task of the detection of a mass unbalance and localization have been done. The above methods give a clear relation between the shaft with the mass unbalance and the caused phenomena in vibration by means of the measurement at both bearings. Theoretical results have been given. The forces in the results are the internal forces, which have been reconstructed as disturbance forces created by the mass unbalance.

From the given Example, it has been theoretically shown, that the mass unbalances on the shaft can be detected. The Estimator is able to estimate the location of a mass unbalance. The method considered can be applicated in the similar area of the problem with the linear and nonlinear dynamic effect from a mass unbalance by the suitable design of an Estimator. Anyway, the suggested methods are very significant not only for the further theoretical research and development but also for the transfer in experiments. Furthermore, one of the important thing to be mentioned is the noise problem. Actually, because noise gives the distortion of the result very often. The another thing is the physical modelling of the given system. So, the noise effects and the modelling of system have to be researched in the future.

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## Appendix

Using the abbreviation $ii = i - j_k + 1, jj = j - j_k + 1$, the sum of the matrices, with accordance to equations (2) and (3), can be described as follows.

$$M_{(g)\ (j_k,j_k)}(i_e) = \sum_{i_e=1}^{N} \left[ \sum_{j_k=1}^{(i_e-1)\ \frac{n}{2}+1} \left( \sum_{i,j=j_k}^{j_k+n-1} M_e(ii,jj) \right) \right]_{(i_e)}$$

$$+M^{0}_{(dime\times dime)} \tag{A 1}$$

$$K_{(g)\ (j_k,j_k)}(i_e) = \sum_{i_e=1}^{N}\left[\sum_{j_k=1}^{(i_e-1)\ \frac{n}{2}+1}\left(\sum_{i,j=j_k}^{j_k+n-1} K_e(ii,jj)\right)\right]_{(i_e)}$$

$$+K^{0}_{(dime\times dime)} \tag{A 2}$$

$$G_{(g)\ (j_k,j_k)}(i_e) = \sum_{i_e=1}^{N}\left[\sum_{j_k=1}^{(i_e-1)\ \frac{n}{2}+1}\left(\sum_{i,j=j_k}^{j_k+n-1} G_e(ii,jj)\right)\right]_{(i_e)}$$

$$+G^{0}_{(dime\times dime)} \tag{A 3}$$

$$D_{(g)\ (j_k,j_k)}(i_e) = \sum_{i_e=1}^{N}\left[\sum_{j_k=1}^{(i_e-1)\ \frac{n}{2}+1}\left(\sum_{i,j=j_k}^{j_k+n-1} D_e(ii,jj)\right)\right]_{(i_e)}$$

$$+D^{0}_{(dime\times dime)}. \tag{A 4}$$

The matrices used in equation(10) are follows
$A =$

$$\underbrace{\begin{bmatrix} 0 & \vdots & I_{(nn)} \\ \cdots & \cdots & \cdots \\ -(M_g)^{-1}K_e & \vdots & -(M_g)^{-1}(D_{dg}+G_g) \end{bmatrix}}_{(64\times 64)}. \tag{A 5}$$

The index i denotes the number of the subsystem. The vector of the order of the excitation and the matrix of nonlinearites,

$$\tilde{u}(t) = \begin{bmatrix} 0 \\ \ldots \\ M_g^{-1}\ f_e \end{bmatrix}_{(64\times 1)} \tag{A 6}$$

$$N_c(Ls_{(i)}) = \begin{bmatrix} 0 \\ \ldots \\ -M_g^{-1}\ Ls_{(i)} \end{bmatrix}_{(64\times 1)}, \tag{A 7}$$

is of (64 x 1).

Where the vector of the excitation consists of graviation and harmonic unbalance , is presented by

$$\begin{cases} f_e = f_{(g,i_e;i=1,..,N)} + f_{(u,i_e=3,4,5)} \\ f_{(g;2)} = f_{(g;30)} = 0 \\ ,f_{(g;6)} = f_{(g;10)} = f_{(g;14)} = \\ f_{(g;18)} = f_{(g;22)} = f_{(g;26)} = -mg, \end{cases} \tag{A 8}$$

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The order of the $f_g$ is of $(32 \times 1)$ and $f_u$ is of $(32 \times 1)$.

$$\begin{cases} f_{(u;17)} = f_{(u;21)} = f_{(u;25)} = \\ -e_m\ \Omega^2\ m_{(ex)}\ sin(\Omega t+\beta) \\ f_{(u;18)} = f_{(u;22)} = f_{(u;26)} = \\ e_m\ \Omega^2\ m_{(ex)}\ cos(\Omega t+\beta), \end{cases} \tag{A 9}$$

where angle of the phase: $\beta$ =0, length of the subsystem of rotor $el = 2m$, Diameter of the subsystem of rotor makes $ed = 0.25m$. The mass of elemental subsystem: $m = \pi\ el\ \rho\frac{ED^2}{4}$, The density is of $\rho = 7860\frac{kg}{m^3}$, excentricity: $e_m$= 0.0001, mass of the excentricity: $m_{(ex)}$= 3 m respectively. The modulus $E\ i_f$ is of $2.1 * 10^5 N/mm^2$. The stiffness of bearing: $K_{beaing} = 15 * 10^5 N/mm^2$. The measurement matrix of order(4 x 64), $C_{(i=1,\ldots 4, j=1,\ldots,64)} = 0$ , except $C_{(1\times 1)} = C_{(2\times 2)} = C_{(29\times 29)} = C_{(30\times 30)} = 1$. The number of the nonlinearities $n_f$ are of 1 and the number of the measurements $m_e$ makes 4. The elementary matrices $K_e, M_e$ and $D_g$ which depend on the geometry, are given in [6, 7, 8]. The weighting matrix $Q_{q2}(i = 1.,,,.66, j = 1.,,,.66)$ and $Q_{q6}(i = 1.,,,.66, j = 1.,,,.66)$ is of:

$$\begin{cases} Q(i,j) = 2.6 \bullet 10^5;\ i=j=1,...,8 \\ Q(i,j) = 2.1 \bullet 10^5;\ i=j=9,...,16 \\ Q(i,j) = 5.7 \bullet 10^5;\ i=j=17,...,24 \\ Q(i,j) = 3.4 \bullet 10^5;\ i=j=25,...,32 \\ Q(i,j) = 15 \bullet 10^5,\ i=j=33,...,45 \\ Q(i,j) = 2.5 \bullet 10^5,\ i=j=46,...,52 \\ Q(i,j) = 1.2 \bullet 10^5;\ i=j=53,...,64 \\ Q(i,j) = 1.5 \bullet 10^7;\ i=j=65,...,66. \end{cases} \tag{A 10}$$